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Research Article

New Nonlinear Controller for a Class of Chaotic Systems Based on Adaptive Backstepping Fuzzy-Immune Control

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An adaptive backstepping fuzzy-immune controller for a class of chaotic systems is proposed. An adaptive backstepping fuzzy method and adaptive laws are used to approximate nonlinear functions and the unknown upper bounds of uncertainty, respectively. The proposed adaptive backstepping fuzzy-immune controller guarantees the stability of a class of chaotic systems while maintaining good tracking performance. The fuzzy-immune algorithm is used for mathematical calculations. The intelligence algorithm consists of the adaptive backstepping fuzzy method and a novel fuzzy-immune scheme which generates optimal parameters for the control schemes. Finally, two simulation examples are given to illustrate the effectiveness of the proposed approach.

1. Introduction

Adaptive fuzzy logic controllers provide a systematic and efficient framework for incorporating linguistic fuzzy information from human experts. In [1], an adaptive fuzzy logic control theory was derived for a class of uncertain nonlinear single-input single-output (SISO) systems. Moreover, many scientists have since dedicated a lot of effort to solving the adaptive fuzzy control problem of uncertain nonlinear systems [2–10]. Furthermore, the stability of uncertain nonlinear systems has been addressed by the integration of fuzzy logic control and the adaptive laws. [11–20]. Subsequently, several methodologies have been instituted for controlling nonlinear systems [20–29]. The primary advantage of adaptive fuzzy control scheme is insensitive to internal uncertainty and external disturbances. Adaptive fuzzy control approaches only can perform desired performance for a simple class of nonlinear systems. If nonlinear systems without satisfying the matching conditions, the adaptive fuzzy control methodologies cannot be implemented.

In the past decade, many adaptive fuzzy control schemes have been developed by combining the backstepping technique [30–32]. The primary advantage of adaptive backstepping fuzzy control is that the matching conditions are not needed. Backstepping is based on the nonlinear stabilization technique of adding an integrator. Adaptive backstepping fuzzy control schemes can provide a systematic framework for tracking or regulation strategies [33–40].

In the past decade, the research area of controlling chaos has received increasing attention. Chaos is a complex nonlinear dynamical system, and it is commonly difficult to exactly predict the behavior of a chaotic system. Recently, many successful methods for controlling chaos have been developed [3, 8]. In the present study, we propose an adaptive backstepping fuzzy-immune controller for a class of chaotic systems. Based on the backstepping algorithm, the fuzzy methodology augmented by an immune algorithm is proposed as a new evolution algorithm, which maintains the advantages of simplicity and easy handling. The four main contributions are (1) an adaptive backstepping fuzzy-immune tracking controller for a class of chaotic systems is proposed, (2) the controller does not require a priori knowledge of the sign of the control coefficient, (3) a novel fuzzy-immune algorithm is used to find the optimal solution, and (4) a correct term can be used to eliminate disturbance.

The rest of this paper is organized as follows. In Section 2, system statement and description of fuzzy systems for chaotic system are presented. The adaptive backstepping fuzzy controller technique and a novel fuzzy-immune mechanism are discussed in Section 3. The results of simulations for chaotic systems are presented to confirm the validity of the proposed control scheme in Section 4. Finally, the conclusions are given in Section 5.

2. System Statement and Description of Fuzzy Systems for Chaotic System

In this paper, we consider a class of chaotic systems that can be shown in strict-feedback systems with nonlinear functions and disturbances

$$\begin{aligned}
\dot{x}_1 &= x_2 + d_1(t),\\
\dot{x}_2 &= x_3 + d_2(t),\\
&\ \vdots\\
\dot{x}_n &= f_n(\overline{x}_n(t)) + g_n(\overline{x}_n(t))u(t) + d_n(t),\\
y &= x_1,
\end{aligned} \tag{2.1}$$

where $\overline{x}_n(t) = [x_1(t), \dots, x_n(t)]^T \in R^n$, $f_n(\overline{x}_n(t))$ and $g_n(\overline{x}_n(t))$ are smooth functions, $u(t)$ and y are control input and output variables, respectively. $d_1(t), \dots, d_n(t)$ denote external disturbance. However, the bound of external disturbance is difficult to obtain in the practical applications. The control objective is to design a stabilizing controller for the system described by (2.1) so that the tracking error converges to zero asymptotically despite the presence of unknown nonlinearities and disturbances.

2.1. Description of Fuzzy Systems

Fuzzy logic systems have been successfully employed to approximate the mathematical models of nonlinear systems. The fuzzy systems can be divided into four parts: fuzzifier, fuzzy rule base, fuzzy inference engine, and defuzzifier. The fuzzy mechanism is described by IF-THEN rules from an input linguistic vector $\overline{x}_n(t)$ to an output variable $f(\overline{x}_n(t))$

$$\begin{aligned} R^i\text{:}\quad & \text{If } x_1 \text{ is } F_{i1},\quad x_2 \text{ is } F_{i2},\ \dots,\ x_n \text{ is } F_{in}, \\ & \text{then } y \text{ is } G_i,\quad i = 1, 2, \dots, m, \end{aligned} \tag{2.2}$$

where m is the number of rules. $F_{i1}, F_{i2}, \dots, F_{in}$ and G_i are the fuzzy sets and $x_1, x_2, \dots, x_n$ are states of system. Using a singleton function, center average defuzzification, and product inference, the fuzzy systems output is

$$\overline{f}(\overline{x}_n) = \frac{\sum_{i=1}^{m} \overline{y}_i \left(\prod_{l=1}^{n} \mu_{F_{il}}(x_l)\right)}{\sum_{i=1}^{m} \left(\prod_{l=1}^{n} \mu_{F_{il}}(x_l)\right)}, \tag{2.3}$$

where $\mu_{F_{il}}(x_l)$ is the membership of x_1 in the fuzzy set F_{i1} and $\overline{y}_i = \max_{y\in R} \mu_{G_i}(\overline{y}_i) = 1$. Then, (2.3) can be rewritten as

$$\overline{f}(\overline{x}_n) = \theta_f^T \varphi_f(\overline{x}_n), \tag{2.4}$$

where $\theta_f = [\overline{y}_1, \overline{y}_2, \dots, \overline{y}_m]^T$ and $\varphi_f(\overline{x}_n(t)) = [\varphi_{11}, \dots, \varphi_{1m}]^T$. We can define φ_{1i} as

$$\varphi_{1i} = \frac{\left(\prod_{l=1}^{n} \mu_{F_{il}}(x_l)\right)}{\sum_{i=1}^{m} \left(\prod_{l=1}^{n} \mu_{F_{il}}(x_l)\right)}, \quad i = 1, \dots, m. \tag{2.5}$$

The logic fuzzy system shown in (2.5) is a universal approximator. It can be proved using the following lemma.

