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Research Article

On the Positioning Problem of a Microscopic Particle Trapped in Optical Tweezers

Carlos Aguilar-Ibañez and Luis I. Rosas-Soriano

CIC-IPN, Avenida Juan de Dios Bátiz s/n, Esq. M. Othón de Mendizabal, U.P.A.L., Col. San Pedro Zacatenco, A.P. 75476, México DF 07700, Mexico

Correspondence should be addressed to Carlos Aguilar-Ibañez, caguilar@cic.ipn.mx

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We solve the positioning problem of a spherical microparticle trapped by Optical Tweezers, under the assumption that the drag viscous force is presented. To do it, we develop two control strategies for the manipulation of this kind of optical trap. The first control strategy is developed assuming that the damping coefficient is known, while in the second strategy this parameter value is only partially known, which in practice it is more realistic due to the difficulty to estimate it. Both strategies are based on the traditional Lyapunov method in conjunction with the use of a saturation function. The stability analysis of both strategies was carried out by using the standard Lyapunov stability theory. Finally, numerical simulations validate the effectiveness of both control approaches in reducing the random position fluctuations produced by the inherent thermal noise.

1. Introduction

Studying, modeling, and controlling the optical tweezers system (OT) has been the subject of sustained interest during the last years [1, 2] because this system is envisioned as a challenging problem, as well as a problem with several potential and actual applications. For instance, the OT system can be used to manipulate micro particles with the advantage of avoiding the necessity of using invasive techniques (see [3–7], to mention only a few references).

The OT system consists of a focused laser beam, able to trap and manipulate one or more micro particles suspended in a colloid or frictionless medium. Researchers of life sciences have used this system in a number of applications where a micro particle must be transported from one position to another [8–11]. Many other examples of the applications of the OT can be found in the specialized literature. For example, in [12] dual beam optical tweezers have been used to measure the distance dependence of colloidal friction coefficients with results in agreement with low-Reynolds numbers calculations. Another useful and common application presented in [8] is to use the OT to stretch a microtubule on its main axis by fixing two beads to its ends, which is a standard procedure to measure static forces on extended tubes, as well as the dynamic effect that happens when these elongations are carried out at high speeds. Despite the existence of many applications of this device, most of them have been developed from a rather practical standpoint, where performance and theoretical aspects were completely ignored or, at best, only slightly treated. However, in practice, moving a microparticle to a very specific location requires the use of techniques offered by new developments in modern Control Theory.

In this work we propose two control strategies for moving a micro particle from one initial position to another final rest position, by manipulating the laser focus of the OT system, under the assumption that a damping force is present in the medium. The first manipulation strategy is based on the fact that the damping coefficient of the medium is known, whilst, in the second one, only an estimation of this parameter is available, which in practice is more realistic due to the difficulty to estimate this coefficient because it is related to the medium density that its own value is temperature environment depending. To justify both strategies we use the traditional Lyapunov method in conjunction with a saturation function. It is important to mention that our control models are based on the previous works of [2, 13], and it is assumed that the optical trapping force is perfectly modeled by a Gaussian function, which is directly related to the potential energy generated by the Gaussian laser beam. Finally, we mention that the corresponding open loop of the selected model can render a micro particle to the origin, if it is located close enough to the geometric center of the beam or inside the stability domain. However, the open loop control model is useless if more complex tasks have to be carried out or the thermal noise has to be considered, as usually happens in actual applications. For instance, a suitable feedback state allows to reduce the position fluctuations produced by the thermal noise, as will be shown in the numerical simulations.

The rest of this work is organized as follows. In Section 2 we present the physical model of the OT system. In Section 3 we develop two alternative approaches to solve the OT system positioning problem. Also, in this section we present the corresponding numerical simulations. The conclusions are presented in Section 4. Finally, the proof of an important lemma for the obtained results is included in the appendix.

2. Nonlinear Dynamics Model of the OT System

The simplified model of the OT system consists of a spherical microparticle of radius r and mass m, which is immersed in a viscous medium and trapped in a potential optical field. The particle is restricted to be inside the (x, y)-plane (see Figure 1). The particle motion is described by the generalized coordinates x and y, while the control actions are represented by the position coordinates of the laser focus (x_0, y_0) (this vector position is related with the geometrical center of the laser beam with a gaussian intensity profile). The corresponding motion equations for the OT control model, already obtained in [2], are given by

$$\begin{aligned} m\ddot{x} + \frac{2\ln(2)p_0(x - x_0)}{a^2}\exp\left[-\ln(2)\left(\frac{(x - x_0)^2}{a^2} + \frac{(y - y_0)^2}{b^2}\right)\right] &= -\beta\dot{x} + F_x(t), \\ m\ddot{y} + \frac{2\ln(2)p_0(y - y_0)}{b^2}\exp\left[-\ln(2)\left(\frac{(x - x_0)^2}{a^2} + \frac{(y - y_0)^2}{b^2}\right)\right] &= -\beta\dot{y} + F_y(t). \end{aligned} \tag{2.1}$$

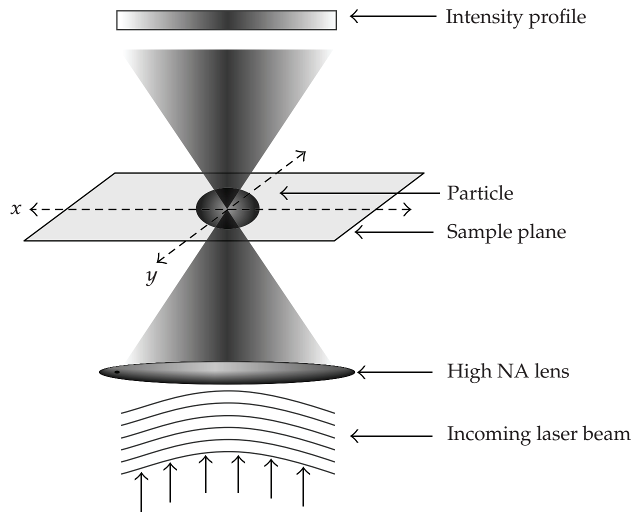


Figure 1: The OT system.

where p_0 is the well depth, a and b are related to the waist of the beam dimensions, $\beta\dot{x}$ and $\beta\dot{y}$ are the drag forces presented in the colloidal medium; with $\beta > 0$, being the damping coefficient, $F_x(t)$ and $F_y(t)$ are the independent external Langevin forces (random thermal), where the random forces will be modeled as white noise signals [14, 15].

