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*Research Article*

# Tool Wear Detection Based on Duffing-Holmes Oscillator

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The cutting sound in the audible range includes plenty of tool wear information. The sound is sampled by the acoustic emission (AE) sensor as a short-time sequence, then worn wear can be detected by the Duffing-Holmes oscillator. A novel engineering method is proposed for determining the chaotic threshold of the Duffing-Holmes oscillator. First, a rough threshold value is calculated by local Lyapunov exponents with a step size 0.1. Second, the exact threshold value is calculated by the Duffing-Holmes system in terms of the law of the golden section. The advantage of the method is low computation cost. The feasibility for tool condition detection is demonstrated by the 27 kinds of cutting conditions with sharp tool and worn tool in turning experiments. The 54 group data sampled as noisy are embedded into the Duffing-Holmes oscillator, respectively. Finally, one chaotic threshold is determined conveniently which can distinguish between worn tool or sharp tool.



## 1. Introduction

Tool wear is a complex phenomenon occurring in metal cutting processes. A worn tool adversely affects the surface finish of the work piece and therefore there is a need to detect tool wear which alerts the operator to the tool wear state, thus, avoiding undesirable product quality. However, accurately determining cutting conditions remains difficult.

Acoustic emission based on tool condition monitoring has been available for approximately 17 years, most of them use analog root mean square of the signal to monitor tool wear or detect breakages. Damodarasamy and Raman [1] combined the radial force, feed force, and AE to model the tool flank wear in a turning operation. Wanqing et al. [2] used a wavelet transform and fractal algorithm to capture the features of the AE signals. Yao et al. [3] used a fuzzy neural network to describe the relation between the monitoring features, which are derived from wavelet-based AE signals, and the tool wear condition. The data processing

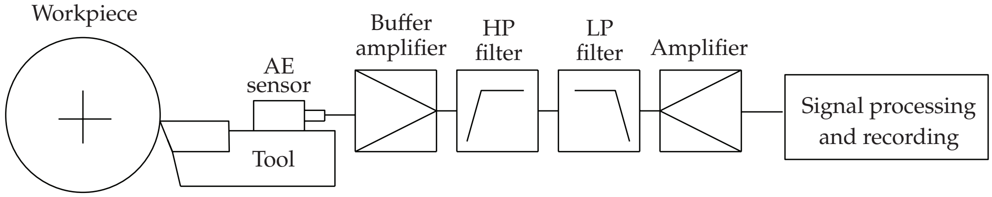



**Figure 1:** AE measurement in metal cutting.

methods have shown acoustic emission signal power to increase with tool wear owing to increased friction effect [4].

Nearly years, chaotic oscillator is used widely to detect weak period signal [5–8]. The weak signal detection is a central problem in the general field of signal processing and the use of chaos theory in weak signal detection is also a topic of interest in chaos control. At present, however, this research is mainly theory and simulation, engineering practice is a few examples. The phase transforms of Duffing-Holmes oscillator are sensitive to periodic signal and periodic interference signals which have larger angular frequency difference from the referential signal, but immune to the random noisy [5, 9]. Since tool wear is a gradual processing during the turning conditions, the cutting sound is composed of periodic signals and a large amount of periodic interference signals and the random noise. Of course, the frequency and amplitude of these signals also are changing gradually along with tool wear except of the random noise. Therefore, the tool wear processing belongs to detect weak periodic signals in strong noisy and very appropriately by Duffing-Holmes oscillator.

Machining tests were carried out on HL-32 NC turning center. This lathe does not have a tailstock. Tungsten carbide finishing tool was used to turn free machining mild steel. The work material was chosen for ease of machining, allowing for generation of surfaces of varying quality without the use of cutting fluids. The experiment equipments are shown in Figure 1. The piezoelectric AE sensor (CAE-150) was mounted on the tool holder. A light coating of petroleum jelly was applied under the sensor to ensure good acoustic emission coupling. Because of high impedance of the sensor, it must be directly connected to a buffer amplifier. Low-frequency noise components, which are inevitably present in AE signal, cannot represent the tool's condition and hence useless. Therefore, those components should be eliminated (highpass filtered) at the earliest possible stage of signal processing to enable usage of full amplitude range of the equipment. The filtered signals were sampled at 4 MHz using a digital storage oscillograph to a PC, see Figure 1. All test data were processed and analyzed by using the Matlab software.

In the experiment, according to the cutting conditions which are presented in Table 1, a sharp tool and a worn tool was used, respectively.

The data sampled by AE, 54 group data, are merged into Duffing-Holmes equation as an exterior perturbation of the chaotic system, respectively. Then, with tool wear, the gradual change sound signal under the background of strong noise can be detected by identifying the phase space trajectory. In terms of the results from theoretical calculation, it is proved that there is a huge difference in the phase space trajectories between the chaotic state and the periodic state, and this difference can be used as the evidence in the chaotic system for the detection of tool wear signal based on Duffing oscillator. Meanwhile, Lyapunov exponents are adopted as threshold value evaluated roughly for chaotic critical state, the law of golden section to determine the threshold is proposed and the threshold in chaotic critical state is