Lemma 2.1 (see [41]). *Let $f(\overline{x}_n)$ be continuous functions defined on a compact set $U \in R^n$ and arbitrary $\varepsilon > 0$, and there exists a fuzzy logic system $\overline{f}(\overline{x}_n)$ in the form of* (2.4) *such that*

$$\sup_{x\in U} \left| \overline{f}(\overline{x}_n) - f_n(\overline{x}_n) \right| \leq \varepsilon. \tag{2.6}$$

After some simple manipulations in (2.4) and (2.5), we can obtain $\overline{g}_n(\overline{x}_n) = \theta_g^T \varphi_g(\overline{x}_n)$ as the approximator of $g_n(\overline{x}_n)$. The nonlinear functions $f_n(\overline{x}_n)$ and $g_n(\overline{x}_n)$ requires successful estimates $\widehat{\theta}_f$ and $\widehat{\theta}_g$ in order to perform the performance shown in (2.6).

Typically, there exists optimal parameter estimates $\overline{\theta}$, and the approximation error is the smallest. The optimal parameter estimate is defined as

$$\overline{\theta}_f = \arg\min_{\theta_f \in \Omega_f} \left\{ \sup_{\overline{x} \in \Omega_{\overline{x}}} \left| f_n(\overline{x}_n) - \theta_f^T \varphi_f \right| \right\},$$
$$\overline{\theta}_g = \arg\min_{\theta_g \in \Omega_g} \left\{ \sup_{\overline{x} \in \Omega_{\overline{x}}} \left| g_n(\overline{x}_n) - \theta_g^T \varphi_g \right| \right\}. \tag{2.7}$$

Based on fuzzy mechanism, (2.1) can be rewritten as below:

$$\begin{aligned} \dot{x}_1 &= x_2 + d_1(t), \\ \dot{x}_2 &= x_3 + d_2(t), \\ &\vdots \\ \dot{x}_n &= \overline{\theta}_f^T \varphi_f(\overline{x}_n) + \overline{\theta}_g^T \varphi_g(\overline{x}_n) u(t) + d_n(t) + \varepsilon_f + \varepsilon_g, \\ y &= x_1, \end{aligned} \tag{2.8}$$

where ε_f and ε_g are internal modeling error variables

$$\begin{aligned} \varepsilon_f &= f_n(\overline{x}_n) - \overline{\theta}_f^T \varphi_f(\overline{x}_n), \\ \varepsilon_g &= g_n(\overline{x}_n) - \overline{\theta}_g^T \varphi_g(\overline{x}_n), \end{aligned} \tag{2.9}$$

Therefore, the mathematical model includes internal modeling error variables and external disturbance. We will discuss our proposed method in the next section.

3. Adaptive Backstepping Fuzzy Controller Technique and Fuzzy-Immune Mechanism

$f_n(\overline{x}_n(t))$ and $g_n(\overline{x}_n(t))$ are the system dynamic functions, these cannot be exactly obtained in general, $d_1(t), \ldots, d_n(t)$ are unknown parameters in practical application Thus, in the adaptive backstepping fuzzy controller (ABFC) system, the fuzzy system is designed to estimate the system dynamic functions.

3.1. Backstepping Design Principle

The design of ABFC for the chaotic dynamic system is described step-by-step as follows:

Step 1. Consider the tracking error

$$\dot{e}_1 = \dot{x}_1 - \dot{r} = x_2 - \dot{r} + d_1 = e_2 + a_1 + d_1, \tag{3.1}$$

where r is the command trajectory. The first Lyapunov function is defined as

$$V_1 = \frac{e_1^2}{2}. \tag{3.2}$$

Differentiating (3.2) with respect to time and it is obtained that

$$\dot{V}_1 = e_1\dot{e}_1 = e_1(e_2 + a_1 + d_1) = e_1e_2 + e_1(a_1 + d_1). \tag{3.3}$$

Define the following stabilizing function:

$$a_1 = -\tau_1 e_1 - \frac{1}{\eta_1^2}e_1, \tag{3.4}$$

where τ_1 and η_1 are positive constants, η_1 represent the attenuation level of disturbances. From substitute (3.3) into (3.4), then we obtain

$$\dot{V}_1 = -\tau_1 e_1^2 + e_1e_2 + \mu_1, \tag{3.5}$$

where $\mu_1 = -(1/\eta_1^2)e_1 + e_1d_1 = -((1/\eta_1)e_1 - (1/2)\eta_1 d_1)^2 + (1/4)\eta_1^2 d_1^2$.

Step 2. The derivative of $e_2 = x_2 - \dot{r} - a_1$ is

$$\dot{e}_2 = \dot{x}_3 + d_2 - \ddot{r} - \frac{\partial a_1(x_2 + d_1)}{\partial x_1} - \frac{\partial a_1\dot{r}}{\partial r_1} = e_3 + a_2 - \frac{\partial a_1 x_2}{\partial x_1} - \frac{\partial a_1\dot{r}}{\partial r_1} + \left(d_2 - \frac{\partial a_1 d_1}{\partial x_1}\right). \tag{3.6}$$

The second Lyapunov function is defined as

$$V_2 = V_1 + \frac{e_2^2}{2}. \tag{3.7}$$

Differentiating (3.7) with respect to time, it is obtained that

$$\dot{V}_2 = e_2e_3 - \tau_1 e_1^2 + e_2\left(e_1 + a_2 - \frac{\partial a_1 x_2}{\partial x_1} - \frac{\partial a_1\dot{r}}{\partial r_1}\right) + \mu_1 + e_2d_2 - e_2\frac{\partial a_1 d_1}{\partial x_1}. \tag{3.8}$$

Define the following stabilizing function:

$$a_2 = -e_1 - \tau_2 e_2 - \frac{1}{\eta_2^2}e_2 + \frac{\partial a_1}{\partial x_1}x_2 + \frac{\partial a_1}{\partial r}\dot{r}, \tag{3.9}$$

where τ_2 and η_2 are positive constants and η_2 represent the attenuation level of disturbances.

