Research Article

Adaptive Fuzzy Tracking Control for a Permanent Magnet Synchronous Motor via Backstepping Approach

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The speed tracking control problem of permanent magnet synchronous motors with parameter uncertainties and load torque disturbance is addressed. Fuzzy logic systems are used to approximate nonlinearities, and an adaptive backstepping technique is employed to construct controllers. The proposed controller guarantees the tracking error convergence to a small neighborhood of the origin and achieves the good tracking performance. Simulation results clearly show that the proposed control scheme can track the position reference signal generated by a reference model successfully under parameter uncertainties and load torque disturbance without singularity and overparameterization.

1. Introduction

Permanent magnet synchronous motors (PMSMs) are of great interest for industrial applications requiring dynamic performance due to their high speed, high efficiency, high power density, and large torque to inertia ratio. Then it is still a challenging problem to control PMSM to get the perfect dynamic performance because the motor dynamic model of PMSM is nonlinear and multivariable, the model parameters such as the stator resistance and the friction coefficient are also not be exactly known. The control of PMSM drivers has recently received wide attention and become an active research area. Some advanced control techniques, such as sliding mode control [1, 2], feedback linearization control [3], adaptive control [2, 4], backstepping principles [5–7], and Fuzzy logic control [8–10], are used to the problems of speed or position control of PMSMs.

Table 1: The denotation of the PMSM parameters.

Parameter	Denotation	Parameter	Denotation
ω	The rotor angular velocity	R_s	The stator resistance
i_d	The d axis current	i_q	The q axis current
L_d	The d axis stator inductor	L_q	The q axis stator inductor
u_d	The d axis voltage	u_q	The q axis voltage
n_p	The pole pair	J	The rotor moment of inertia
T	The electromagnetism torque	T_L	The load torque
B	The viscous friction coefficient	Φ	The magnet flux linkage of inertia

Backstepping is a newly developed technique to control the nonlinear systems with parameter uncertainty, particularly those systems in which the uncertainty does not satisfy matching conditions. Though backstepping is successfully applied to the control of PMSM drivers recently, it usually makes the designed controllers' structure to be very complex.

Fuzzy logic control (FLC) has been found one of the most popular and conventional tools in functional approximations. An FLC [11, 12] has strong ability of handling uncertain information and can be easily used in the control of systems which is ill-defined or too complex to have a mathematical model. It provides an effective way to design control system that is one of important applications in the area of control engineering.

In this paper, an adaptive fuzzy control approach is proposed for speed tracking control of PMSM drive system via the backstepping technique. During the controller design process, fuzzy logic systems are employed to approximate the nonlinearities, the adaptive fuzzy controllers are constructed via backstepping. The designed fuzzy controller can track the reference signal quite well even the existence of the parameter uncertainties and load torque disturbance. Compared with the existing controller design schemes via backstepping, the proposed method is very simple and the proposed controller has a simple structure.

2. Mathematical Model of the PMSM Drive System and Preliminaries

In this section, some preparatory knowledge of a PMSM will be introduced. The following assumptions are made in the derivation of the mathematical model of a PMSM [13].

Assumption 2.1. Saturation and iron losses are neglected although it can be taken into account by parameter changes.

Assumption 2.2. The back emf is sinusoidal.

The model of a PMSM can be described in the well known $(d-q)$ frame through the Park transformation as follows. The stator d, q equations in the rotor frame are expressed as follows [14]:

$$J\frac{d\omega}{dt} = T - T_L - B\omega = \frac{3}{2}n_p\left[(L_d - L_q)i_d i_q + \Phi i_q\right] - B\omega - T_L,$$

$$L_d\frac{di_d}{dt} = -R_s i_d + n_p\omega L_q i_q + u_d, \qquad (2.1)$$

$$L_q\frac{di_q}{dt} = -R_s i_q - n_p\omega L_d i_d - n_p\omega\Phi + u_q.$$

The denotation of the PMSM parameters is shown in Table 1.