**Table 1:** Experimental cutting conditions.

| No. | Speed $r/\text{min}$ | Depth of cut mm | Feed $\text{mm}/r$ | No. | Speed $r/\text{min}$ | Depth of cut mm | Feed $\text{mm}/r$ |
|---|---|---|---|---|---|---|---|
| 1 | 1500 | 1 | 0.1 | 15 | 1000 | 0.2 | 0.05 |
| 2 | 1500 | 0.5 | 0.1 | 16 | 800 | 1 | 0.05 |
| 3 | 1500 | 0.2 | 0.1 | 17 | 800 | 0.5 | 0.05 |
| 4 | 1000 | 1 | 0.1 | 18 | 800 | 0.2 | 0.05 |
| 5 | 1000 | 0.5 | 0.1 | 19 | 1500 | 1 | 0.02 |
| 6 | 1000 | 0.2 | 0.1 | 20 | 1500 | 0.5 | 0.02 |
| 7 | 800 | 1 | 0.1 | 21 | 1500 | 0.2 | 0.02 |
| 8 | 800 | 0.5 | 0.1 | 22 | 1000 | 1 | 0.02 |
| 9 | 800 | 0.2 | 0.1 | 23 | 1000 | 0.5 | 0.02 |
| 10 | 1500 | 1 | 0.05 | 24 | 1000 | 0.2 | 0.02 |
| 11 | 1500 | 0.5 | 0.05 | 25 | 800 | 1 | 0.02 |
| 12 | 1500 | 0.2 | 0.05 | 26 | 800 | 0.5 | 0.02 |
| 13 | 1000 | 1 | 0.05 | 27 | 800 | 0.2 | 0.02 |
| 14 | 1000 | 0.5 | 0.05 | | | | |

evaluated more accurately. Melniko's function also can be used to calculate the threshold for chaos, but Melniko's function only determines the threshold from order to chaos, but Lyapunov exponents can determine the threshold from chaos to order [7, 10, 11]. We describe a means for tool wear whether or not a system is chaotic. When the tool is sharp, the Duffing-Holmes oscillator is chaos in state space trajectory, when the tool is wear, the Duffing-Holmes oscillator takes on periodic trajectory from chaos to order in state space.

## 2. Principle detecting weak signal based on Duffing-Holmes oscillator

The Duffing-Holmes is the second differential equation containing the item of the power five, which can be motivated by exterior stimulations to engender oscillation movement and then generate chaotic trajectory or periodic trajectory; its dynamic equation is as follows:

$$x''(t) + 0.5x'(t) - x^3(t) + x^5(t) = r\sin(t) + (\text{input}), \tag{2.1}$$

where 0.5 denotes the ratio of damping, $r\sin(t)$ is the forced periodic terms, which is the reference signal and as an internal signal, $-x^3(t) + x^5(t)$ term is the nonlinear recovery force in system 1, the kinematical state of the system mainly depends on this recovery force term $r\sin(t)$. *Input* terms are the signal measured which is imported to the dynamic system as the supplement of special parameters of chaotic oscillator; we can adjust the amplitude $r$ of the reference signal to the special value as in the chaotic critical state. The value is called threshold value in the chaotic system 1. If a weak periodic signal is merged into system 1, so long as the threshold is adjusted appropriately, the behavior of the Duffing-Holmes will be changed dramatically from chaotic states to periodic states. For example, let *input* terms be $f(t) = 0.2\ \sin(t)$, then the Duffing-Holmes equation is

$$x''(t) + 0.5x'(t) - x^3(t) + x^5(t) = r\sin(t) + f(t), \tag{2.2}$$

when $f(t)$ contains a weak white noisy, that is, $f(t) = 0.2\sin(t) + 0.01$ rand, rand is a random white noisy $(0 \sim 1)$, *input* terms $f(t)$ are a low-amplitude periodicsignal with white noisy, then the Duffing-Holmes equation is

$$x''(t) + 0.5x'(t) - x^3(t) + x^5(t) = r\sin(t) + f(t), \tag{2.3}$$

when one weak periodic noisy signal is merged into *input* terms $f(t)$, that is,

$$\begin{aligned} \hat{f}_i &= f_i, \quad i \neq 16 + 128k, \\ \hat{f}_{16+128k} &= f_{15+128k} + 0.04, \quad k = 0, 1, 2, 3, \ldots, \end{aligned} \tag{2.4}$$

then

$$x''(t) + 0.5x'(t) - x^3(t) + x^5(t) = r\sin(t) + f(t). \tag{2.5}$$

Let dynamical system 2, 3, and 4 initial point $x'(0) = 0$, $x(0) = 0$, then set threshold $r_d = 0.52544$ as the critical state for the system 2, integrated with Runge-Kutta method of fourth order with a fixed step size $t = 0.01$ second. Total time is 16 seconds. The phase space in systems 2 and 3 takes on periodic state trajectory, but phase space in system 4 is chaotic trajectory.

When a strong noise $[0.2\sin(t)]$ without white noisy or with white noisy is added to the Duffing-Holmes system, both systems 2 and 3 take on the periodic state. It means the random noisy is not influenced on the state of the dynamic system. Once the strong noisy contains a weak periodic noise signal, the behaviors of system 4 is changed immediately from a large-scale periodic state to a chaotic state. The temporal waveform of $f(t)$, the phase orbit, and the temporal waveforms of systems 3 and 4 are shown in Figures 2 and 3. In other words, the Duffing-Holmes takes on some immunity to random noisy [12] and strong sensitivity to some weak periodic signal. Since 0.01 rand term is too small, it is not obvious in the temporal waveform.