From substitute (3.9) into (3.8), then we obtain

$$\dot{V}_2 = -\tau_1 e_1^2 - \tau_2 e_2^2 + e_2e_3 + \mu_1 + \mu_2, \tag{3.10}$$

where

$$\mu_2 = -\frac{1}{\eta_2^2}e_2^2 + e_2\left(d_2 - \frac{\partial a_1}{\partial x_1}d_1\right) = -\left[\frac{1}{\eta_2}e_2 - \frac{1}{2}\eta_2\left(d_2 - \frac{\partial a_1}{\partial x_1}d_1\right)\right]^2 + \frac{1}{4}\eta_2^2\left(d_2 - \frac{\partial a_1}{\partial x_1}d_1\right)^2. \tag{3.11}$$

Step i

After some simple manipulations and the same method, the derivative of generalized error is

$$\dot{e}_i = x_{i+1} + \left[d_i - \sum_{l=1}^{i-1}\left(\frac{\partial a_{i-1}}{\partial x_l}\right)d_l\right] - \frac{d^i r}{dr} - \left(\sum_{l=1}^{i-1}\frac{\partial a_{i-1}x_{l+1}}{\partial x_l} + \sum_{l=1}^{i-1}\frac{\partial a_{i-1}}{\partial r^{(l-1)}}r^l\right). \tag{3.12}$$

The ith Lyapunov function is defined as

$$V_i = V_{i-1} + \frac{e_i^2}{2}. \tag{3.13}$$

Differentiating (3.13) with respect to time, it is obtained that

$$\dot{V}_i = e_i e_{i+1} - \sum_{l=1}^{i}\tau_l e_l^2 + \sum_{l=1}^{i}\eta_l, \tag{3.14}$$

where

$$\mu_l = -\left[\frac{1}{\eta_l}e_l - \frac{1}{2}\eta_l\left(d_l - \sum_{k=1}^{l-1}\frac{\partial a_{l-1}}{\partial x_k}d_k\right)\right]^2 + \frac{1}{4}\eta_l^2\left(d_l - \sum_{k=1}^{l-1}\frac{\partial a_{l-1}}{\partial x_k}d_k\right)^2. \tag{3.15}$$

Define the following stabilizing function:

$$a_i = -e_{i-1} - \tau_i e_i - \frac{1}{\eta_i^2}e_i + \sum_{l=1}^{i-1}\frac{\partial a_{i-1}}{\partial x_l}x_{l+1} + \sum_{l=1}^{i-1}\frac{\partial a_{i-1}}{\partial r^{(l-1)}}r^l. \tag{3.16}$$

Step n

The final control law will be determined in the final step, and the derivative of step generalized error can be described as

$$\begin{aligned}
\dot{e}_n &= \dot{x}_n - \frac{d^n r}{dr} - \left(\sum_{l=1}^{n-1} \frac{\partial a_{n-1} x_{l+1}}{\partial x_l} + \sum_{l=1}^{n-1} \frac{\partial a_{n-1}}{\partial r^{(l-1)}} r^l\right) \\
&= \overline{\theta}_f^T \varphi_f(\overline{x}_n) + \overline{\theta}_g^T \varphi_g(\overline{x}_n) u - \frac{d^n r}{dr} - \left(\sum_{l=1}^{n-1} \frac{\partial a_{n-1} x_{l+1}}{\partial x_l} + \sum_{l=1}^{n-1} \frac{\partial a_{n-1}}{\partial r^{(l-1)}} r^l\right) \\
&\quad + d_n - \sum_{l=1}^{n-1} \left(\frac{\partial a_{n-1}}{\partial x_l}\right) d_l + \varepsilon_f + \varepsilon_g u,
\end{aligned} \tag{3.17}$$

$$u = \frac{1}{\widehat{\theta}_g^T \varphi_g} \left(a_n + \frac{d^n r}{dr}\right), \quad \widehat{\theta}_g^T \varphi_g \neq 0, \tag{3.18}$$

$$a_n = -e_{n-1} - \tau_n e_n - \frac{1}{\eta_n^2} e_n - \widehat{\theta}_f^T \varphi_f(\overline{x}_n) + \sum_{l=1}^{n-1} \frac{\partial a_{n-1}}{\partial x_l} x_{l+1} + \sum_{l=1}^{n-1} \frac{\partial a_{n-1}}{\partial r^{l-1}} r^l, \tag{3.19}$$

where η_n is the given correct factor and τ_n is a positive constant.

3.2. *Fuzzy-Immune Mechanism Design*

Assume that the number of the kth-generation antigens is $\varepsilon(k)$, the output of the helper T-cells, stimulated by antigens, is $T_H(k)$, and the suppressor T-cells affect B-cells to the amount of $T_s(k)$ [42, 43]

$$T_e(k) = T_H(k) - T_s(k), \tag{3.20}$$

where $T_H(k) = h_1 \varepsilon(k)$ and $T_s(k) = h_2 f(T_e(k), \Delta T_e(k)) \varepsilon(k)$. The feedback control rules are defined as

$$\begin{aligned}
u(k) &= h_1 e(k) - h_2 f(u(k), \Delta u(k)) e(k) \\
&= h_1 \left[1 - \frac{h_2}{h_1} f(u(k), \Delta u(k))\right] e(k) \\
&= h_1 \left[1 - \hbar f(u(k), \Delta u(k))\right] e(k),
\end{aligned} \tag{3.21}$$

where h_1, h_2, and λ are scaling factors and $\hbar = h_2/h_1$ is utilized to control the stabilization effect. Then, we propose a novel correct constriction coefficient to improve the performance of the immune mechanism. In this, the constriction coefficient can be expressed as follows:

$$\hbar = \hbar_{\max} - \frac{\hbar_{\max} - \hbar_{\min}}{k_{\max}} \times k_{\text{now}}, \tag{3.22}$$

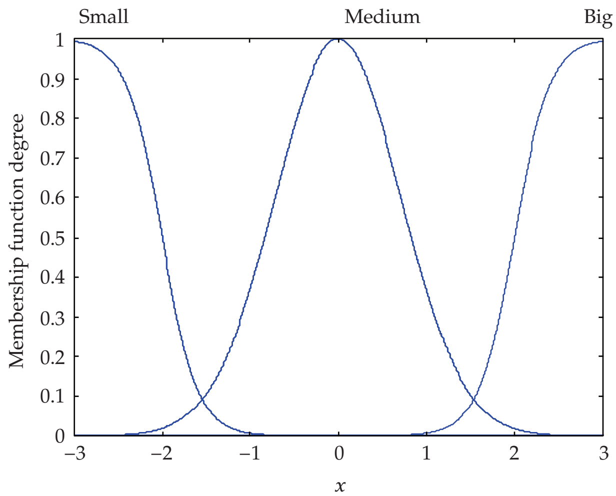

Figure 1: Membership functions for $u(k)$.

where $\hbar_{\max}$ and $\hbar_{\min}$ denote the maximum and minimum of $\hbar$, respectively. $k_{\max}$ denotes the total number of evolution generations, and k_{now} denotes the current number of evolution generations. Let $\text{de}(k)$ denote the difference between the $e(k)$'s for two consecutive iterations, that is, at iteration k:

$$\text{de}(k) = e(k) - e(k-1) \geq 0. \tag{3.23}$$

Based on the above inferences, the following nine fuzzy rules are suggested.

