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Research Article

Existence and Iteration of Monotone Positive Solution of BVP for an Elastic Beam Equation

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This paper is concerned with the existence of monotone positive solution of boundary value problem for an elastic beam equation. By applying iterative techniques, we not only obtain the existence of monotone positive solution but also establish iterative scheme for approximating the solution. It is worth mentioning that the iterative scheme starts off with zero function, which is very useful and feasible for computational purpose. An example is also included to illustrate the main results.

1. Introduction

It is well-known that beam is one of the basic structures in architecture. The deformations of an elastic beam in equilibrium state can be described by the following equation of deflection curve:

$$\frac{d^2}{dx^2}\left(EI_z\frac{d^2u}{dx^2}\right) = q(x), \tag{1.1}$$

where E is Yang's modulus constant, I_z is moment of inertia with respect to z axes, and $q(x)$ is loading at x. If the loading of beam considered is in relation to deflection and rate of change of deflection, we need to study more general equation

$$u^{(4)}(x) = f\left(x, u(x), u'(x)\right). \tag{1.2}$$

According to different forms of supporting, various boundary value problems (BVPs) should be considered.

Owing to its importance in engineering, physics, and material mechanics, BVPs for elastic beam equations have attracted much attention from many authors see, for example, [1–15] and the references therein. However, almost all of the papers we mentioned focused their attention on the existence of solutions or positive solutions. In the existing literature, there are few papers concerned with the computational methods of solutions or positive solutions. It is worth mentioning that Zhang [16] obtained the existence of positive solutions and established iterative schemes for approximating the solutions for an elastic beam equation with a corner. The main tools used were monotone iterative techniques. For monotone iterative methods, one can refer [17–19] and the references therein.

Motivated greatly by the above-mentioned excellent works, in this paper we investigate the existence and iteration of monotone positive solution for the following elastic beam equation BVP

$$\begin{gathered} u^{(4)}(t) = f(t, u(t), u'(t)), \quad t \in (0,1), \\ u(0) = u'(1) = u''(0) = u'''(1) = 0. \end{gathered} \tag{1.3}$$

In material mechanics, the equation in (1.3) describes the deflection or deformation of an elastic beam under a certain force. The boundary conditions in (1.3) mean that the elastic beam is simply fixed at the end $t = 0$ and fastened with a sliding clamp at the end $t = 1$. By applying iterative techniques, we not only obtain the existence of monotone positive solution but also establish iterative scheme for approximating the solution. It is worth mentioning that the iterative scheme starts off with zero function, which is very useful and feasible for computational purpose. An example is also included to illustrate the main results.

Throughout this paper, we always assume that the following condition is satisfied:

(A) $f \in C([0,1] \times [0,+\infty) \times [0,+\infty), [0,+\infty))$.

2. Preliminary

In order to obtain the main results of this paper, we first present several fundamental lemmas in this section.

Lemma 2.1. *Let $y \in C[0,1]$. Then, the BVP*

$$\begin{gathered} u^{(4)}(t) = y(t), \quad t \in (0,1), \\ u(0) = u'(1) = u''(0) = u'''(1) = 0, \end{gathered} \tag{2.1}$$

has a unique solution

$$u(t) = \int_0^1 G(t,s) y(s) ds, \quad t \in [0,1], \tag{2.2}$$

where

$$G(t,s) = \frac{1}{6} \begin{cases} s\left(6t - 3t^2 - s^2\right), & 0 \le s \le t \le 1, \\ t\left(6s - 3s^2 - t^2\right), & 0 \le t \le s \le 1. \end{cases} \tag{2.3}$$

Lemma 2.2. *For any* $(t, s) \in [0,1] \times [0,1]$, *we have*

$$0 \leq \frac{\partial G(t,s)}{\partial t} \leq s(1-t), \qquad \frac{1}{3}t^2 s \leq G(t,s) \leq \frac{1}{2}\left(2t - t^2\right)s. \tag{2.4}$$

Proof. For any fixed $s \in [0,1]$, it is easy to know that

$$\frac{\partial G(t,s)}{\partial t} = \begin{cases} s(1-t), & 0 \leq s \leq t \leq 1, \\ s(1-t) - \dfrac{1}{2}(s-t)^2, & 0 \leq t \leq s \leq 1, \end{cases} \tag{2.5}$$

which shows that

$$0 \leq \frac{\partial G(t,s)}{\partial t} \leq s(1-t), \quad \text{for } (t,s) \in [0,1] \times [0,1], \tag{2.6}$$

and so,

$$G(t,s) = \int_0^t \frac{\partial G(\tau,s)}{\partial \tau} d\tau \leq \int_0^t s(1-\tau)d\tau = \frac{1}{2}\left(2t - t^2\right)s, \quad \text{for } (t,s) \in [0,1] \times [0,1]. \tag{2.7}$$