## 3. Threshold calculated based on Lyapunov exponents

Lyapunov exponents are frequently computed measure for the characteristic of chaotic dynamics [10, 11, 13]. It describes a method for diagnosing whether or not a system is chaotic. To confirm the existence of the weak periodic signal to be detected and the amplitude of the signal, we need to define a proper index for denoting the change in the states of the chaos detection system. The index should be sensitive to a weak periodic signal, but insensitive to the random noise from the viewpoint of statistical characteristics. Thus, the dynamic properties of a certain system are reflected statistically by Lyapunov exponents which are described as follows [14–16].

Dynamic system $x''(t) + 0.5x'(t) - x^3(t) + x^5(t) = r\cos(t)$ is transformed below:

$$\begin{aligned} y(t) &= x'(t), \\ y'(t) &= -0.5\,y(t) + x^3(t) - x^5(t) + r\cos(t). \end{aligned} \tag{3.1}$$

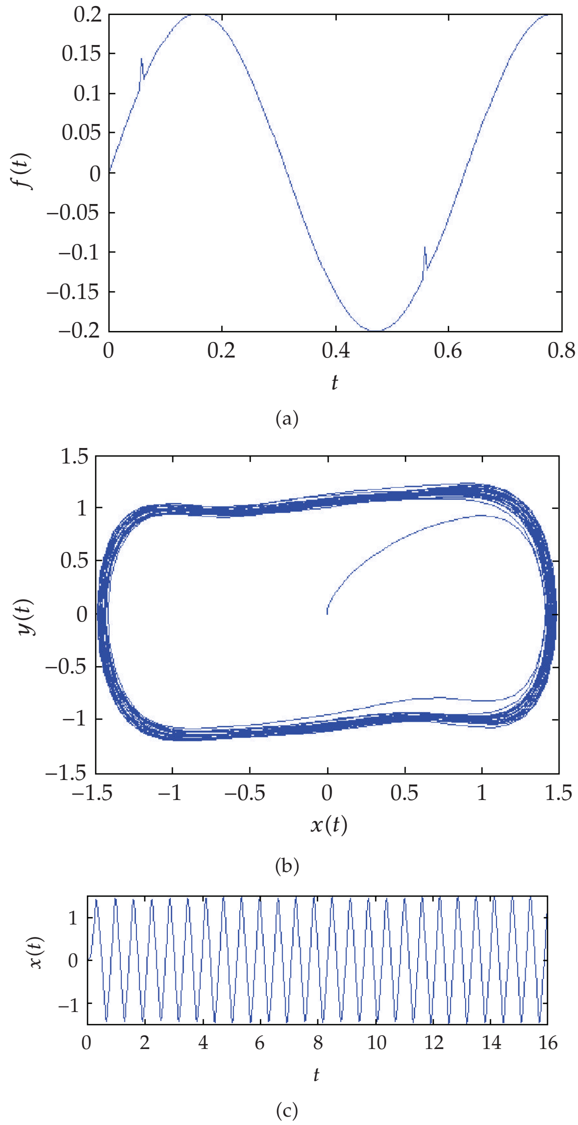



**Figure 2:** Dynamic character with weak periodic noisy signal, $y(t) = x'(t)$.

To a two-dimensional plane $x(t),\ y(t) = x'(t)$, two Lyapunov exponents can be solved in system 5. When the system is in the large-scale periodic state, both of the two Lyapunov exponents are negative. When the system is in the chaotic state, at least one of the two Lyapunov exponents of the system is positive which has behaviors of the chaos. Therefore, the detection system is established on the basis of Lyapunov exponents.

Let initial condition $x(0) = 1$, $x'(0) = 1$, with about typical 30 points in the region $r = [0.5, 1]$ chosen to calculate the Lyapunov exponents (LE), the computation precision of $r$ is two digits after the decimal dot, see Table 2. LE curve are plotted in Figure 4. $r = 0.70$, system 5 takes on the chaotic state, and $r = 0.78$, system 5 takes on the periodic state. They are shown in Figures 5 and 6.