Comment 1. It is easy to show by simple linearization that the above system has one stable equilibrium point, defined by $(x_0, y_0, 0, 0)$, which is locally exponentially stable, and a set of unstable equilibrium points given by $(x \to \pm\infty, y \to \pm\infty, 0, 0)$. It means that if the particle is located far enough from the centroid (x_0, y_0), then it cannot be trapped by the Gaussian potential, unless we move the centroid to a neighborhood near the particle. In other words, the above system is locally stable in an open loop.

In order to manage the above model in a simple way, we define the following variables:

$$\begin{aligned} x_1 &= x, \qquad x_2 = \dot{x}, \qquad x_r = x_1 - x_0, \\ y_1 &= y, \qquad y_2 = \dot{y}, \qquad y_r = y_1 - y_0 \end{aligned} \tag{2.2}$$

and the positive constants

$$k_s = \frac{2\ln(2)p_0}{a^2 m}, \qquad \gamma = \frac{\beta}{m}, \qquad \alpha = \frac{1}{m}. \tag{2.3}$$

Thus, system (2.1) can be rewritten as

$$\begin{aligned}
\dot{x}_1 &= x_2,\\
\dot{x}_2 &= -\frac{1}{2}k_s x_r \exp\left[-\left(\frac{x_r^2}{a^2}+\frac{y_r^2}{b^2}\right)\right]-\gamma x_2+\alpha F_x(t),\\
\dot{y}_1 &= y_2,\\
\dot{y}_2 &= -\frac{1}{2}k_s y_r \exp\left[-\left(\frac{x_r^2}{a^2}+\frac{y_r^2}{b^2}\right)\right]-\gamma y_2+\alpha F_y(t),
\end{aligned} \tag{2.4}$$

evidently, x_r and y_r are the new control inputs of system (2.4). From now on, we use q and $\dot{q}$ to indicate the vector position and the vector velocity of the system, that is, $q = [x_1, y_1]$ and $\dot{q} = [x_2, y_2]$.

Remark 2.1. Notice that the dynamic model (2.4) is nonaffine in the control variables x_r and y_r. Consequently, many standard control strategies cannot be directly applied to manipulate this kind of system. Also, if the control variables x_r and y_r are large enough, then the system loses controllability. Physically, if the particle is located far enough from the laser beam focus, it is not possible to trap it. However, this inconvenience can be overcome by using a bonded control or a method based on saturation functions. Fortunately, in the opposite case, that is, when the control variables x_r and y_r are small enough (close to zero), the OT system behaves as a linear system.

We want the reader to notice that system (2.4) is said to be an open loop system if the controllers x_0 and y_0 remain constant. In this case, whenever the particle belongs inside its respective domain of attraction, the open loop system asymptotically attracts it to the origin ($q = 0, \dot{q} = 0$). However, to carry out any control maneuver task, like changing the position of the particle or making the system more robust against external perturbations such as thermal noise, we need to apply some control strategy, because the attraction force exerted by the optical trap is actually a very weak force that decreases exponentially to zero, as long as the particle moves away from the laser focus (x_0, y_0).

Before we go any further, we must provide some important considerations related to our OT controlled model, that help us to establish the scope of this work.

(C.1) The microparticle is restricted to solely move inside the (x, y) plane. The motions along the z-axis are discarded.

(C.2) We consider the case where the gradient force, generated by the Gaussian potential energy (induced by the laser beam), is stronger than the scattering force. That is, the scattering force is not included in the controlled model.

It is important to mention that in almost any work related to the control of the OT system, the model setup is proposed in such a way that the scattering force is neglected inside the region, where the system is controlled. In other words, what is looking for is having a model robust enough, where the gradient force always dominates over the scattering force (for a detailed treatment of this issue, see [16]). In fact, using feedback control helps to obtaining a more robust system because it reduces the nondesired effects of non modeled dynamics, like the scattering and stochastic forces. However, from the view of physics the scattering force should be taken into account, in some cases, as mentioned in [17, 18].

We end this section by establishing the control objective of this work.

Problem Statement

In this work we solve the positioning problem of the OT system. This problem consists of placing a particle trapped by an OT system in a final rest position, given by $q = [x_f, y_f]$ and $\dot{q} = 0$, starting from any initial position.

3. Stabilization of the OT System

In this section we solve the aforementioned control problem. To this end, we propose two simple control strategies, both based on the application of the Lyapunov Control Theory in conjunction with a saturation function. The first control strategy is based on the fact that the damping coefficient β is known with high precision, while in the second one this restriction is omitted. Since the stability analysis of the nonlinear dynamic model (2.4) is generally very difficult to obtain by means of the Lyapunov Method when the thermal noise signals are present, then for simplicity we assume that external perturbations are $F_x(t) = 0$ and $F_y(t) = 0$. To evaluate the performance of the closed loop system in the presence of the thermal forces we carried out some numerical simulations and we devoted a section where we discuses the effect of these forces in our controlled model.

First of all, to avoid large values in the control variable x_r and y_r, we use a saturation function, defined as follows.