To simplify the previous method mode, the following notations are introduced:

$$x_1 = \omega, \qquad x_2 = i_q, \qquad x_3 = i_d,$$

$$a_1 = \frac{3n_p\Phi}{2}, \qquad a_2 = \frac{3n_p(L_d - L_q)}{2},$$

$$b_1 = -\frac{R_s}{L_q}, \qquad b_2 = -\frac{n_pL_d}{L_q}, \qquad b_3 = -\frac{n_p\Phi}{L_q}, \qquad b_4 = \frac{1}{L_q}, \tag{2.2}$$

$$c_1 = -\frac{R_s}{L_d}, \qquad c_2 = \frac{n_pL_q}{L_d}, \qquad c_3 = \frac{1}{L_d}.$$

By using these notations, the dynamic model of a PMSM motor can be described by the following differential equations:

$$\dot{x}_1 = \frac{a_1}{J}x_2 + \frac{a_2}{J}x_2x_3 - \frac{B}{J}x_1 - \frac{T_L}{J},$$

$$\dot{x}_2 = b_1x_2 + b_2x_1x_3 + b_3x_1 + b_4u_q, \tag{2.3}$$

$$\dot{x}_3 = c_1x_3 + c_2x_1x_2 + c_3u_d.$$

The control objective is to design an adaptive fuzzy controller such that the state variable x_1 tracks the given reference signal x_d and all signals of the resulting closed-loop system are uniformly ultimately bounded. In this paper, we adopt the singleton fuzzifier, product inference, and the center-defuzzifier to deduce the following fuzzy rules:

$$R_i : \text{IF } x_1 \text{ is } F_1^i \text{ and} \ldots \text{and } x_n \text{ is } F_n^i \text{ THEN } y \text{ is } B^i \quad (i = 1, 2, \ldots, N), \tag{2.4}$$

where $x = [x_1, \ldots, x_n]^T \in R^n$, and $y \in R$ are the input and output of the fuzzy system, respectively, F_i^j and B^i are fuzzy sets in R. The fuzzy inference engine performs a mapping from fuzzy sets in R^n to fuzzy set in R based on the IF-THEN rules in the fuzzy rule base and the compositional rule of inference. The fuzzifier maps a crisp point $x = [x_1, \ldots, x_n]^T \in R^n$ into a fuzzy set A_x in R. The defuzzifier maps a fuzzy set in R to a crisp point in R. Since the strategy of singleton fuzzification, center-average defuzzification, and product inference is used, the output of the fuzzy system can be formulated as

$$y(x) = \frac{\sum_{j=1}^N W_j \prod_{i=1}^n \mu_{F_i^j}(x_i)}{\sum_{j=1}^N \left[\prod_{i=1}^n \mu_{F_i^j}(x_i)\right]}, \tag{2.5}$$

where W_j is the point at which fuzzy membership function $\mu_{B^j}(W_j)$ achieves its maximum value, and it is assumed that $\mu_{B^j}(W_j) = 1$. Let $p_j(x) = \prod_{i=1}^n \mu_{F_i^j}(x_i) / \sum_{j=1}^N [\prod_{i=1}^n \mu_{F_i^j}(x_i)]$, $S(x) = [p_1(x), p_2(x), \ldots, p_N(x)]^T$ and $W = [W_1, \ldots, W_N]^T$, then the fuzzy logic system above can be

rewritten as

$$y(x) = W^T S(x). \tag{2.6}$$

If all memberships are taken as Gussian functions, then the following lemma holds.

Lemma 2.3 (see [15]). *Let $f(x)$ be a continuous function defined on a compact set Ω. Then for any scalar $\varepsilon > 0$, there exists a fuzzy logic system in the form (2.6) such that*

$$\sup_{x \in \Omega} |f(x) - y(x)| \le \varepsilon. \tag{2.7}$$

3. Adaptive Fuzzy Controller with the Backstepping Technique

For the system (2.3), the backstepping design procedure contains 3 steps. At each design step, a virtual control function α_i ($i = 1, 2$) will be constructed by using an appropriate Lyapunov function V. At the last step, a real controller is constructed to control the system. In the following, we will give the procedure of the backstepping design.

Step 1. For the reference signal x_d, define the tracking error variable as $z_1 = x_1 - x_d$. From the first subsystem of (2.3), the error dynamic system is given by $\dot{z}_1 = (a_1/J)x_2 + (a_2/J)x_2x_3 - (B/J)x_1 - T_L/J - \dot{x}_d$.