On the other hand, it follows from the expression of $G(t,s)$ that

$$\begin{aligned} G(t,s) &= \frac{1}{6}s\left(6t - 3t^2 - s^2\right) \geq \frac{1}{6}s\left(6t - 4t^2\right) \geq \frac{1}{3}t^2 s, \quad 0 \leq s \leq t \leq 1, \\ G(t,s) &= \frac{1}{6}t\left(6s - 3s^2 - t^2\right) \geq \frac{1}{6}t\left(6s - 4s^2\right) \geq \frac{1}{3}t^2 s, \quad 0 \leq t \leq s \leq 1. \end{aligned} \tag{2.8}$$

□

Let $E = C^1[0,1]$ be equipped with the norm $\|u\| = \max\{\|u\|_\infty, \|u'\|_\infty\}$. Then, E is a Banach space. Denote

$$K = \left\{ u \in E : u(t) \geq \frac{2}{3}t^2 \|u\|_\infty, \ u'(t) \geq 0, \ \text{for } t \in [0,1] \right\}. \tag{2.9}$$

Then, it is easy to verify that K is a cone in E. Note that this induces an order relation $\lesssim$ in E by defining $u \lesssim v$ if and only if $v - u \in K$. Now, we define an operator T on K by

$$(Tu)(t) = \int_0^1 G(t,s) f\left(s, u(s), u'(s)\right) ds, \quad t \in [0,1]. \tag{2.10}$$

Obviously, fixed points of T are monotone and nonnegative solutions of the BVP (1.3).

Lemma 2.3. *$T : K \to K$ is completely continuous.*

Proof. First, we prove $T(K) \subset K$. Suppose that $u \in K$. In view of Lemma 2.2, on the one hand,

$$\begin{aligned}
0 &\le (Tu)(t) \\
&= \int_0^1 G(t,s) f\left(s, u(s), u'(s)\right) ds \\
&\le \frac{1}{2}\left(2t - t^2\right) \int_0^1 s f\left(s, u(s), u'(s)\right) ds \\
&\le \frac{1}{2} \int_0^1 s f\left(s, u(s), u'(s)\right) ds, \quad t \in [0,1],
\end{aligned} \tag{2.11}$$

which shows that

$$\|Tu\|_\infty \le \frac{1}{2} \int_0^1 s f\left(s, u(s), u'(s)\right) ds. \tag{2.12}$$

On the other hand,

$$\begin{aligned}
(Tu)(t) &= \int_0^1 G(t,s) f\left(s, u(s), u'(s)\right) ds \\
&\ge \frac{1}{3} t^2 \int_0^1 s f\left(s, u(s), u'(s)\right) ds, \quad t \in [0,1],
\end{aligned} \tag{2.13}$$

which together with (2.12) implies that

$$(Tu)(t) \ge \frac{2}{3} t^2 \|Tu\|_\infty, \quad t \in [0,1]. \tag{2.14}$$

Again, by Lemma 2.2, we have

$$(Tu)'(t) = \int_0^1 \frac{\partial G(t,s)}{\partial t} f\left(s, u(s), u'(s)\right) ds \ge 0, \quad t \in [0,1]. \tag{2.15}$$

So, it follows from (2.14), (2.15) and the definition of K that $T(K) \subset K$.

Next, we show that T is a compact operator. Let $D \subset K$ be a bounded set. Then, there exists $M_1 > 0$ such that $\|u\| \le M_1$ for any $u \in D$. For any $\{y_k\}_{k=1}^\infty \subset T(D)$, there exist $\{x_k\}_{k=1}^\infty \subset D$ such that $y_k = Tx_k$. Denote

$$M_2 = \sup\left\{ f(t,x,y) : (t,x,y) \in [0,1] \times [0,M_1] \times [0,M_1] \right\}. \tag{2.16}$$