Definition 3.1. Let $M > 0$ be a strictly positive real number. We say that $\sigma_M(w) : R \to R$ is a saturation function, if it satisfies

(1) $\sigma_M(w) = 0 \Leftrightarrow w = 0$,

(2) $w\sigma_M(w) \geq 0$ for all $w \in R$,

(3) $\sigma_M(-w) = -\sigma_M(w)$ for all $w \in R$,

(4) $-M \leq \sigma_M(w) \leq M$ for all $w \in R$.

3.1. *First Control Approach*

Here we develop a simple control strategy for changing the position of the particle, assuming that the parameter β is known. To this end, we first introduce a suitable change of variables to write system (2.4) as two first-order differential equations. Then, using the most simple quadratic function we derive the needful controllers that assure the asymptotical convergence at the origin.

Let us introduce the following auxiliary variables:

$$w_x = \gamma(x_1 - x_f) + x_2, \qquad w_y = \gamma(y_1 - y_f) + y_2. \tag{3.1}$$

So, system (2.4) can be easily expressed as

$$\dot{w}_x = -\frac{1}{2}k_s x_r \exp\left[-\left(\frac{x_r^2}{a^2} + \frac{y_r^2}{b^2}\right)\right],$$
$$\dot{w}_x = -\frac{1}{2}k_s y_r \exp\left[-\left(\frac{x_r^2}{a^2} + \frac{y_r^2}{b^2}\right)\right]. \tag{3.2}$$

Notice that if the variables w_x and w_y are brought to zero, we have

$$w_x = 0 = \gamma(x_1 - x_f) + \dot{x}_1, \qquad w_y = 0 = \gamma(y_1 - y_f) + \dot{y}_1. \tag{3.3}$$

Therefore, as $\gamma > 0$, we have

$$\lim_{t\to\infty} x_1 = x_f, \qquad \lim_{t\to\infty} x_2 = 0, \qquad \lim_{t\to\infty} y_1 = y_f, \qquad \lim_{t\to\infty} y_2 = 0. \tag{3.4}$$

From (3.3) and (3.4), we can design a control strategy that brings system (3.2) to the resting equilibrium point ($w_x = 0, w_y = 0$). In other words, we propose a control strategy which forces the motion of system (3.2), starting from any arbitrary initial condition $(w_x(0), w_y(0))$ towards the resting equilibrium point.

We propose the following very simple feedback controllers

$$x_r = -\sigma_a(w_x) \,\widehat{=}\, -\varepsilon\sigma_a\left(\frac{1}{\lambda}(\gamma(x_1 - x_f) + x_2)\right),$$
$$y_r = -\sigma_b(w_y) \,\widehat{=}\, -\varepsilon\sigma_b\left(\frac{1}{\lambda}(\gamma(y_1 - y_f) + y_2)\right), \tag{3.5}$$

where ε and λ are strictly positive constants. Now, substituting the two controllers (3.5) into system (3.2), we have the following closed loop system:

$$\dot{w}_x = -\frac{1}{2}\varepsilon k_s \sigma_a\left(\frac{1}{\lambda}w_x\right)e^{-(x_r^2/a^2 + y_r^2/b^2)},$$
$$\dot{w}_y = -\frac{1}{2}\varepsilon k_s \sigma_b\left(\frac{1}{\lambda}w_y\right)e^{-(x_r^2/a^2 + y_r^2/b^2)}. \tag{3.6}$$

To show that the above system is asymptotically stable, we introduce the following candidate Lyapunov function:

$$V = \frac{1}{2}w_x^2 + \frac{1}{2}w_y^2. \tag{3.7}$$

Then, the time derivative of V along of the trajectories of system (3.6) produces

$$\dot{V} = -\frac{1}{2}\varepsilon k_s\left(w_x\sigma_a\left(\frac{1}{\lambda}w_x\right) + w_y\sigma_b\left(\frac{1}{\lambda}w_y\right)\right)e^{-(\varepsilon^2\sigma_a((1/\lambda)w_x)^2/a^2 + \varepsilon^2\sigma_b((1/\lambda)w_y)^2/b^2)}. \tag{3.8}$$