Choose Lyapunov function candidate as $V_1 = (J/2)z_1^2$, then the time derivative of V_1 is given by

$$\dot{V}_1 = J z_1 \dot{z}_1 = z_1(a_1 x_2 + a_2 x_2 x_3 - B x_1 - T_L - J \dot{x}_d). \tag{3.1}$$

As the parameters B, T_L, and J are unknown, they cannot be used to construct the control signal. Thus, let $\widehat{B}, \widehat{T}_L$, and $\widehat{J}$ be their estimations of B, T_L, and J, respectively. The corresponding adaptation laws will be determined later. Now, construct the virtual control law α_1 as

$$\alpha_1(Z_1) = \frac{1}{a_1}\left(-k_1 z_1 + \widehat{B} x_1 + \widehat{T}_L + \widehat{J} \dot{x}_d\right), \tag{3.2}$$

where $k_1 > 0$ is a design parameter and $Z_1 = [x_1, x_d, \dot{x}_d, \widehat{B}, \widehat{T}_L, \widehat{J}]^T$. Defining $z_2 = x_2 - \alpha_1$ and substituting (3.2) into (3.1) yield

$$\dot{V}_1 = -k_1 z_1^2 + a_1 z_1 z_2 + a_2 z_1 x_2 x_3 + z_1\left(\widehat{B} - B\right)x_1 + z_1\left(\widehat{T}_L - T_L\right) + z_1\left(\widehat{J} - J\right)\dot{x}_d. \tag{3.3}$$

Step 2. Differentiating z_2 and using the second subsystem of (2.3) give

$$\dot{z}_2 = \dot{x}_2 - \dot{\alpha}_1 = b_1 x_2 + b_2 x_1 x_3 + b_3 x_1 + b_4 u_q - \dot{\alpha}_1. \tag{3.4}$$

Now, choose the Lyapunov function candidate as $V_2 = V_1 + (1/2)z_2^2$. Obviously, the time derivative of V_2 is given by

$$
\begin{aligned}
\dot{V}_2 &= \dot{V}_1 + z_2\dot{z}_2 \\
&= -k_1z_1^2 + a_2z_1x_2x_3 + z_1\left(\widehat{B} - B\right)x_1 + z_1\left(\widehat{T}_L - T_L\right) + z_1\left(\widehat{J} - J\right)\dot{x}_d + z_2(f_2 + b_4u_q),
\end{aligned}
\tag{3.5}
$$

where

$$
\begin{aligned}
\dot{\alpha}_1 &= \frac{\partial\alpha_1}{\partial x_1}\dot{x}_1 + \sum_{i=0}^{1}\frac{\partial\alpha_1}{\partial x_d^{(i)}}x_d^{(i+1)} + \frac{\partial\alpha_1}{\partial\widehat{B}}\dot{\widehat{B}} + \frac{\partial\alpha_1}{\partial\widehat{T}_L}\dot{\widehat{T}}_L + \frac{\partial\alpha_1}{\partial\widehat{J}}\dot{\widehat{J}} \\
&= \frac{\partial\alpha_2}{\partial x_1}\left(\frac{a_1}{J}x_2 + \frac{a_2}{J}x_2x_3 - \frac{B}{J}x_1 - \frac{T_L}{J}\right) + \sum_{i=0}^{1}\frac{\partial\alpha_1}{\partial x_d^{(i)}}x_d^{(i+1)} + \frac{\partial\alpha_1}{\partial\widehat{B}}\dot{\widehat{B}} + \frac{\partial\alpha_1}{\partial\widehat{T}_L}\dot{\widehat{T}}_L + \frac{\partial\alpha_1}{\partial\widehat{J}}\dot{\widehat{J}},
\end{aligned}
\tag{3.6}
$$

$$
f_2(Z_2) = a_1z_1 + b_1x_2 + b_2x_1x_3 + b_3x_1 - \dot{\alpha}_1,
$$

$$
Z_2 = \left[x_1, x_2, x_3, x_d, \dot{x}_d, \ddot{x}_d, \widehat{B}, \widehat{T}_L, \widehat{J}\right]^T.
$$

Apparently, there are two nonlinear terms in (3.5), that is, $a_2z_1x_2x_3$ and f_2, therewithal, f_2 contains the derivative of $\dot{\alpha}_1$. This will make the classical adaptive backstepping design become very complex and troubled, and the designed control law u_q will have the complex structure. To avoid this trouble in design procedure and simplify the control signal structure, we will employ the fuzzy logic system to approximate the nonlinear function f_2. As shown later, the design procedure of u_q becomes simple and u_q has the simple structure. According to Lemma 2.3, for any given $\varepsilon_2 > 0$, there exists a fuzzy logic system $W_2^TS_2(Z_2)$ such that

$$
f_2(Z_2) = W_2^TS_2(Z_2) + \delta_2(Z_2),
\tag{3.7}
$$

with $\delta_2(Z)$ being the approximation error and satisfying $|\delta_2| \le \varepsilon_2$. Consequently, a simple method computing produces the following inequality:

$$
z_2f_2 = z_2\left(W_2^TS_2 + \delta_2\right) \le \frac{1}{2l_2^2}z_2^2\|W_2\|^2S_2^2 + \frac{1}{2}l_2^2 + \frac{1}{2}z_2^2 + \frac{1}{2}\varepsilon_2^2.
\tag{3.8}
$$