Then, for any positive integer k, it follows from Lemma 2.2 that

$$\begin{aligned}
|y_k(t)| &= |(Tx_k)(t)| \\
&= \left| \int_0^1 G(t,s) f\left(s, x_k(s), x_k'(s)\right) ds \right| \\
&\le \frac{1}{2}\left(2t - t^2\right) \int_0^1 s f\left(s, x_k(s), x_k'(s)\right) ds \\
&\le \frac{M_2}{4}, \quad t \in [0,1],
\end{aligned} \tag{2.17}$$

which indicates that $\{y_k\}_{k=1}^{\infty}$ is uniformly bounded. Similarly, we have

$$\begin{aligned}
|y_k'(t)| &= |(Tx_k)'(t)| \\
&= \left| \int_0^1 \frac{\partial G(t,s)}{\partial t} f\left(s, x_k(s), x_k'(s)\right) ds \right| \\
&\le (1-t) \int_0^1 s f\left(s, x_k(s), x_k'(s)\right) ds \\
&\le \frac{M_2}{2}, \quad t \in [0,1].
\end{aligned} \tag{2.18}$$

This shows that $\{y_k'\}_{k=1}^{\infty}$ is uniformly bounded, which implies that $\{y_k\}_{k=1}^{\infty}$ is equicontinuous. By Arzela-Ascoli theorem, we know that $\{y_k\}_{k=1}^{\infty}$ has a convergent subsequence in $C[0,1]$. Without loss of generality, we may assume that $\{y_k\}_{k=1}^{\infty}$ converges in $C[0,1]$.

On the other hand, for any $\epsilon > 0$, by the uniform continuity of $\partial G(t,s)/\partial t$, we know that there exists a $\delta > 0$ such that for any $t_1, t_2 \in [0,1]$ with $|t_1 - t_2| < \delta$,

$$\left| \frac{\partial G(t_1,s)}{\partial t} - \frac{\partial G(t_2,s)}{\partial t} \right| < \frac{\epsilon}{M_2 + 1}, \quad s \in [0,1]. \tag{2.19}$$

So, for any positive integer k, $t_1, t_2 \in [0,1]$ with $|t_1 - t_2| < \delta$, we get

$$\begin{aligned}
|y_k'(t_1) - y_k'(t_2)| &= |(Tx_k)'(t_1) - (Tx_k)'(t_2)| \\
&= \int_0^1 \left| \frac{\partial G(t_1,s)}{\partial t} - \frac{\partial G(t_2,s)}{\partial t} \right| f\left(s, x_k(s), x_k'(s)\right) ds \\
&< \frac{M_2 \epsilon}{M_2 + 1} \\
&< \epsilon,
\end{aligned} \tag{2.20}$$

which shows that $\{y_k'\}_{k=1}^{\infty}$ is equicontinuous. Again, it follows from Arzela-Ascoli theorem that $\{y_k'\}_{k=1}^{\infty}$ has a convergent subsequence in $C[0,1]$. Therefore, $\{y_k\}_{k=1}^{\infty}$ has a convergent subsequence in K.

Finally, we prove that T is continuous. Suppose that $u_m,\ u \in K$ and $\|u_m - u\| \to 0$ $(m \to \infty)$. Then, there exists $M_3 > 0$ such that for any positive integer m, $\|u_m\| \leq M_3$. Denote

$$M_4 = \sup\{f(t,x,y) : (t,x,y) \in [0,1] \times [0,M_3] \times [0,M_3]\}. \tag{2.21}$$

Then, for any positive integer m and $t \in [0,1]$, by Lemma 2.2, we have

$$\begin{aligned} &G(t,s)f(s,u_m(s),u_m'(s)) \leq \frac{M_4}{2}(2t-t^2)s \leq \frac{M_4}{2}s, \quad s \in [0,1], \\ &\frac{\partial G(t,s)}{\partial t} f(s,u_m(s),u_m'(s)) \leq M_4(1-t)s \leq M_4 s, \quad s \in [0,1]. \end{aligned} \tag{2.22}$$

So, it follows from Lebesgue dominated convergence theorem that

$$\begin{aligned} \lim_{m\to\infty}(Tu_m)(t) &= \lim_{m\to\infty}\int_0^1 G(t,s)f(s,u_m(s),u_m'(s))ds \\ &= \int_0^1 G(t,s)f(s,u(s),u'(s))ds \\ &= (Tu)(t), \quad t \in [0,1], \\ \lim_{m\to\infty}(Tu_m)'(t) &= \lim_{m\to\infty}\int_0^1 \frac{\partial G(t,s)}{\partial t} f(s,u_m(s),u_m'(s))ds \\ &= \int_0^1 \frac{\partial G(t,s)}{\partial t} f(s,u(s),u'(s))ds \\ &= (Tu)'(t), \quad t \in [0,1], \end{aligned} \tag{2.23}$$

which implies that T is continuous. Therefore, $T : K \to K$ is completely continuous. □