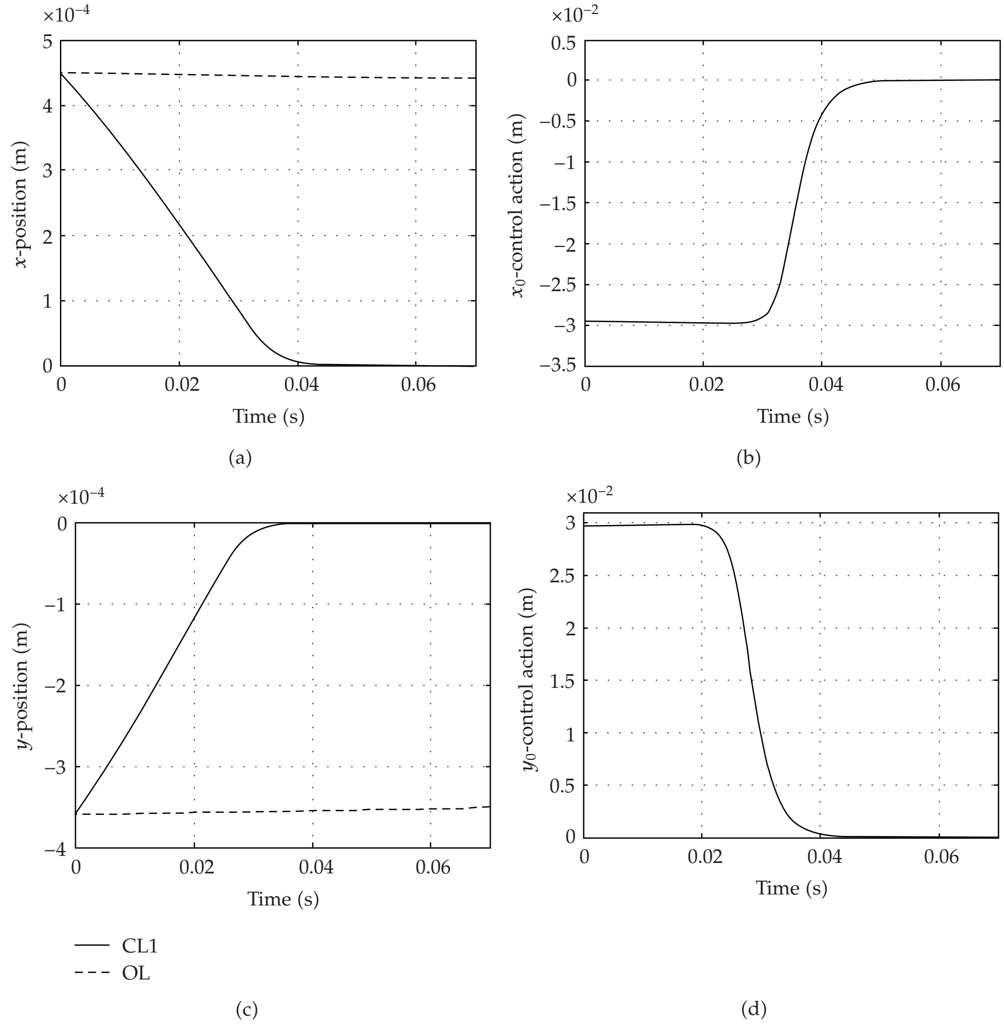


Figure 2: OT system in closed loop response to the first control strategy. For comparison, the corresponding open loop response is shown as a dotted line. As expected, the closed loop response outperforms the open loop response.

Now, from Definition 3.1 we have that $s\sigma_a(s/\lambda) > 0$; for all $s \neq 0$ and $\lambda > 0$, hence $\dot{V} < 0$. (Note that $0 < e^{-2\varepsilon^2} \leq e^{-(\varepsilon^2\sigma_a((1/\lambda)w_x)^2/a^2+\varepsilon^2\sigma_b((1/\lambda)w_x)^2/b^2)} \leq 1$.) That is, the solution of system (3.6) asymptotically converges to $(w_x = 0, w_y = 0)$. Therefore, from (3.4), as time goes to infinity, we also have that $q \to q_f$ and $\dot{q} \to 0$. Also, we can show that the obtained closed loop system is locally exponentially stable. This fact can be seen by a simple linearization of system (3.6).

We finish this section by introducing the following proposition.

Proposition 3.2. *Let us consider the nonlinear system* (2.4) *in closed loop with the controllers*

$$x_r = -\varepsilon\sigma_a\left(\frac{1}{\lambda}\left(\gamma\left(x_1 - x_f\right) + x_2\right)\right), \qquad y_r = -\varepsilon\sigma_b\left(\frac{1}{\lambda}\left(\gamma\left(y_1 - y_f\right) + y_2\right)\right), \tag{3.9}$$

where $\varepsilon > 0$ and $\lambda > 0$. Then the resulting closed loop system is globally asymptotically stable. Besides, the closed loop system is locally exponentially stable.

Numerical Simulation

The performance of the proposed control strategy, summarized in Proposition 3.2, was tested with a numerical simulation. The experiment was designed as follows. The system physical parameters were fixed as

$$\begin{aligned} a = 600\,\mu\text{m}, \qquad b = 600\,\mu\text{m}, \qquad p_0 = 1 \times 10^{-14}\,\text{J}, \\ m = 3.68 \times 10^{-14}\,\text{kg}, \qquad \beta = 3.7 \times 10^{-8}\,\text{Ns/m}, \end{aligned} \tag{3.10}$$

where β was estimated using the Stokes formula, $\beta = 6\pi\eta r$, where $\eta \cong 10^{-3}\,\text{kg/(ms)}$ is the water viscosity and $r = 2\,\mu\text{m}$ is the particle radius (see [2, 11] for details); the initial condition was set as $(x(0) = 4.5 \times 10^{-4},\ y(0) = -3.6 \times 10^{-4},\ \dot{x}(0) = 0,\ \dot{y}(0) = 0)$, the selected feedback saturation function was $\sigma_m(s) = m\tanh(s)$, $\varepsilon = 1$, and $\lambda = a$. Figure 2 shows the performance of the closed loop system in comparison to the corresponding open loop system; in both cases the task goal was bringing the system to the origin, assuming that the initial condition was close enough to the origin. Recalling that x_0 and y_0 are the control actions and are related with the geometric center of the laser focus; in other words, they represent how the OT focus must be manipulate to achieve the final desires position (x_f, y_f). As we can see from this figure, the closed loop strategy achieves faster settling time than the corresponding open loop strategy, this is one of the main advantages of using feedback state over open loop system. (For simplicity, we use CL1 to refer the closed loop response of the first control strategy, i.e., system (2.4) in closed loop with (3.9). Similarly, OP denotes the corresponding open loop response, that is, when $x_0 = 0$ and $y_0 = 0$.)

3.2. *Second Control Approach*

Here we present an alternative control strategy for achieving positioning of the OT system, under the assumption that, as already mentioned in the introduction, β is an uncertain parameter. This is well justified since, in an actual application, the damping coefficient is related with the medium density, which varies as the surrounding or environment temperature does.