(1) If $u(k)$ is big and $\Delta u(k)$ is big, then $f(u(k), \Delta u(k))$ is small.

(2) If $u(k)$ is big and $\Delta u(k)$ is small, then $f(u(k), \Delta u(k))$ is medium.

(3) If $u(k)$ is small and $\Delta u(k)$ is big, then $f(u(k), \Delta u(k))$ is medium.

(4) If $u(k)$ is medium and $\Delta u(k)$ is big, then$f(u(k), \Delta u(k))$ is small.

(5) If $u(k)$ is big and $\Delta u(k)$ is medium, then$f(u(k), \Delta u(k))$ is small.

(6) If $u(k)$ is medium and $\Delta u(k)$ is medium, then $f(u(k), \Delta u(k))$ is medium.

(7) If $u(k)$ is medium and $\Delta u(k)$ is small, then $f(u(k), \Delta u(k))$ is medium.

(8) If $u(k)$ is small and $\Delta u(k)$ is medium, then $f(u(k), \Delta u(k))$ is big.

(9) If $u(k)$ is small and $\Delta u(k)$ is small, then $f(u(k), \Delta u(k))$ is big.

The membership functions for $u(k)$, $\Delta u(k)$, and $f(u(k), \Delta u(k))$ are shown in Figures 1–3, respectively.

The output of the controller has the following expression:

$$u = \frac{1}{\widehat{\theta}_g^T \varphi_g} \left(a_n + \frac{d^n r}{dr} \right) \left\{ h_1 \left[1 - \hbar f(u(k), \Delta u(k)) \right] e_n \right\}. \tag{3.24}$$

The proposed method is trying to find more efficient ways of utilizing immune mechanism to correct controller input, and the correct item is effective in keeping out

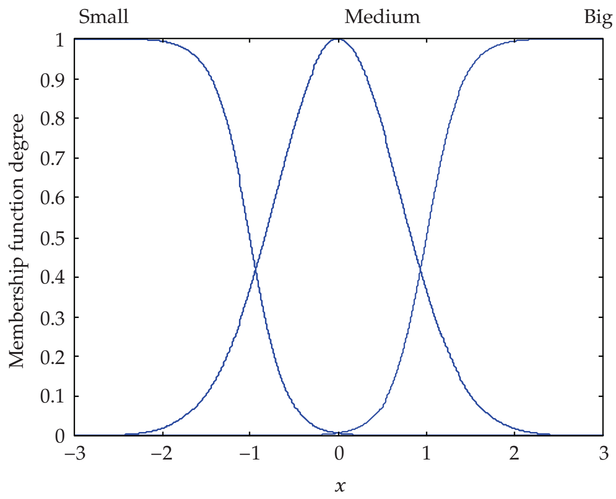

Figure 2: Membership functions for $\Delta u(k)$.

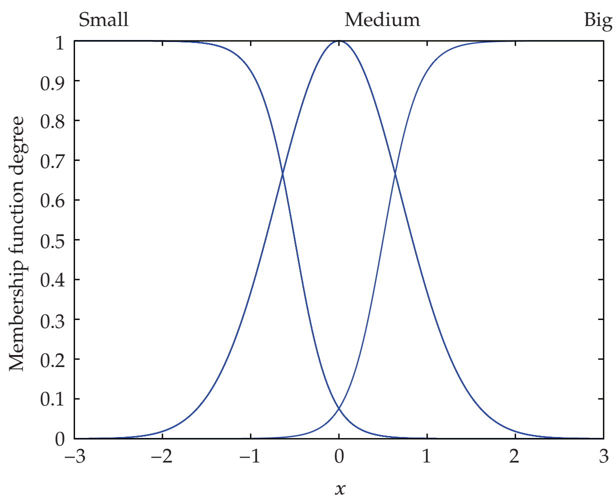

Figure 3: Membership functions for $f(u(k), \Delta u(k))$.

perturbation by external or internal. Then, we have the following theorem to gain the objective.

Theorem 3.1. *Consider the nonlinear systems* (2.1) *and with the controller* u *is given by* (3.18)*. By utilizing parameter adjusting law* (3.25) *and* (3.26)

$$\dot{\hat{\theta}}_f = P^{-1}\varphi_f(\overline{x}_n)e_n, \tag{3.25}$$

$$\dot{\hat{\theta}}_g = Q^{-1}\varphi_g(\overline{x}_n)e_n u, \tag{3.26}$$

then the proposed adaptive backstepping fuzzy-immune control scheme can guarantee the following properties:

(1) *The asymptotic stability of the system is guaranteed.*

(2) *The tracking error can be described as*

$$\sum_{l=1}^{n}\int_{0}^{T}\left(-\tau_l e_l^2\right)dt \le V(0)+\sum_{l=1}^{n}\int_{0}^{T}\left(\frac{1}{4}\eta_l^2\left(d_l-\sum_{k=1}^{l-1}\frac{\partial a_{l-1}}{\partial x_k}d_k\right)^2\right)dt. \tag{3.27}$$

Proof. Firstly, consideration of fuzzy approximating error, the Lyapunov functional is set as (3.28)

$$V=V_{n-1}+\frac{1}{2}e_n^2+\frac{1}{2}\tilde{\theta}_f^T P\tilde{\theta}_f+\frac{1}{2}\tilde{\theta}_g^T Q\tilde{\theta}_g, \tag{3.28}$$

where P and Q are positive symmetric matrices.