It follows immediately from substituting (3.8) into (3.5) that

$$
\begin{aligned}
\dot{V}_2 \le &-\sum_{i=1}^{2}k_iz_i^2 + a_2z_1x_2x_3 + z_1\left(\widehat{B} - B\right)x_1 + z_1\left(\widehat{T}_L - T_L\right) \\
&+ z_1\left(\widehat{J} - J\right)\dot{x}_d + \frac{1}{2l_2^2}z_2^2\|W_2\|^2S_2^2 + \frac{1}{2}l_2^2 + \frac{1}{2}z_2^2 + \frac{1}{2}\varepsilon_2^2 + z_2b_4u_q.
\end{aligned}
\tag{3.9}
$$

The control input u_q is designed as

$$u_q = \frac{1}{b_4}\left(-k_2 z_2 - \frac{1}{2}z_2 - \frac{1}{2l_2^2}z_2\hat{\theta}S_2^2\right), \tag{3.10}$$

where $\hat{\theta}$ is the estimation of the unknown constant θ which will be specified later. Using equality (3.10), the derivative of V_2 becomes as

$$\dot{V}_2 \leq -\sum_{i=1}^{2}k_i z_i^2 + a_2 z_1 x_2 x_3 + z_1\left(\hat{B} - B\right)x_1 + z_1\left(\hat{T}_L - T_L\right)$$

$$+ z_1\left(\hat{J} - J\right)\dot{x}_d + \frac{1}{2l_2^2}z_2^2\left(\|W_2\|^2 - \hat{\theta}\right)S_2^2 + \frac{1}{2}l_2^2 + \frac{1}{2}\varepsilon_2^2. \tag{3.11}$$

Step 3. At this step, we will construct the control law u_d. To this end, define $z_3 = x_3$ and choose the following Lyapunov function candidate as $V_3 = V_2 + (1/2)z_3^2$. Then the derivative of V_3 is given by

$$\dot{V}_3 = \dot{V}_2 + z_3\dot{z}_3 \leq -\sum_{i=1}^{2}k_i z_i^2 + a_2 z_1 x_2 x_3 + z_1\left(\hat{B} - B\right)x_1 + z_1\left(\hat{T}_L - T_L\right)$$

$$+ z_1\left(\hat{J} - J\right)\dot{x}_d + \frac{1}{2l_2^2}z_2^2\left(\|W_2\|^2 - \hat{\theta}\right)S_2^2 + \frac{1}{2}l_2^2 + \frac{1}{2}\varepsilon_2^2 + z_3(f_3(Z_3) + c_3 u_d), \tag{3.12}$$

where $f_3(Z_3) = a_2 z_1 x_2 + c_1 x_3 + c_2 x_1 x_2$ and $Z_3 = [x_1, x_2, x_3, x_d]^T$. Similarly, by Lemma 2.3 the fuzzy logic system $W_3^T S_3(Z_3)$ is utilized to approximate the nonlinear function f_3 such that for given $\varepsilon_3 > 0$,

$$z_3 f_3 \leq \frac{1}{2l_3^2}z_3^2\|W_3\|^2 S_3^2 + \frac{1}{2}l_3^2 + \frac{1}{2}z_3^2 + \frac{1}{2}\varepsilon_3^2. \tag{3.13}$$

Substituting (3.13) into (3.12) gives

$$\dot{V}_3 = \dot{V}_2 + z_3\dot{z}_3 \leq -\sum_{i=1}^{2}k_i z_i^2 + \frac{1}{2l_2^2}z_2^2\left(\|W_2\|^2 - \hat{\theta}\right)S_2^2 + \sum_{i=2}^{3}\frac{1}{2}\left(l_i^2 + \varepsilon_i^2\right)$$

$$+ z_1\left(\hat{B} - B\right)x_1 + z_1\left(\hat{T}_L - T_L\right) + z_1\left(\hat{J} - J\right)\dot{x}_d + \frac{1}{2l_3^2}z_3^2\|W_3\|^2 S_3^2 + \frac{1}{2}z_3^2 + c_3 z_3 u_d. \tag{3.14}$$

Now design u_d as

$$u_d = \frac{-1}{c_3}\left(k_3 z_3 + \frac{1}{2}z_3 + \frac{1}{2l_3^2}z_3\hat{\theta}S_3^2\right). \tag{3.15}$$