We start by proposing the following feedback controllers:

$$\begin{aligned} x_r &= -\varepsilon\sigma_a\left(\frac{1}{\varepsilon}\left(k_1\left(x_1 - x_f\right) + k_2 x_2\right)\right) \hat{=} -\varepsilon\sigma_a\left(\frac{1}{\varepsilon}v_x\right), \\ y_r &= -\varepsilon\sigma_b\left(\frac{1}{\varepsilon}\left(k_1\left(y_1 - y_f\right) + k_2 y_2\right)\right) \hat{=} -\varepsilon\sigma_b\left(\frac{1}{\varepsilon}v_y\right), \end{aligned} \tag{3.11}$$

where the strictly positive gains ε, k_1 and k_2 will be determinate latter. Naturally, for this case v_x and v_y can be seen as the new outputs of the closed loop system.

To show that system (2.4), in closed loop with the feedback (3.11), is asymptotically stable for some gains ε, k_1, and k_2, we rewrite the closed loop system as an exponentially stable linear system, perturbed by two nonlinear residual functions of the outputs v_x and v_y.

Next, we shape a convenient candidate Lyapunov function in agreement with Lemma 3.3. In fact, the Lyapunov function is a quadratic function of both states x and y. Finally, to assure that the closed loop system is asymptotically stable, we show that the time derivative of the proposed Lyapunov function is strictly negative.

After some simple manipulation it is easy to verify that the closed loop system, defined by (2.4) and (3.11), can be expressed as state space representation, given by

$$\begin{aligned}\dot{x} &= Ax + Br_x(v_x, v_y),\\ \dot{y} &= Ay + Br_x(v_x, v_y),\end{aligned} \tag{3.12}$$

where

$$x = \begin{bmatrix} x_1 - x_f \\ x_2 \end{bmatrix}, \qquad y = \begin{bmatrix} y_1 - y_f \\ y_2 \end{bmatrix}, \qquad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \qquad A = \begin{bmatrix} 0 & 1 \\ -\dfrac{k_1}{2} & -\dfrac{k_2}{2} - \gamma \end{bmatrix}, \tag{3.13}$$

$$\begin{aligned} r_x(v_x, v_y) &:= \frac{1}{2}\left(v_x - \varepsilon\sigma_a\left(\frac{1}{\varepsilon}v_x\right)e^{-w^2}\right),\\ r_y(v_x, v_y) &:= \frac{1}{2}\left(v_y - \varepsilon\sigma_b\left(\frac{1}{\varepsilon}v_y\right)e^{-w^2}\right),\end{aligned} \tag{3.14}$$

with $w^2 = (\sigma_a^2(v_x)/a^2 + \sigma_b^2(v_y)/b^2)$.

Before presenting the closed loop stability analysis, we introduce the useful Lemma 3.3, which allows us to shape the candidate Lyapunov function.

Lemma 3.3. *Let x, y, A and B be defined as* (3.13). *Selecting P and Q, as*

$$P = \begin{bmatrix} \gamma k_1 & k_1 \\ k_1 & k_2 \end{bmatrix}, \qquad Q = \begin{bmatrix} k_1^2 & k_1k_2 \\ k_1k_2 & k_2^2 + 2(\gamma k_2 - k_1) \end{bmatrix}, \tag{3.15}$$

where k_1 and k_2 satisfy $\gamma k_2 - k_1 > 0$. Then, P and Q are strictly positive matrices which satisfy

$$\begin{gathered} PA + A^T P + Q = 0,\\ x^T PB = v_x, \qquad y^T PB = v_y. \end{gathered} \tag{3.16}$$

Besides

$$Q - K \geq 0, \tag{3.17}$$

where K is given by

$$K = \begin{bmatrix} k_1^2 & k_1k_2 \\ k_1k_2 & k_2^2 \end{bmatrix}. \tag{3.18}$$

Proof. See the appendix. □

Continuing with the stability analysis, we introduce the following candidate Lyapunov function:

$$V = \frac{1}{2}x^T P x + \frac{1}{2}y^T P y, \tag{3.19}$$

where matrix $P = P^T > 0$ is selected according to Lemma 3.3.

Then, the time derivative of V along the trajectories of system (3.12) leads to

$$\dot{V} = x^T\left(PA + A^T P\right)x + y^T\left(PA + A^T P\right)y + 2x^T PBr_x(v_x, v_y) + 2y^T PBr_y(v_x, v_y). \tag{3.20}$$

Now, by simple algebra, we have the following two equalities:

$$\begin{aligned} 2x^T PBr_x(v_x, v_y) &= v_x^2 - \varepsilon v_x \sigma_a\left(\frac{1}{\varepsilon}v_x\right)e^{-w^2} = x^T Kx - \varepsilon v_x \sigma_a\left(\frac{1}{\varepsilon}v_x\right)e^{-w^2}, \\ 2x^T PBr_y(v_x, v_y) &= v_y^2 - \varepsilon v_y \sigma_b\left(\frac{1}{\varepsilon}v_y\right)e^{-w^2} = y^T Ky - \varepsilon v_y \sigma_b\left(\frac{1}{\varepsilon}v_y\right)e^{-w^2}. \end{aligned} \tag{3.21}$$

Hence, substituting the above equalities into the time derivative of V (3.20), we have

$$\dot{V} = -x^T(Q-K)x - y^T(Q-K)y - \varepsilon v_x \sigma_a\left(\frac{1}{\varepsilon}v_x\right)e^{-w^2} - \varepsilon v_y \sigma_b\left(\frac{1}{\varepsilon}v_y\right)e^{-w^2}. \tag{3.22}$$

According to Lemma 3.3 (see definition of Q and (3.17)), we have that the last equality can be read as

$$\dot{V} = -x_2^2(\gamma k_2 - k_1) - y_2^2(\gamma k_2 - k_1) - \varepsilon v_x \sigma_a\left(\frac{1}{\varepsilon}v_x\right)e^{-w^2} - \varepsilon v_y \sigma_b\left(\frac{1}{\varepsilon}v_y\right)e^{-w^2}. \tag{3.23}$$

The last equality implies that $\dot{V}$ is strictly negative definite (note that, from definition of w and a being small and positive, we have $0 < \exp[-2a] \le \exp[-w^2] < 1$). since $s\sigma_a((1/\varepsilon)s) \ge 0$ for all $s \in R$. Hence, the closed loop system is asymptotically stable. On the other hand, the simple linearization of the close-loop system (3.12) produces

$$\dot{x} = Ax, \qquad \dot{y} = Ay, \tag{3.24}$$

where, evidently, A is a Hurtwitz matrix. That is to say, the closed loop system is also locally exponentially stable and robust with respect to the small external perturbation.