Differentiating (3.28) with respect to time, it is obtained that

$$\begin{aligned}\dot{V}&=\dot{V}_{n-1}+e_n\dot{e}_n+\tilde{\theta}_f^T P^{-1}\dot{\tilde{\theta}}_f+\tilde{\theta}_g^T Q^{-1}\dot{\tilde{\theta}}_g\\&=e_{n-1}e_n+e_n\left(\hat{\theta}_g^T\varphi_g(\overline{x}_n)u-\frac{d^n r}{dr}-a_n\right)+\sum_{l=1}^{n-1}\left(\mu_l-\tau_l e_l^2\right)\\&\quad+\tilde{\theta}_f^T P\left(P^{-1}\varphi_f(\overline{x}_n)e_n-\dot{\hat{\theta}}_f\right)+\tilde{\theta}_g^T Q\left(Q^{-1}\varphi_g(\overline{x}_n)e_n u-\dot{\hat{\theta}}_f\right)\\&\quad+e_n\left(\left(\overline{\theta}_f^T\varphi_f(\overline{x}_n)+a_n-\left(\sum_{l=1}^{n-1}\frac{\partial a_{n-1}x_{l+1}}{\partial x_l}+\sum_{l=1}^{n-1}\frac{\partial a_{n-1}}{\partial r^{(l-1)}}r^l+d_n-\sum_{l=1}^{n-1}\left(\frac{\partial a_{n-1}}{\partial x_l}\right)d_l+\varepsilon_f+\varepsilon_g u\right)\right)\right).\end{aligned} \tag{3.29}$$

Define

$$\dot{V}_n=\dot{V}_{n-1}+e_n\left\{\overline{\theta}_f^T\varphi_f(\overline{x}_n)+a_n-\left(\sum_{l=1}^{n-1}\frac{\partial a_{n-1}x_{l+1}}{\partial x_l}+\sum_{l=1}^{n-1}\frac{\partial a_{n-1}}{\partial r^{(l-1)}}r^l+d_n-\sum_{l=1}^{n-1}\left(\frac{\partial a_{n-1}}{\partial x_l}\right)d_l+\varepsilon_f+\varepsilon_g u\right)\right\}. \tag{3.30}$$

Equation (3.29) can be rewritten as

$$\dot{V}=\dot{V}_n+e_n\left(\hat{\theta}_g^T\varphi_g(\overline{x}_n)u-\frac{d^n r}{dr}-a_n\right)+\tilde{\theta}_f^T P\left(P^{-1}\varphi_f(\overline{x}_n)e_n-\dot{\hat{\theta}}_f\right)+\tilde{\theta}_g^T Q\left(Q^{-1}\varphi_g(\overline{x}_n)e_n u-\dot{\hat{\theta}}_g\right), \tag{3.31}$$

Define $\dot{V}=-\sum_{l=1}^{n}\tau_l e_l^2+\sum_{l=1}^{n}\mu_l$.

By using (3.18) and adaptive laws (3.25)-(3.26), (3.31) can be written as

$$\dot{V} = -\sum_{l=1}^{n}\tau_l e_l^2 + \sum_{l=1}^{n}\mu_l = \sum_{l=1}^{n}\left\{-\left[\frac{1}{\eta_l}e_l - \frac{1}{2}\eta_l\left(d_l - \sum_{k=1}^{l-1}\frac{\partial a_{l-1}}{\partial x_k}d_k\right)\right]^2 + \frac{1}{4}\eta_l^2\left(d_l - \sum_{k=1}^{l-1}\frac{\partial a_{l-1}}{\partial x_k}d_k\right)^2\right\} - \sum_{l=1}^{n}\tau_l e_l^2. \tag{3.32}$$

Integrating (3.32) from $t = 0$ to T yields

$$V(T) - V(0) \leq \sum_{l=1}^{n}\int_0^T\left(-\tau_l e_l^2 + \frac{1}{4}\eta_l^2\left(d_l - \sum_{k=1}^{l-1}\frac{\partial a_{l-1}}{\partial x_k}d_k\right)^2\right)dt. \tag{3.33}$$

Furthermore, one can derive that

$$\sum_{l=1}^{n}\int_0^T\left(-\tau_l e_l^2\right)dt \leq V(0) + \sum_{l=1}^{n}\int_0^T\left(\frac{1}{4}\eta_l^2\left(d_l - \sum_{k=1}^{l-1}\frac{\partial a_{l-1}}{\partial x_k}d_k\right)^2\right)dt. \tag{3.34}$$

This completes the proof of the theorem. □

4. Illustrative Examples

In this section, two examples are provided to illustrate the usefulness of our method.

Example 4.1. Consider the following Duffing-Holmes chaotic system [40]:

$$\begin{aligned} \dot{x}_1 &= x_2 + d_1(t), \qquad d_1(t) = 0.1\sin t + 0.2\cos t, \\ \dot{x}_2 &= -ax_2 - bx_1^3 - cx_1 + q\cos(\omega t) + (1 + \cos x_1)u + d_2(t), \qquad d_2(t) = 0.1\cos t, \end{aligned} \tag{4.1}$$

where a, b, c and q are constants, ω is the frequency, $d_1(t)$ and $d_2(t)$ are external disturbance, and u is the control effort. For observing chaotic unpredictable behavior, the open-loop system behavior with $u = 0$, $d_1(t) = d_2(t) = 0$ was simulated with $a = 0.1$, $b = 1.0$, $c = 1$, $q = 12$, and $\omega = 1.0$. The phase plane plots from an initial condition point $(0, 0)$ are shown in Figure 4. The proposed controller is designed to force the system output to track the given desired trajectory $y_m = \sin t$. Now, choose $y = x_1$, and there exist external disturbances when the chaotic systems have the form of strict-feedback. It is assumed that the external disturbance $d_1(t) = 0.1\sin t + 0.2\cos t$ and $d_2(t) = 0.1\cos t$. By Lemma 2.1, we use using some fuzzy rules

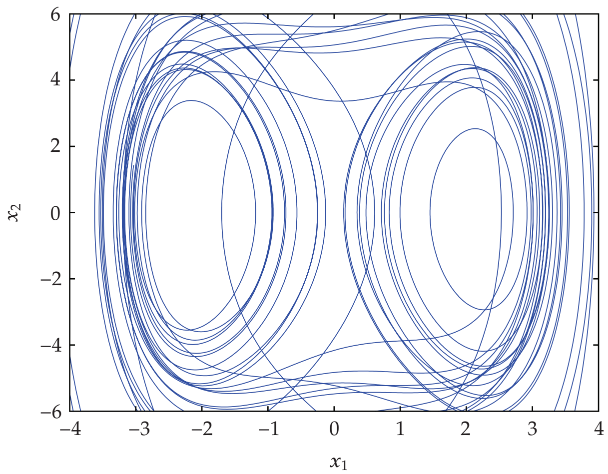

Figure 4: State trajectories of the Duffing equation.