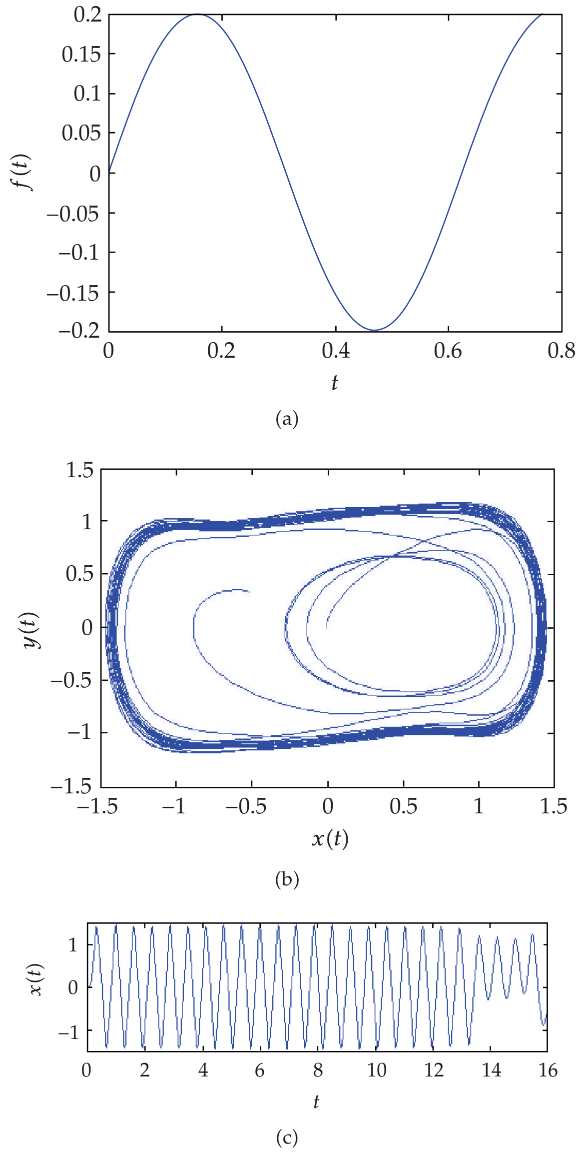



**Figure 3:** Dynamic character without weak periodic noisy signal, $y(t) = x'(t)$.

Obviously, LE changed from positive to negative correspond to region $r = [0.733, 0.734]$ based on the chaotic system extreme sensitivity to parameters changed. If the threshold $r$ is equal to 0.733, because computation precision of $r$ is only three effective digits after decimal dot, the sensitivity from chaos to periodic is not enough. Above, computation cost spends about 3 hours for typical 30 point sets of $r$ with Matlab. In order to improve sensitivity of system 5, however, if the computation precision of $r$ is risen 4 digits after decimal dot, namely, $r = [0.5200, 0.9800]$, time interval 0.01 second and 1000 steps, the computation cost will spend about 30 hours with Matlab. The more high sensitivity is, the more long computation time is.

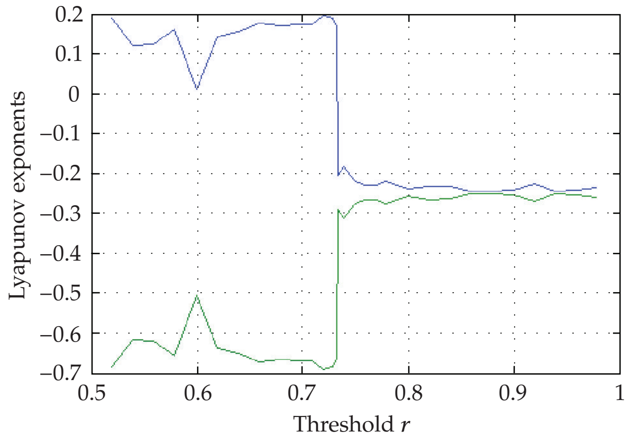



**Figure 4:** The relational curve of LE and $r$.

**Table 2:** Lyapunov exponents in Duffing-Holmes.

| No. | $r$ | Max LE | Min LE | No. | $r$ | Max LE | Min LE |
|---|---|---|---|---|---|---|---|
| 1 | 0.52 | 0.185 | −0.685 | 16 | 0.74 | −0.18527 | −0.31473 |
| 2 | 0.54 | 0.11914 | −0.61914 | 17 | 0.75 | −0.22179 | −0.27821 |
| 3 | 0.56 | 0.12314 | −0.62314 | 18 | 0.76 | −0.23059 | −0.26941 |
| 4 | 0.58 | 0.15765 | −0.65765 | 19 | 0.77 | −0.23088 | −0.26912 |
| 5 | 0.6 | 0.00782 | −0.50782 | 20 | 0.78 | −0.22217 | −0.27783 |
| 6 | 0.62 | 0.13867 | −0.63867 | 21 | 0.8 | −0.24112 | −0.25888 |
| 7 | 0.64 | 0.15349 | −0.65349 | 22 | 0.82 | −0.23407 | −0.26893 |
| 8 | 0.66 | 0.17321 | −0.67321 | 23 | 0.84 | −0.23449 | −0.26551 |
| 9 | 0.68 | 0.16921 | −0.66921 | 24 | 0.86 | −0.24647 | −0.25353 |
| 10 | 0.7 | 0.17056 | −0.67056 | 25 | 0.88 | −0.24773 | −0.25227 |
| 11 | 0.71 | 0.17226 | −0.67226 | 26 | 0.9 | −0.24394 | −0.25606 |
| 12 | 0.72 | 0.19317 | −0.69317 | 27 | 0.92 | −0.22882 | −0.27118 |
| 13 | 0.73 | 0.185 | −0.685 | 28 | 0.94 | −0.24693 | −0.25307 |
| 14 | 0.733 | 0.164 | −0.664 | 29 | 0.96 | −0.24271 | −0.25729 |
| 15 | 0.734 | −0.20655 | −0.29345 | 30 | 0.98 | −0.23848 | −0.26152 |