Summarizing the previous discussion, we introduce the following proposition.

Proposition 3.4. *The nonlinear system* (2.4) *in closed loop with the feedback* (3.11) *is globally asymptotically stable, if the gains k_1 and k_2 are selected provided that $\gamma k_2 - k_1 > 0$. Besides, the closed loop system is locally exponentially stable.*

Remark 3.5. The positiveness of $\gamma k_2 - k_1$ can be easily assured by selecting the gain k_1 small enough in comparison with the gain k_2,without knowing the exact values of γ.

Remark 3.6. We believe that the restriction in Remark 3.5 is not strong enough, because it is always possible to select another set of positive gains k_1 and k_2, keeping the closed loop asymptotically stable. This could be shown (at least locally) by using simple linearization. However, the proof of this conjecture is beyond the scope of this paper, due to the fact that we would need to shape another Lyapunov function or use the small gain theorem.

Numerical Simulation

The performance of the second control strategy was tested by a second numerical experiment. The initial conditions and the physical parameter values were the same as in the first experiment. The gains of the controllers were fixed as $k_1 = 1/a$ and $k_2 = 2/(a\overline{\beta})$, with $\overline{\beta} = 3 \times 10^{-8}$, while the saturation function was selected as $\sigma_a(s) = \tanh(s)$. The control task was bringing the system to the origin. Figure 3 shows the performance of the closed loop system in comparison to the corresponding open loop system. Once again the closed loop strategy achieves faster settling time than the corresponding open loop strategy. Notice that both system position coordinates with their respective controllers asymptotically converge to the rest position after 0.11 seconds elapse. Also, as in Figure 2, x_0 and y_0 are related with the geometric center of the laser focus. (For simplicity, CL2 denotes the closed loop response of the second control strategy, i.e., system (2.4) in closed loop by (3.11); OP denotes the corresponding open loop response.) From the numerical simulations we can see that the first control strategy has a better settle time than the second one. It was expected since in the first strategy the damping coefficient is perfectly know.

3.3. Stabilization in the Presence of Thermal Noise

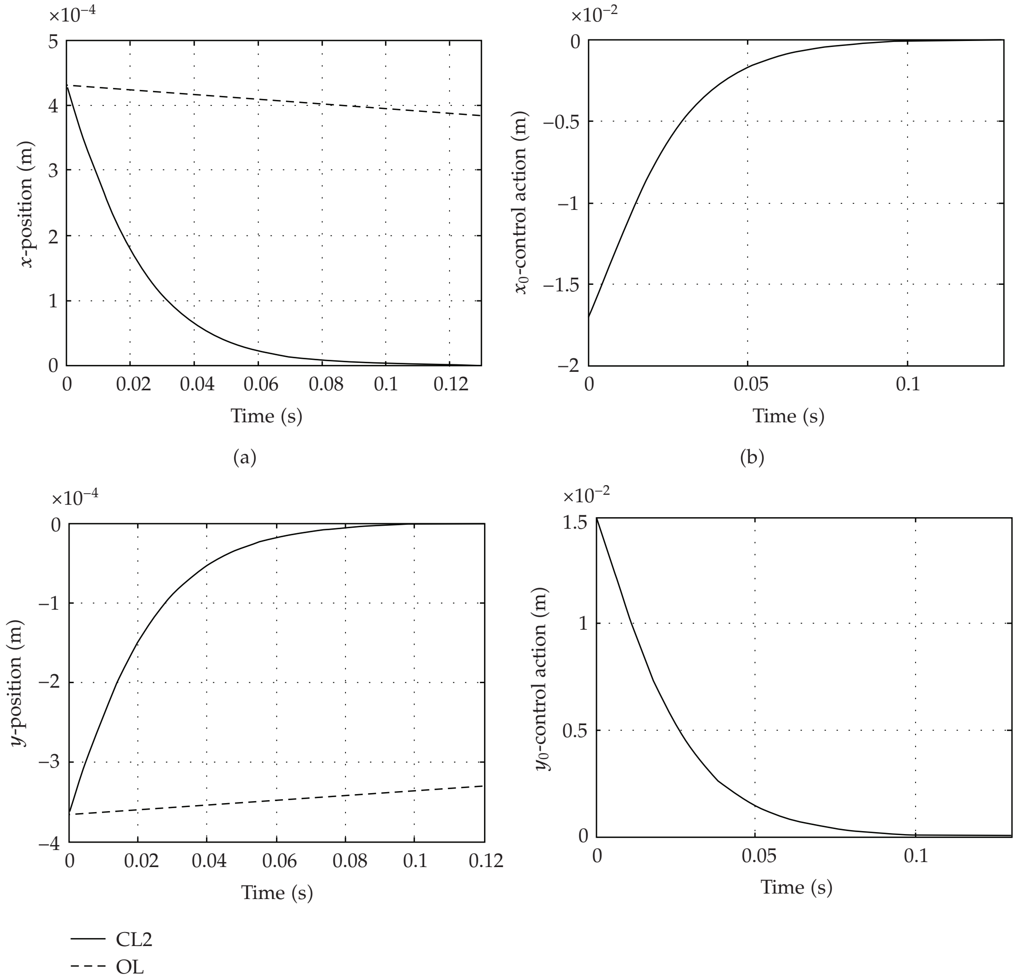
In this section we consider the inherent effect of the thermal noise in our control model. To this purpose, we designed a numerical simulation, where the external random thermal noise is included in the model. That is, the independent external random perturbations $F_x(t)$ and $F_y(t)$ (referred to as the Langevin random thermal forces) are modeled as