for approximation of the function $f_n(\overline{x}_n(t))$ and $g_n(\overline{x}_n(t))$. The membership functions of the fuzzy sets are expressed as

$$\mu_{F_i^1}(x_i) = \frac{1}{1+\exp(5\times(x_i+2))},$$
$$\mu_{F_i^2}(x_i) = \exp\left[-(x_i+1)^2\right],$$
$$\mu_{F_i^3}(x_i) = \exp\left[-x_{\mathrm{i}}^2\right], \tag{4.2}$$
$$\mu_{F_i^4}(x_i) = \exp\left[-(x_i-1)^2\right],$$
$$\mu_{F_i^5}(x_i) = \frac{1}{1+\exp(5\times(x_i-2))}.$$

The initial membership functions for $u(k)$, $\Delta u(k)$, and $f(u(k), \Delta u(k))$ are shown in Figures 1–3, respectively. In order to evaluate the performance of the adaptive backstepping fuzzy-immune control applied to the above system (4.1), it was compared to fuzzy adaptive control (FAC) [40]. Figures 5 and 6 show the simulation curves of the two controls for x_1 and x_2, for the system described by (4.1) under the initial conditions $x(0) = [2, 2]$ with external disturbance $d_1(t) = 0.1\sin t + 0.2\cos t$ and $d_2(t) = 0.1\cos t$. In order to compare the stabilization and tracking performance, we consider the example introduced in [40]. FAC and the proposed method require 3.5~4.0 seconds and 2.5~3.0 seconds to track the reference signal, respectively. The proposed scheme can also suppress system uncertainty and disturbance, and its ISE (integral square error criterion) is lower than FAC. Figures 7 and 8 show x_1 and x_2 for the nonlinear system described by (4.1) under initial states $[-1.5, -1.5]$, $[5, -2]$, and $[-5, 2]$ with external disturbance $d_1(t) = 0.1\sin t + 0.2\cos t$ and $d_2(t) = 0.1\cos t$, respectively. Finally, all simulation results are given in Table 1.

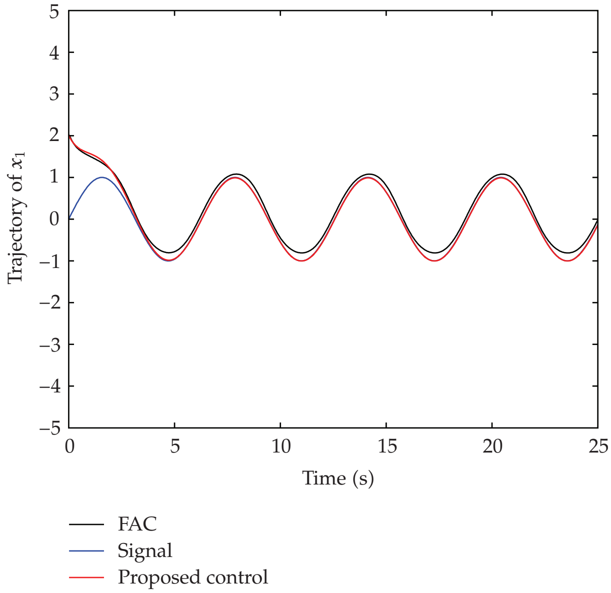

Figure 5: The tracking result for x_1.

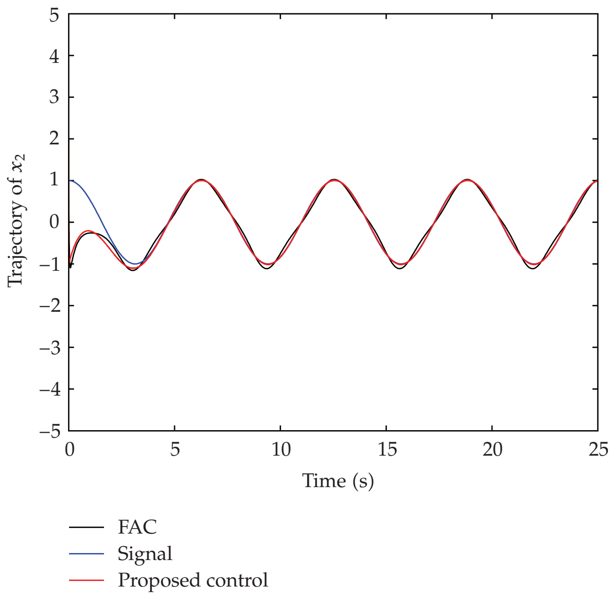

Figure 6: The tracking result for x_2.

Table 1: Experiments results for FAC and proposed control methods.

	Time (sec)	ISE	Performance
FAC [40]	3.5~4.0	0.30	Affected by noise
Proposed	2.5~3.0	0.05	Rejects noise

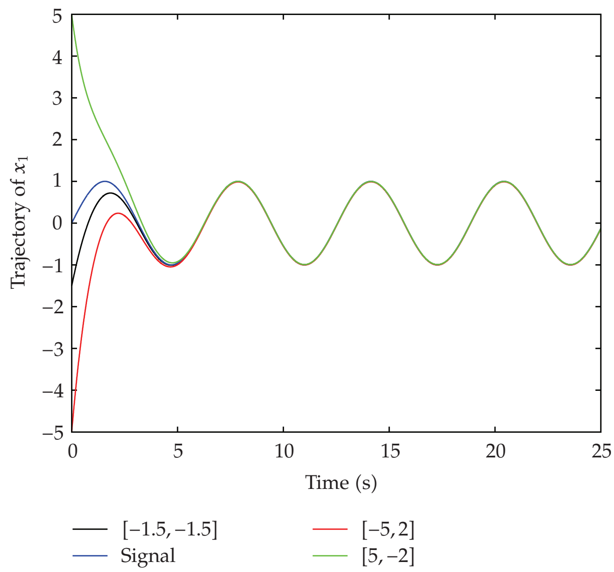

Figure 7: Trajectories of state x_1 under initial conditions.