Then, define $\theta = \max\{\|W_2\|^2, \|W_3\|^2\}$. Then, combining (3.14) with (3.15) results in

$$
\begin{aligned}
\dot{V}_4 &\leq -\sum_{i=1}^{3} k_i z_i^2 + \sum_{i=2}^{3} \frac{1}{2}\left(l_i^2 + \varepsilon_i^2\right) + z_1\left(\hat{B} - B\right)x_1 + z_1\left(\hat{T}_L - T_L\right) \\
&\quad + z_1\left(\hat{J} - J\right)\dot{x}_d + \sum_{i=2}^{3} \frac{1}{2l_i^2} z_i^2 \left(\|W_i\|^2 - \hat{\theta}\right) S_i^T(Z_i) S_i(Z_i) \\
&\leq -\sum_{i=1}^{3} k_i z_i^2 + \sum_{i=2}^{3} \frac{1}{2}\left(l_i^2 + \varepsilon_i^2\right) + z_1\left(\hat{B} - B\right)x_1 + z_1\left(\hat{T}_L - T_L\right) \\
&\quad + z_1\left(\hat{J} - J\right)\dot{x}_d + \sum_{i=2}^{3} \frac{1}{2l_i^2} z_i^2 S_i^T(Z_i) S_i(Z_i)\left(\theta - \hat{\theta}\right).
\end{aligned}
\tag{3.16}
$$

At the present stage, to estimate the unknown constants $B, T_L, J,$ and θ, define the adaptive variables as follows:

$$
\begin{aligned}
\tilde{T}_L &= \hat{T}_L - T_L, \\
\tilde{B} &= \hat{B} - B, \\
\tilde{J} &= \hat{J} - J, \\
\tilde{\theta} &= \hat{\theta} - \theta.
\end{aligned}
\tag{3.17}
$$

In order to determine the corresponding adaptation laws, choose the following Lyapunov function candidate:

$$
V = V_3 + \frac{1}{2r_1}\tilde{T}_L^2 + \frac{1}{2r_2}\tilde{B}^2 + \frac{1}{2r_3}\tilde{J}^2 + \frac{1}{2r_4}\tilde{\theta}^2,
\tag{3.18}
$$

where $r_i, i = 1, 2, 3, 4$ are positive constant. By differentiating V and taking (3.16)–(3.18) into account, one has

$$
\begin{aligned}
\dot{V} &\leq -\sum_{i=1}^{3} k_i z_i^2 + \sum_{i=2}^{3} \frac{1}{2}\left(l_i^2 + \varepsilon_i^2\right) + z_1\tilde{B}x_1 + z_1\tilde{T}_L \\
&\quad + z_1\tilde{J}\dot{x}_d - \sum_{i=2}^{3} \frac{1}{2l_i^2} z_i^2 \tilde{\theta} S_i^T(Z_i) S_i(Z_i) + \frac{1}{r_1}\tilde{T}_L\dot{\hat{T}}_L + \frac{1}{r_2}\tilde{B}\dot{\hat{B}} + \frac{1}{r_3}\tilde{J}\dot{\hat{J}} + \frac{1}{r_4}\tilde{\theta}\dot{\hat{\theta}} \\
&= -\sum_{i=1}^{3} k_i z_i^2 + \sum_{i=2}^{3} \frac{1}{2}\left(l_i^2 + \varepsilon_i^2\right) + \frac{1}{r_1}\tilde{T}_L\left(r_1 z_1 + \dot{\hat{T}}_L\right) \\
&\quad + \frac{1}{r_2}\tilde{B}\left(r_2 z_1 x_1 + \dot{\hat{B}}\right) + \frac{1}{r_3}\tilde{J}\left(r_3 z_1 \dot{x}_d + \dot{\hat{J}}\right) + \frac{1}{r_4}\tilde{\theta}\left[-\sum_{i=2}^{3} \frac{r_4}{2l_i^2} z_i^2 S_i^T(Z_i) S_i(Z_i) + \dot{\hat{\theta}}\right].
\end{aligned}
\tag{3.19}
$$

According to (3.19), the corresponding adaptive laws are chosen as follows:

$$\dot{\hat{T}}_L = -r_1 z_1 - m_1 \tilde{T}_L,$$

$$\dot{\hat{B}} = -r_2 z_1 x_1 - m_2 \hat{B},$$

$$\dot{\hat{J}} = -r_3 z_1 \dot{x}_d - m_3 \hat{J}, \tag{3.20}$$

$$\dot{\hat{\theta}} = \sum_{i=2}^{3} \frac{r_4}{2 l_i^2} z_i^2 S_i^T(Z_i) S_i(Z_i) - m_4 \hat{\theta},$$

where m_i for $i = 1, 2, 3, 4$ and l_i for $i = 2, 3$ are positive constant.