## 4. Threshold computation combined the law of golden section with Lyapunov exponents

First, rough region of the system threshold $r$ is estimated by Lyapunov exponents with computation precision to be one digit after decimal dot, the calculating process only spends about 40 minutes in the region $r = [0, 1]$ with step size 0.1. Whatever any kinds of weak external signal merged, the region of $r = [0.7, 0.8]$ is always sensitivity region changed from chaotic state to large periodic state in system 5. Since the law of the golden section can search optimizing solution quickly [12], the threshold value is determined by the golden section accurately in the region $r = [0.7, 0.8]$. The Duffing-Holmes oscillator is below:

$$x''(t) + 0.5x'(t) - x^3(t) + x^5(t) = r\cos(t). \tag{4.1}$$

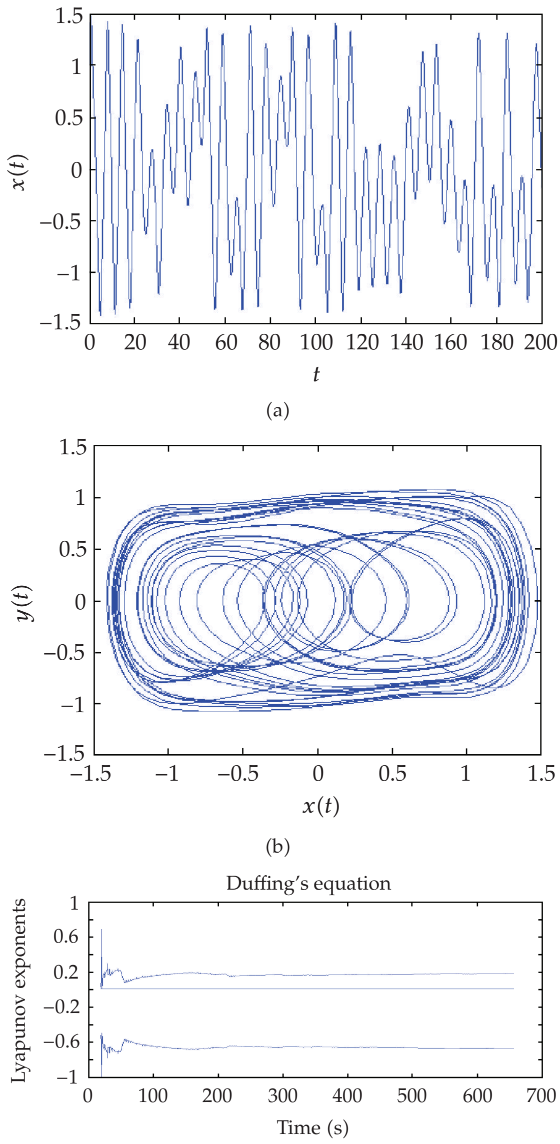



**Figure 5:** $r = 0.70$, system character and LE.

Let initial condition $x(0) = 1,\ x'(0) = 1$, the computation precision of threshold value is six digits after the decimal dot in system 6. The method is as follows:

(1) because 0.7 corresponds to chaotic state and 0.8 corresponding periodic state, $r = 0.75$ is the middle value between 0.7 and 0.8.

(2) because $r = 0.75$ corresponds to periodic states, the region of $r$ is $[0.7, 0.75]$. Then, $r$ is accumulated from 0.7 to 0.75 with the step 0.01 up to 0.71 which corresponds to chaotic state and 0.72 which corresponds to periodic state. 0.715 is the middle value between 0.71 and 0.72.

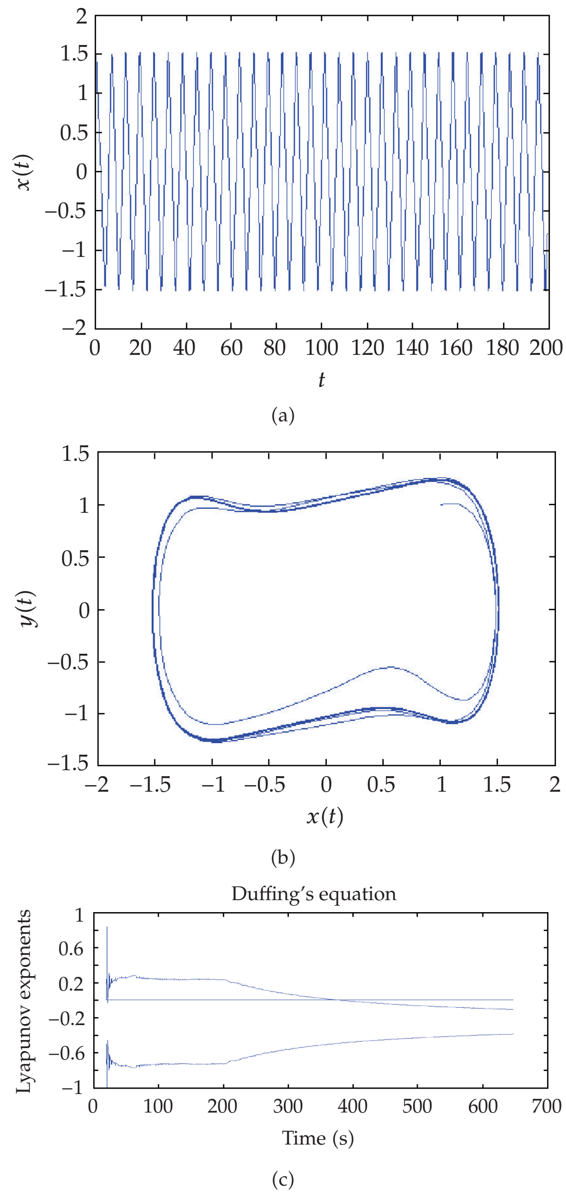



**Figure 6:** $r = 0.78$, system character and LE.

(3) because $r = 0.715$ corresponds to chaotic state, the region of $r$ is taken $[0.715, 0.72]$. Then, $r$ is accumulated from 0.715 to 0.72 with the step 0.001 up to $r = 0.717$ which corresponds to chaotic state and 0.718 which corresponds to periodic state, 0.7175 is the middle value between 0.717 and 0.718.