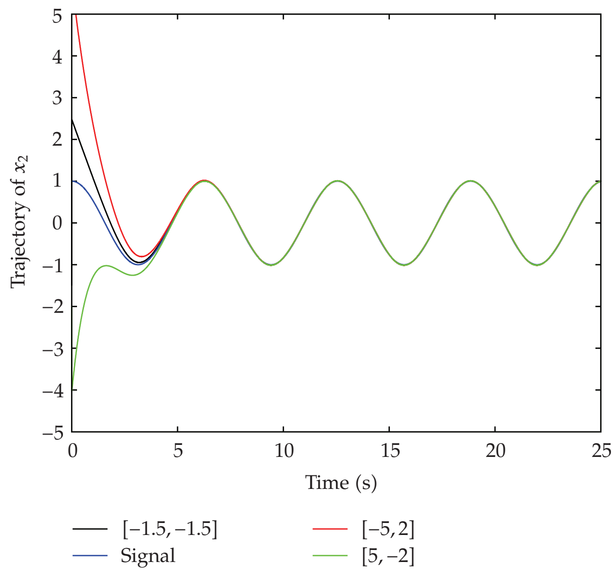

Figure 8: Trajectories of state x_2 under initial conditions.

Example 4.2. As a second example, consider the following forced chaotic attractor of the modified Chua circuit. The dynamics of the systems can be described as [44]

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \left[f + (1 + \sin(x_1) u(t)) \right] + \begin{bmatrix} d_1(t) \\ d_2(t) \\ d_3(t) \end{bmatrix}. \tag{4.3}$$

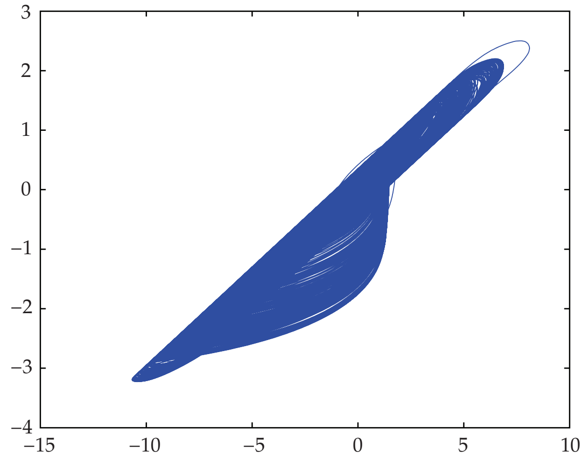

Figure 9: Chaotic attractor of the modified Chua circuit.

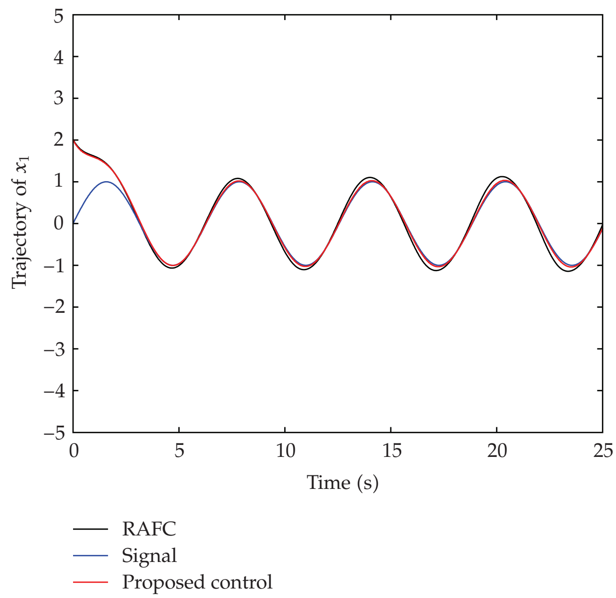

Figure 10: The tracking result for x_1.

In this simulation, let the sampling time equal 0.01 and the initial system states be $[2, 1, 3]$. Now, let $d_1(t)$, $d_2(t)$ and $d_3(t)$ are external disturbance. If $d_1(t) = d_2(t) = d_3(t) = u(t) = 0$ and $f = (14/1805)x_1 - (168/9025)x_2 + (1/38)x_3 - (2/45) \times ((28/321)x_1 + (7/95)x_2 + x_3)^3$, then the system (4.3) is chaotic system and the trajectories of the state variables x_1, x_2, and x_3 are shown in Figure 9. By Lemma 2.1, we use some fuzzy rules for approximation of the function $f_n(\overline{x}_n(t))$ and $g_n(\overline{x}_n(t))$. In order to control this chaotic system, the proposed control is utilized and the corresponding adaptation laws are also applied such that the system output y tracks a desired trajectory. In addition, let the desired output be $y_m = \sin(t)$.

Simulated results are demonstrated in Figures 10, 11, 12, 13, 14 and 15. The initial membership functions for $u(k)$, $\Delta u(k)$ and $f(u(k), \Delta u(k))$ are shown in Figures 1–3,

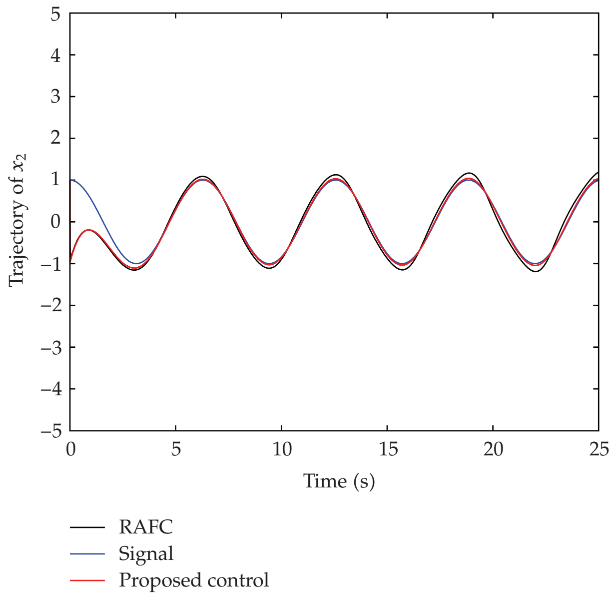

Figure 11: The tracking result for x_2.