4. Stability Analysis

In this section, the stability analysis of the resulting closed-loop system will be addressed. Substituting (3.20) into (3.19) yields

$$\dot{V} \le -\sum_{i=1}^{3} k_i z_i^2 + \sum_{i=2}^{3} \frac{1}{2}\left(l_i^2 + \varepsilon_i^2\right) - \frac{m_1}{r_1}\tilde{T}_L\hat{T} - \frac{m_2}{r_2}\tilde{B}\hat{B} - \frac{m_3}{r_3}\tilde{J}\hat{J} - \frac{m_4}{r_4}\tilde{\theta}\hat{\theta}. \tag{4.1}$$

For the term $-\tilde{T}_L\hat{T}$, one has

$$-\tilde{T}_L\hat{T}_L = -\tilde{T}_L\left(\tilde{T}_L + T_L\right) \le -\frac{1}{2}\tilde{T}_L^2 + \frac{1}{2}T_L^2. \tag{4.2}$$

Similarly, we have

$$-\tilde{B}\hat{B} \le -\frac{1}{2}\tilde{B}^2 + \frac{1}{2}B^2,$$

$$-\tilde{J}\hat{J} \le -\frac{1}{2}\tilde{J}^2 + \frac{1}{2}J^2, \tag{4.3}$$

$$-\tilde{\theta}\hat{\theta} \le -\frac{1}{2}\tilde{\theta}^2 + \frac{1}{2}\theta^2.$$

Consequently, by using these inequalities, (4.1) can be rewritten in the following form:

$$\dot{V} \le -\sum_{i=1}^{3} k_i z_i^2 - \frac{m_1}{2r_1}\tilde{T}_L^2 - \frac{m_2}{2r_2}\tilde{B}^2 - \frac{m_3}{2r_3}\tilde{J}^2 - \frac{m_4}{2r_4}\tilde{\theta}^2$$

$$+ \sum_{i=2}^{3} \frac{1}{2}\left(l_i^2 + \varepsilon_i^2\right) + \frac{m_1}{2r_1}T_L^2 + \frac{m_2}{2r_2}B^2 + \frac{m_3}{2r_3}J^2 + \frac{m_4}{2r_4}\theta^2 \tag{4.4}$$

$$\le -a_0 V + b_0,$$

where $a_0 = \min\{2k_1/J, 2k_2, 2k_3, m_1, m_2, m_3, m_4\}$ and $b_0 = \sum_{i=2}^{3}(1/2)(l_i^2 + \varepsilon_i^2) + (m_1/2r_1)T_L^2 + (m_2/2r_2)B^2 + (m_3/2r_3)J^2 + (m_4/2r_4)\theta^2$. Furthermore, (4.4) implies that

$$V(t) \leq \left(V(t_0) - \frac{b_0}{a_0}\right)e^{-a_0(t-t_0)} + \frac{b_0}{a_0} \leq V(t_0) + \frac{b_0}{a_0}, \quad \forall t \geqslant t_0. \tag{4.5}$$

As a result, all $z_i(i = 1,2,3)$, $\tilde{T}_L, \tilde{B}, \tilde{J}$ and $\tilde{\theta}$ belong to the compact set

$$\Omega = \left\{\left(z_i, \tilde{T}_L, \tilde{B}, \tilde{J}, \tilde{\theta}\right) \mid V \leq V(t_0) + \frac{b_0}{a_0}, \forall t \geqslant t_0\right\}. \tag{4.6}$$

Namely, all the signals in the closed-loop system are bounded. Especially, from (4.5) we have

$$\lim_{t \to \infty} z_1^2 \leq \frac{2b_0}{a_0}. \tag{4.7}$$

From the definitions of a_0 and b_0, it is clear that to get a small tracking error we can take r_i large and l_i and ε_i small enough after giving the parameters k_i and m_i.