(4) because $r = 0.7175$ corresponds to periodic state, the region of $r$ is $[0.717, 0.7175]$. Then, $r$ is accumulated from 0.717 to 0.7175 with the step 0.0001 up to 0.7173 which corresponds to chaotic state and 0.7174 which corresponds to periodic state. 0.71735 is the middle value between 0.7173 and 0.7174.

**Table 3:** Threshold $r$ based on the golden section in the region $r = [0.7, 0.8]$.

| Chaos, $r_1$ | Phase plane | Periodic, $r_2$ | Phase plane | Golden section, $r$ | Phase plane |
|---|---|---|---|---|---|
| 0.7 |  | 0.8 |  | $\frac{r_1 + r_2}{2}$ | |
| 0.71 |  | 0.72 |  | 0.75 |  |
| 0.717 |  | 0.718 |  | 0.715 |  |
| 0.7173 |  | 0.7174 |  | 0.7175 |  |
| 0.71732 |  | 0.71733 |  | 0.71735 |  |
| 0.717329 |  | 0.717330 |  | 0.717325 |  |

(5) because $r = 0.71735$ corresponds to periodic state, the region of $r$ is $[0.7173, 0.71735]$. Then, $r$ is accumulated from 0.7173 to 0.71735 with the step 0.00001 up to 0.71732 which corresponds to chaotic state and 0.71733 which corresponds to periodic state. 0.717325 is the middle value between 0.71732 and 0.71733.

(6) because $r = 0.717325$ corresponds to periodic state, the region of $r$ is $[0.717325, 0.71733]$. Then, $r$ is accumulated from 0.717325 to 0.71733 with the step 0.000001 up to 0.717329 which corresponds to chaotic state and 0.717330 which corresponds to periodic state.

(7) final, the threshold value calculated is 0.717329. When a weak periodic signal is merged into system 6, the system takes on the large-scale periodic state. Calculating process is shown in Table 3.

The computation processing only spends about 10 minutes for computation precision to be six digits after the decimal dot. The method has important meaning for engineering practice. 30th steps calculated yield the search optimization threshold value. This is the most amounts of the point sets in the case.

## 5. Experiment work

The sound signal of sharp tool sampled by AE as an initial condition is merged into the Duffing-Holmes system 6 which is in the chaotic critical state, (its phase plane changes from the chaotic state to the large-scale periodic state), the movement state of the system will

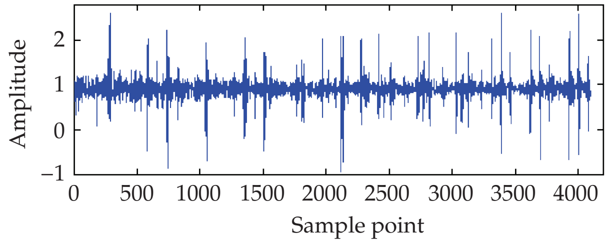



**Figure 7:** Waveform of sharp tool in first condition.

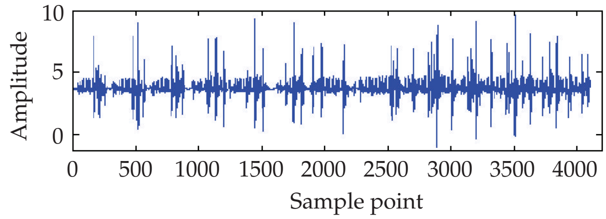



**Figure 8:** Waveform of wear tool in first condition.

transit immediately from the chaotic state to the large-scale periodic state. The simulation of systems 3 an 4 above has only one input signal, however, for this practice engineering, since the sharp tool and wear tool have 27 groups data, respectively, see systems 7 and 8, the threshold in the both the systems must satisfy to distinguish sharp tool and wear tool in 54 group data. The dynamic system 6 is transformed to systems 7 and 8. When the data of sharp tool are embedded to the chaotic system 7, the phase space is chaotic state; however, when the data of wear tool are embedded to the chaotic system 8, phase space change is the large-scale periodic state. The method based on the change of the dynamic behaviors of a chaotic system (chaotic state, periodic state) has been proposed for recognizing, where there exists a signal to be detected in a system, and greatly immune to the random noise of arbitrary zero average value with unknown probability distribution. The threshold value $r$ should firstly be determined in system 7, which is the critical problem of wear signal chaotic detection. The algorithm to determine the threshold value, using Lyapunov exponents method based on the golden section is detailed as follows:

$$x''(t) + 0.5x'(t) - x^3(t) + x^5(t) = r\cos(t) + \text{sharp\_1},$$
$$x''(t) + 0.5x'(t) - x^3(t) + x^5(t) = r\cos(t) + \text{wear\_1}. \tag{5.1}$$