$$F_x(t) = \sqrt{4k_b T\beta}\eta_1(t), \qquad F_y(t) = \sqrt{4k_b T\beta}\eta_2(t), \tag{3.25}$$

where k_b is the Boltzmann's constant, β is the drag coefficient (estimated by the Stokes formula), T is the absolute temperature, $\eta_1(t)$ and $\eta_2(t)$ are independent white noise signals, that is, for all t, t',

$$\langle \eta_i(t) \rangle = 0, \qquad \langle \eta_i(t)\eta_j(t') \rangle = \delta_{ij}\delta(t - t'), \tag{3.26}$$

with the dimension of $\eta_i(t)$ given by $\sqrt{s}$, for more details about this topic, we recommend [14, 15] and Chapters 3 and 10 of Risken's book [19]. The numerical simulations in presence of

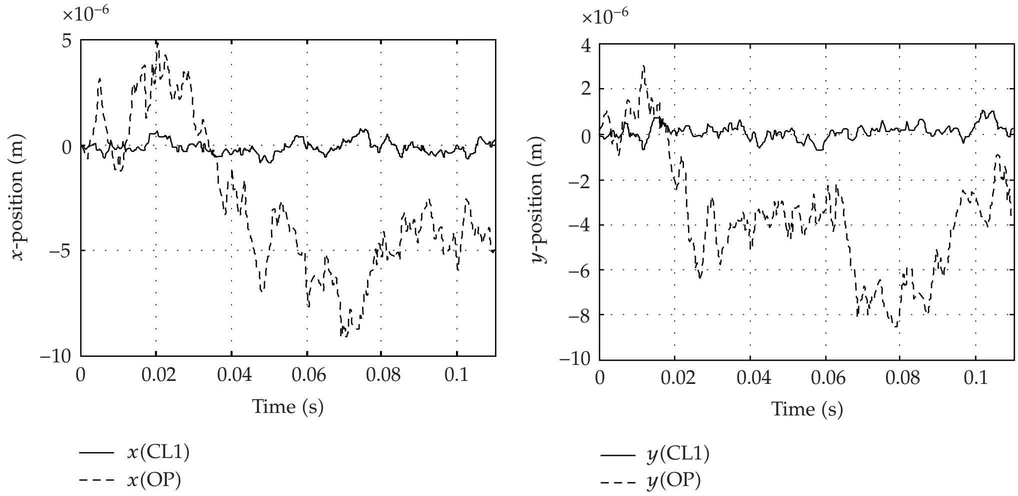


(a) (b) (c) (d)

Figure 3: OT system in closed loop response to the second control strategy in a stabilization task. As we can see the system effectively reaches the final rest position, $(x_f = 0, y_f = 0)$.

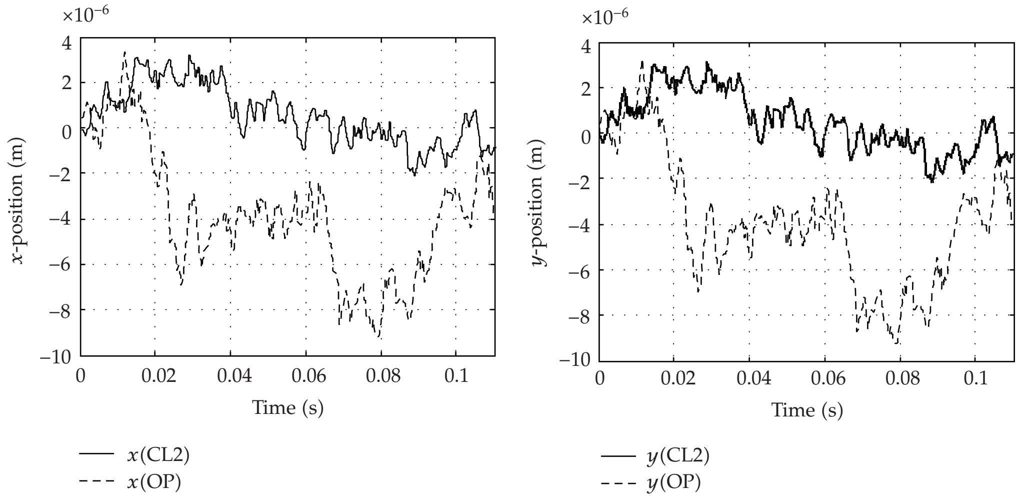
the thermal noise were carried out by using the same physical parameters, as were presented in (3.10). For simplicity, we take k_bT approximately as 3.8×10^{-21} Nm. Hence, we have that $\sqrt{4k_bT\beta} = 2.5 \times 10^{-10}\,\mathrm{N}/\sqrt{s}$.

Once again, the first and second control approaches (see (2.4) with (3.9) and (2.4) with (3.11)) were simulated in the presence of the thermal noise to highlight the advantages of using feedback state. The control task consists of keeping all the states closed enough to the origin, under the assumption that the system was initialized at the rest equilibrium point. To reduce the time execution of the stochastic processes, we used a simple version of Euler-numerical method, with a sampling time of 1×10^{-7} [s]. In Figure 4 we show both, the open loop and the closed loop responses to the first control approach. To this end, we use the same control gains, as in Section 3.1. From this figure we see that in closed loop the thermal force effect is less dominant than in the open loop. This is one of the main advantages of using feedback state in comparison with the open loop response. In Figure 5 we show, once again,



(a) (b)

Figure 4: Closed-loop response of the OT system to the first control approach with the corresponding open loop response (dotted-line), when the OT system is subject to the external thermal forces.