5. Simulation

To illustrate the effectiveness of the proposed results, the simulation will be done for the PMSM motor with the parameters:

$$J = 0.00379\,\text{Kgm}^2, \qquad R_s = 0.68\,\Omega, \qquad L_d = 0.00315\,\text{H},$$

$$L_q = 0.00285\,\text{H}, \qquad B = 0.001158\,\text{Nm}/(\text{rad}/\text{s}), \qquad \Phi = 0.1245\,\text{Wb}, \qquad n_p = 3. \tag{5.1}$$

Then, the proposed adaptive fuzzy controllers are used to control this PMSM motor. Given the reference signal is $x_d = 30$ and the control parameters are chosen as follows:

$$k_1 = 2.5, \qquad k_2 = k_3 = 50, \qquad r_1 = r_2 = r_3 = r_4 = 2.5,$$

$$m_1 = m_2 = m_3 = m_4 = 0.0005, \qquad l_2 = l_3 = 5. \tag{5.2}$$
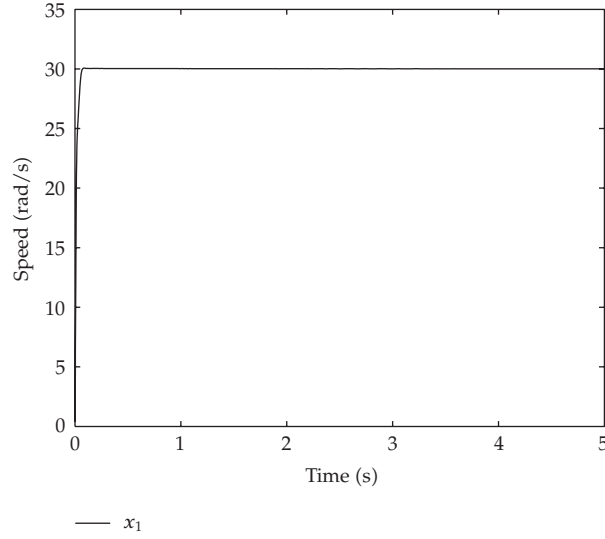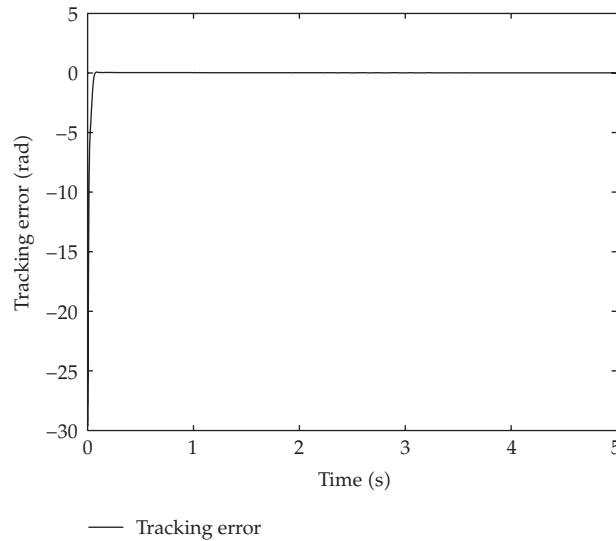
Figure 1: The curve of the rotor speed x_1.
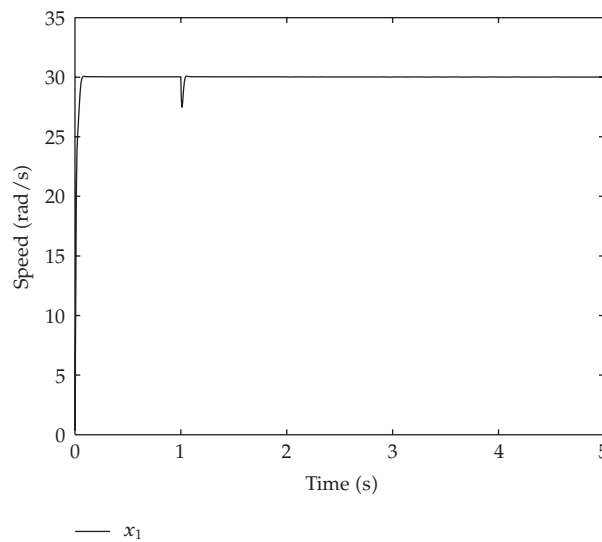
The fuzzy membership functions are chosen as

$$\mu_{F_i^1} = \exp\left[\frac{-(x+5)^2}{2}\right], \qquad \mu_{F_i^2} = \exp\left[\frac{-(x+4)^2}{2}\right],$$

$$\mu_{F_i^3} = \exp\left[\frac{-(x+3)^2}{2}\right], \qquad \mu_{F_i^4} = \exp\left[\frac{-(x+2)^2}{2}\right],$$