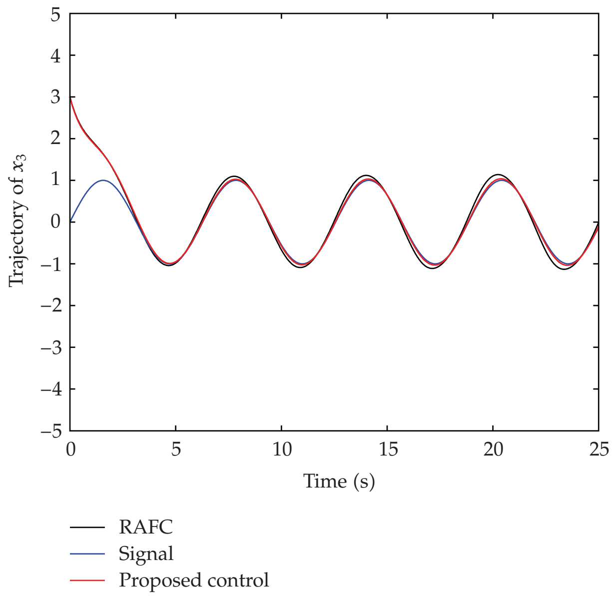

Figure 12: The tracking result for x_3.

respectively. In order to evaluate the performance of the adaptive backstepping fuzzy-immune control applied to the above system (4.3), it was compared to robust adaptive fuzzy controller (RAFC) [44]. Figures 10–12 show the simulation curves for the system described by (4.3) under the initial conditions $x(0) = [2, 1, 3]$ with external disturbance $d_1(t) = 0.1 \cos t + 0.4 \cos t$, $d_2(t) = 0.2 \sin t$ and $d_3(t) = 0.1 \sin t$. RFAC and the proposed method require 3.0~3.2 seconds and 2.5~3.0 seconds to track the reference signal, respectively. The

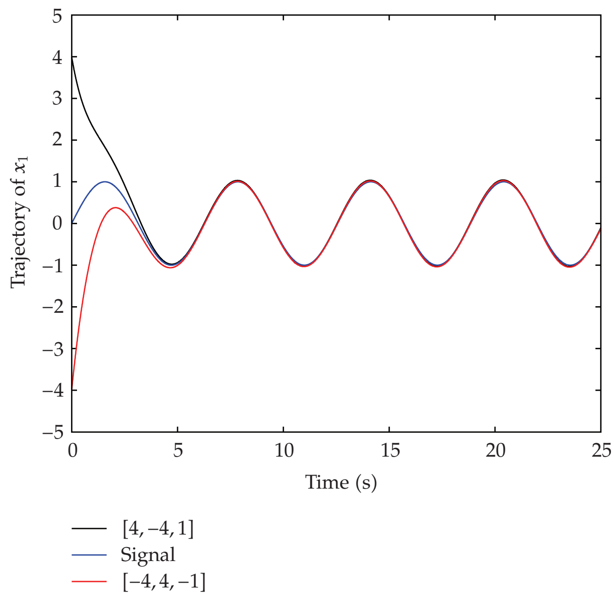

Figure 13: Trajectories of state x_1 under initial conditions.

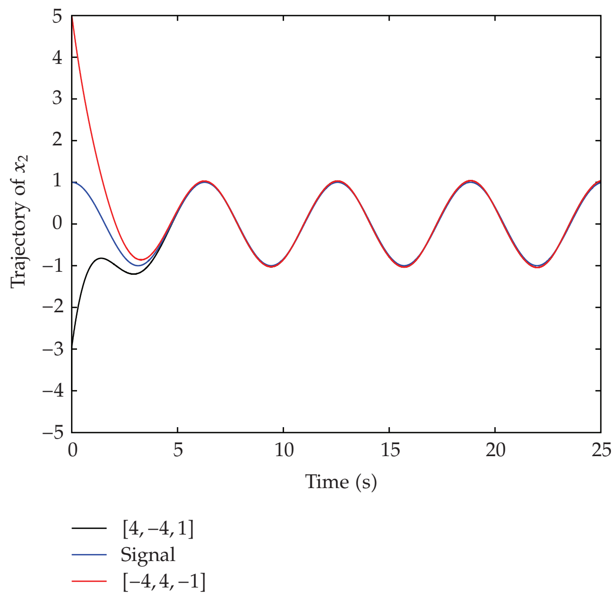

Figure 14: Trajectories of state x_2 under initial conditions.

proposed scheme can also suppress disturbance, and its ISE (integral square error criterion) is lower than RAFC. Figures 13–15 show the controlled stabilization of x_1, x_2, and x_3 for the nonlinear system described by (4.3) under initial states $[4, -4, 1]$, and $[-4, 4, -1]$ with external disturbance $d_1(t) = 0.1\cos t + 0.4\cos t$, $d_2(t) = 0.2\sin t$ and $d_3(t) = 0.1\sin t$. It seems to be satisfactory. Finally, all simulation results are given in Table 2.

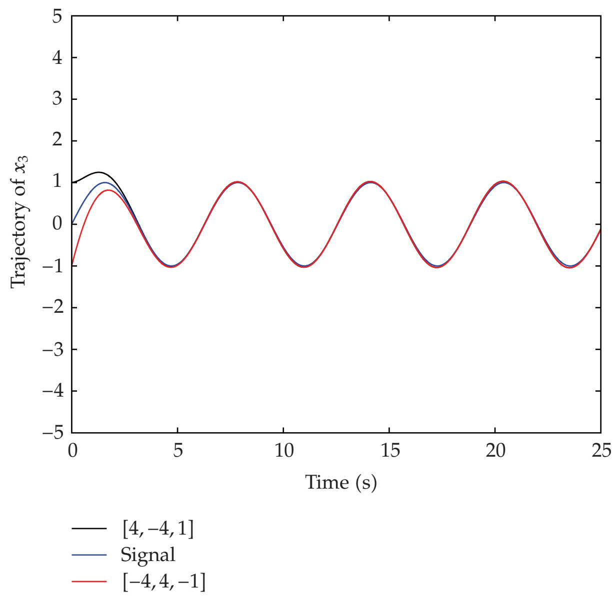

Figure 15: Trajectories of state x_3 under initial conditions.

Table 2: Experiments results for RAFC and proposed control methods.

	Time (sec)	ISE	Performance
RAFC [44]	3.0~3.2	0.2	Affected by noise
Proposed	2.5~3.0	0.05	Rejects noise

5. Conclusion

A hybrid optimization algorithm that combines the adaptive backstepping principle and the fuzzy-immune algorithm for a class of chaotic dynamical systems was presented. The four main contributions of this paper are: (1) an adaptive backstepping fuzzy-immune tracking control method for a class of chaotic systems is designed, (2) the controller does not require a priori knowledge of the sign of the control coefficient, (3) fuzzy-immune algorithm is used to self-adjustment controller's coefficient for the optimal solution, and (4) a correct term is introduced to eliminate internal uncertainty and external disturbance. The proposed hybrid intelligence adaptive backstepping fuzzy-immune controller guarantees closed-loop stability while maitaining the desired tracking performance. Simulation results show that the proposed controller scheme provides better tracking performance than those of two existing methods.

Acknowledgment

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