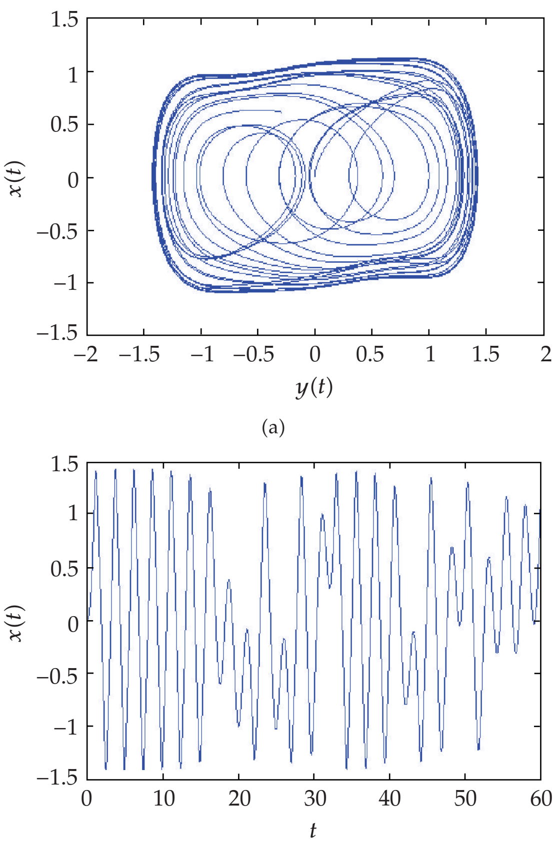
Since the signal amplitude merged is too bigger than interior perturbation force $r$, the signal sampled is decreased 100 times, thus, signals embedded to Duffing-Holmes are weak perturbation noisy, see systems 9 and 10. The interior perturbation force $r$ is still main signal in the dynamic systems 9 and 10. Sharp_1 signal and wear_1 are shown in Figures 7 and 8 in time domain the initial condition is $[0, 0]$ in systems 9 and 10 frequency sampled is 0.001 second. We set up a chaotic oscillator sensitive to weak periodic signals based on the Duffing-Holmes equation (5.2), and poising the system at its critical state

$$x''(t) + 0.5x'(t) - x^3(t) + x^5(t) = r\cos(t) + 0.01\ \text{sharp\_1}, \tag{5.2}$$

$$x''(t) + 0.5x'(t) - x^3(t) + x^5(t) = r\cos(t) + 0.01\ \text{wear\_1}. \tag{5.3}$$

**Table 4:** The critical value for 27 group sharp tool data.

| No. | $r$ | No. | $r$ | No. | $r$ |
|---|---|---|---|---|---|
| 1 | 0.724896 | 10 | 0.724276 | 19 | 0.72371 |
| 2 | 0.724633 | 11 | 0.724203 | 20 | 0.723873 |
| 3 | 0.724716 | 12 | 0.724249 | 21 | 0.723819 |
| 4 | 0.723738 | 13 | 0.723154 | 22 | 0.723945 |
| 5 | 0.724714 | 14 | 0.724004 | 23 | 0.723782 |
| 6 | 0.724467 | 15 | 0.724051 | 24 | 0.723759 |
| 7 | 0.724533 | 16 | 0.723862 | 25 | 0.723975 |
| 8 | 0.724526 | 17 | 0.724141 | 26 | 0.723982 |
| 9 | 0.724561 | 18 | 0.723814 | 27 | 0.72379 |




(a)

(b)

**Figure 9:** Sharp_1 phase plane and time domain.

Here, we meet a problem. When the computation precision of the threshold $r$ is not appropriate, dynamic system 10 is not stable. In other words, perhaps one group data is chaotic and another group data is periodic state in all wear tools. Behavior of the dynamic system is changed with the threshold difference, see Figure 4. In order to decrease computation cost with Matlab, we fix a step size 0.1 in the region $r = [0, 1]$, the system trends

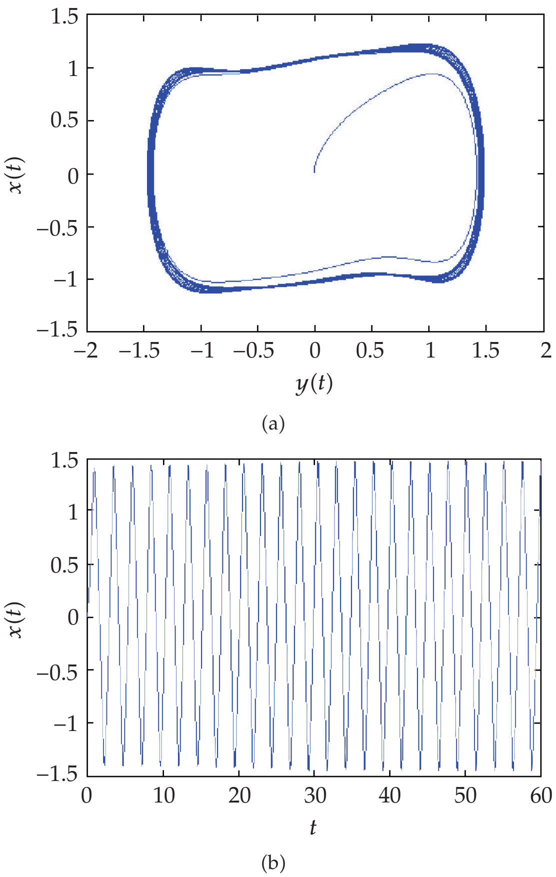



**Figure 10:** Wear_1 phase plane and time domain.

of dynamic behavior can be get roughly, this computing process spends about 40 minutes. In fact, the region $r = [0.7, 0.8]$ is the region from chaotic state to periodic state for Duffing-Holmes oscillator, no matter what any exterior weak periodic signals are merged into the system, the $r = [0.7, 0.8]$ can be used directly as ruler.