(a) (b)

Figure 5: Closed-loop response of the OT system to the second controller in comparison with the corresponding open loop response (dotted-line), when the external thermal forces are presented in the model.

the open loop and the closed loop responses to the second control approach. We use the same control gains, as were proposed in Section 3.2. As expected, the first strategy outperforms the second one.

Finally, Figure 6 shows the closed loop response of both OT changing position control strategies in the presence of the thermal noise. The control goal was bringing the particle from the origin to the final position ($x_f = 1.2 \times 10^{-4}$ m, $y_f = -1.4 \times 10^{-4}$ m). Once again, we can note that the first control strategy outperforms the second one.

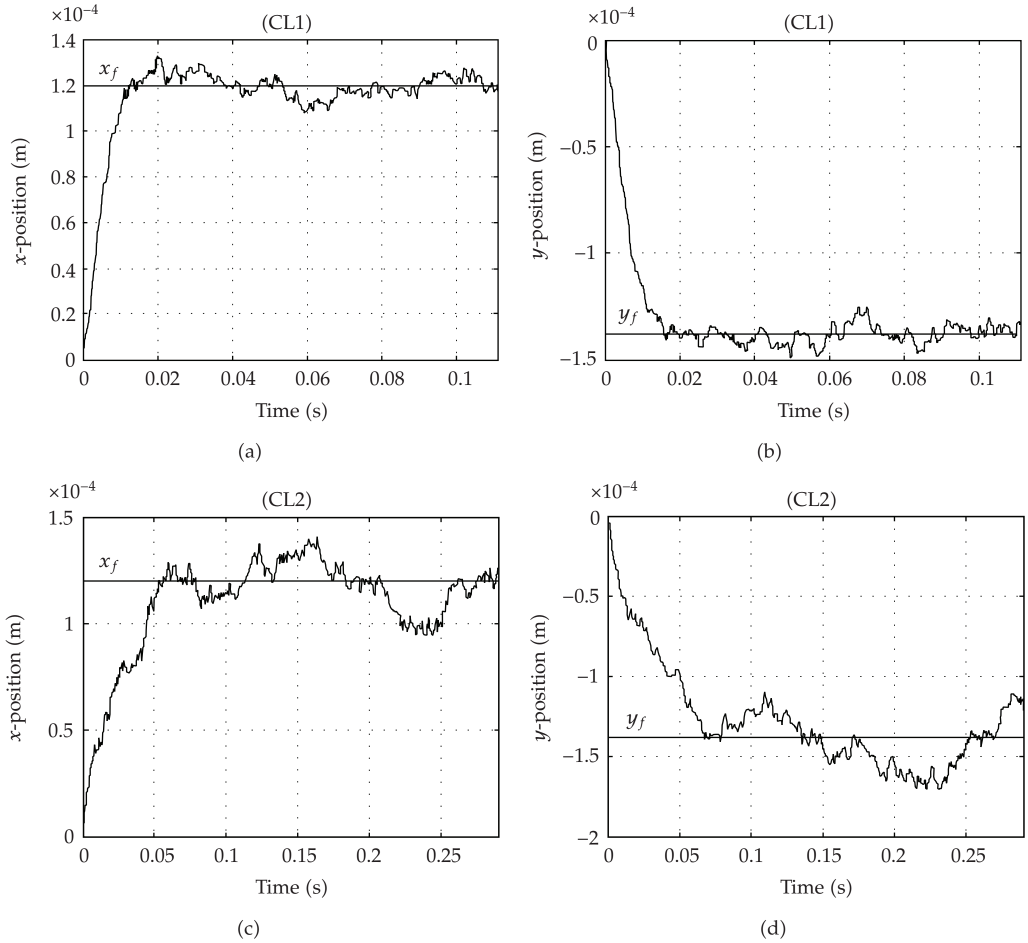


Figure 6: Comparison of the closed loop response of both OT changing position control strategies in the presence of thermal noise.

4. Conclusions

In this work we have presented two simple control strategies to move a spherical micro particle trapped in an OT system from one initial condition to a final rest position. The first control strategy is based on the fact that the damping coefficient presented in the medium is known, while in the second strategy this condition is omitted. Both manipulation strategies have been derived by using the traditional Lyapunov method in conjunction with the use of a saturation function. Numerical simulations have been carried out to show the performance and effectiveness of the proposed steering strategy. From the obtained numerical simulations, we claim that both control strategies respond quite well even when the random thermal noise is presented in the model. As a matter of fact, it is possible to show that both strategies achieve asymptotic average convergence at the origin by using more sophisticated arguments from the Stochastic Control Theory. In practice, this is a rather important property, because it allows to carry out experiments where a located particle needs to be moved to a very specific location in spite of external random thermal noise.

Appendix

Proof of Lemma 3.3. We show that P and Q are strictly definite positive. Computing the determinants of P and Q, we have

$$\det(P) = k_1\left(-k_1 + \gamma k_2\right), \qquad \det(Q) = 2k_1^2\left(-k_1 + \gamma k_2\right). \tag{A.1}$$

From the assumption $\gamma k_2 - k_1 > 0$, we have that both determinants are positive and k_1 and γ are positive, therefore P and Q are positive. □

Now, the first matrix equation in (3.15) can be easily tested by substituting the respective values of the given matrices P, Q, and A (defined previously in the aforementioned Lemma 3.3). In the same way, we can check that the equalities in (3.15) hold.

Finally, from the definitions of Q and K, given, respectively, in (3.15) and (3.18), it follows that $w^T(Q-K)w = 2(\gamma k_2 - k_1)w_2^2 \geq 0$, for all $w^T = [w_1, w_2]$.

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