$$\mu_{F_i^5} = \exp\left[\frac{-(x+1)^2}{2}\right], \qquad \mu_{F_i^6} = \exp\left[\frac{-(x-0)^2}{2}\right],$$

$$\mu_{F_i^7} = \exp\left[\frac{-(x-1)^2}{2}\right], \qquad \mu_{F_i^8} = \exp\left[\frac{-(x-2)^2}{2}\right], \qquad (5.3)$$

$$\mu_{F_i^9} = \exp\left[\frac{-(x-3)^2}{2}\right], \qquad \mu_{F_i^{10}} = \exp\left[\frac{-(x-4)^2}{2}\right],$$

$$\mu_{F_i^{11}} = \exp\left[\frac{-(x-5)^2}{2}\right].$$

The simulation is carried out under the zero initial condition for two cases. In the first case, $T_L = 1.5$ and in the second case,

$$T_L = \begin{cases} 1.5, & 0 \le t \le 1, \\ 3, & t \ge 1. \end{cases} \qquad (5.4)$$
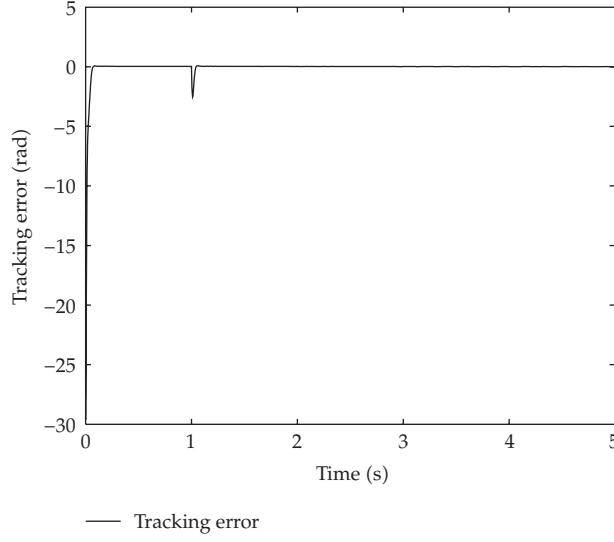
Figure 2: The curve of the speed tracking error.

Figure 3: The curve of the rotor position x_1.

Figures 1 and 2 show the simulation results for case 1, and Figures 3 and 4 show the simulation results for the second case. From these figures, it is seen clearly that the tracking performance has been achieved very well. This means that the proposed controller can track the reference signal satisfactorily even under parameter uncertainties and load torque disturbance.

6. Conclusion

Based on adaptive fuzzy control approach and backstepping technique, an adaptive fuzzy control scheme is proposed to control a permanent magnet synchronous motor. The proposed

Figure 4: The curve of the speed tracking error.

controllers guarantee that the tracking error converges to a small neighborhood of the origin and all the closed-loop signals are bounded. Simulation results illustrate the effectiveness of the presented method.

Appendix

The Controller of PMSM Based on Conventional Backstepping

The controller of PMSM based on conventional backstepping technique is briefly reviewed here. The detailed design procedure is in [7]. The controllers based on conventional backstepping are shown as follows:

$$
u_q = \frac{1}{b_4}\left(-k_2 z_2 + b_4\left(\frac{\dot{\widehat{J}}}{a_1}\left(-k_1 z_1 + \widehat{F} x_1 + \widehat{\Gamma} + \dot{x}_d\right) + \frac{\widehat{J}}{a_1}\left(\dot{\widehat{F}} x_1 + \dot{\widehat{\Gamma}} + \ddot{x}_d + k_1 \dot{x}_d\right)\right.\right.
$$
$$
\left.\left. + \left(\widehat{F} - k_1\right)\left(x_2 + \frac{a_2}{a_1} x_2 x_3\right) - \frac{\widehat{J}}{a_1}\left(\widehat{F} - k_1\right)\left(\widehat{F} x_1 + \widehat{\Gamma}\right) - (b_1 x_2 + b_2 x_1 x_3 + b_3 x_1)\right)\right),
$$
$$
u_d = \frac{1}{c_3}\left(-k_3 z_3 - c_1 x_3 - c_2 x_1 x_2 - \frac{a_2}{\widehat{J}} c_3 z_1 x_2\right).
$$

$$(A.1)$$

Comparing the presented method with the above controller designed via conventional back-stepping, it can be seen that the proposed controllers based on adaptive fuzzy backstepping have a very simple structure. And the simulation results illustrate the effectiveness of the presented method in this paper.

Acknowledgments

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