If we took the critical value of each group data as the threshold value, we would get 27 difference threshold values. However, we must get one threshold value for all 27 group data. For the reason, the range of the threshold value will be enlarged, that is, the threshold value will be decreased. We take the minimum threshold value in all 27 group sharp tool data. Using the law of the golden section in the region $r = [0.7, 0.8]$ for each group data of sharp tool, their critical value are calculated, see Table 4. Obviously, minimum is 0.723710.

When the amplitude of interior perturbation force $r$ is equal to 0.723710, System 9 is critical state from chaotic to periodic, one of 27 groups data of sharp tool is merged to system 9, the system shows chaotic state, and when one of 27 groups of wear tool is merged to system 10, the system shows larger scale periodic state. For first group data, system 9 and system 10 are showed Figures 9 and 10.

The computation precision is six digits after decimal dot for the threshold determined accurately, it is enough sensitivity for distinguish wear tool or sharp tool. Of course, the

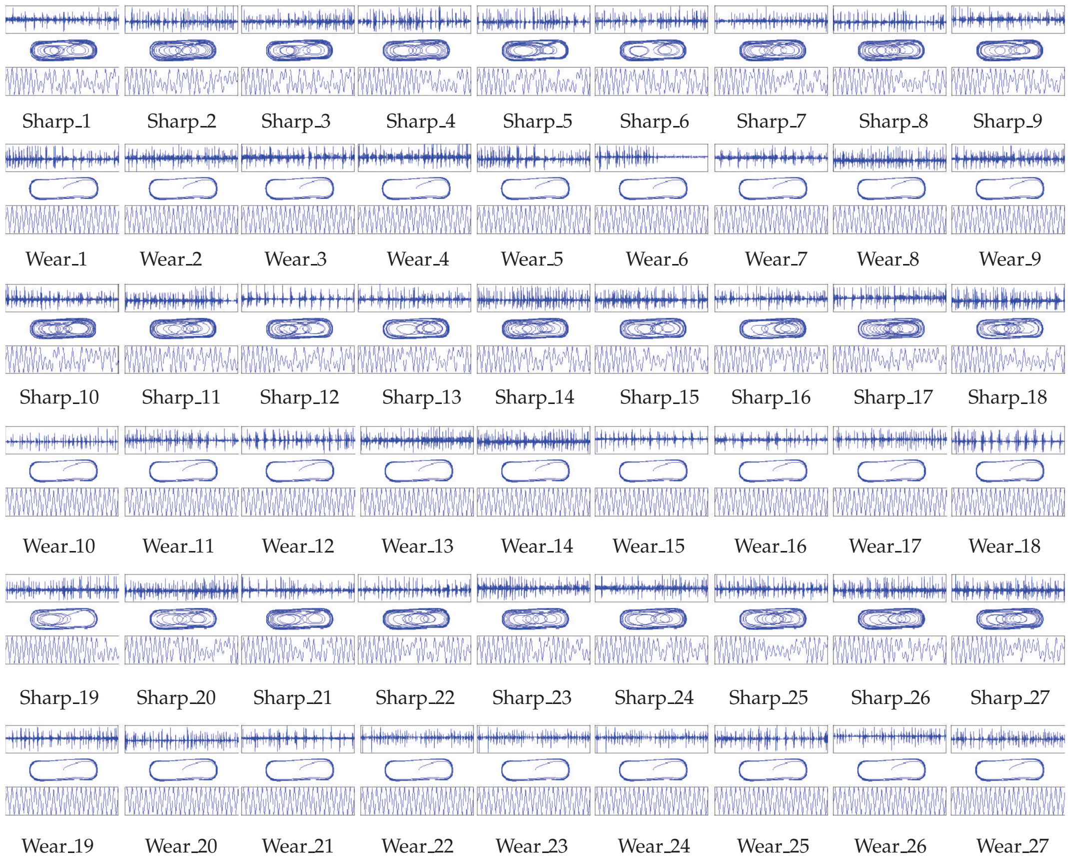



**Figure 11:** Map of time domain to be detected signal, phase plane, and state of time domain on system 9.

difference engineering problem may choose the difference computation precision for the threshold value in chaotic system model.

Finally, 0.723710 is a threshold value detected wear tool. All 54 group data sampled is merged to system 9, respectively, time domain map of each group data sampled, system phase plane, time domain map of system state; they are shown in Figure 11.

## 6. Conclusion

Currently, Duffing-Holmes oscillator is the area of most intense research activity for developing weak signal detected. The method which was described in this paper can be used as a valuable tool for the tool condition monitoring. In comparison to conventional weak signal detected, the advantages of tool wear detected based on Duffing-Holmes oscillator were shown. Compared to the Lyapunov exponents calculated determining the threshold of system chaotic critical state, the law of the golden section spends the less time and useful engineering meaning. The computation precision of the threshold can be calculated conventionally to satisfy the sensitivity of wear tool detected.

For the future development of the presented techniques in laboratory, several 10 approaches are to be tested. For example, relationship between the computation precision of the threshold and sensitivity of the chaotic critical state for difference engineering problem.

Since Runge-Kutta method of fourth order is one kind of approximate solution method for dynamic equation, a difference time-step size will impact the computation precision for the threshold value.

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