3. Main Results

Theorem 3.1. *Assume that $f(t,0,0) \not\equiv 0$ for $t \in (0,1)$, and there exists a constant $a > 0$ such that*

$$f(t,u_1,v_1) \leq f(t,u_2,v_2) \leq 2a, \quad 0 \leq t \leq 1,\ 0 \leq u_1 \leq u_2 \leq a,\ 0 \leq v_1 \leq v_2 \leq a. \tag{3.1}$$

If we construct a iterative sequence $v_{n+1} = Tv_n$, $n = 0, 1, 2, \ldots$, *where* $v_0(t) \equiv 0$ *for* $t \in [0,1]$, *then* $\{v_n\}_{n=1}^{\infty}$ *converges to* v^* *in* $C^1[0,1]$, *which is a monotone positive solution of the BVP* (1.3) *and satisfy*

$$0 < v^*(t) \leq a \quad \text{for } t \in (0,1], \qquad 0 \leq (v^*)'(t) \leq a \quad \text{for } t \in [0,1]. \tag{3.2}$$

Proof. Let $K_a = \{u \in K : \|u\| \leq a\}$. We assert that $T : K_a \to K_a$. In fact, if $u \in K_a$, then

$$0 \leq u(s) \leq \max_{0 \leq s \leq 1} u(s) \leq \|u\| \leq a, \quad 0 \leq u'(s) \leq \max_{0 \leq s \leq 1} u'(s) \leq \|u\| \leq a, \quad \text{for } s \in [0,1], \tag{3.3}$$

which together with the condition (3.1) and Lemma 2.2 implies that

$$\begin{gathered} 0 \leq (Tu)(t) = \int_0^1 G(t,s) f\big(s, u(s), u'(s)\big) ds < a, \quad t \in [0,1], \\ 0 \leq (Tu)'(t) = \int_0^1 \frac{\partial G(t,s)}{\partial t} f\big(s, u(s), u'(s)\big) ds \leq a, \quad t \in [0,1]. \end{gathered} \tag{3.4}$$

Hence, we have shown that $T : K_a \to K_a$.

Now, we assert that $\{v_n\}_{n=1}^{\infty}$ converges to v^* in $C^1[0,1]$, which is a monotone positive solution of the BVP (1.3) and satisfies

$$0 < v^*(t) \leq a \quad \text{for } t \in (0,1], \qquad 0 \leq (v^*)'(t) \leq a \quad \text{for } t \in [0,1]. \tag{3.5}$$

Indeed, in view of $v_0 \in K_a$ and $T : K_a \to K_a$, we have $v_n \in K_a$, $n = 1, 2, \ldots$. Since the set $\{v_n\}_{n=0}^{\infty}$ is bounded and T is completely continuous, we know that the set $\{v_n\}_{n=1}^{\infty}$ is relatively compact.

In what follows, we prove that $\{v_n\}_{n=0}^{\infty}$ is monotone by induction. First, by Lemma 2.2, we have

$$\begin{aligned} &\frac{1}{3} t^2 \int_0^1 s f(s,0,0) ds \\ &\quad \leq v_1(t) - v_0(t) = (Tv_0)(t) = \int_0^1 G(t,s) f(s,0,0) ds \leq \frac{1}{2} \int_0^1 s f(s,0,0) ds, \quad t \in [0,1], \end{aligned} \tag{3.6}$$

which implies that

$$v_1(t) - v_0(t) \geq \frac{2}{3} t^2 \|v_1 - v_0\|_{\infty}, \quad t \in [0,1]. \tag{3.7}$$

At the same time, it is obvious that

$$v_1'(t) - v_0'(t) = v_1'(t) = (Tv_0)'(t) = \int_0^1 \frac{\partial G(t,s)}{\partial t} f(s,0,0) ds \geq 0, \quad t \in [0,1]. \tag{3.8}$$

It follows from (3.7) and (3.8) that $v_1 - v_0 \in K$, which shows that $v_0 \lesssim v_1$. Next, we assume that $v_{k-1} \lesssim v_k$. Then, in view of Lemma 2.2 and (3.1), we have

$$\begin{aligned}
v_{k+1}(t) - v_k(t) &= \int_0^1 G(t,s)\left[f\left(s, v_k(s), v_k'(s)\right) - f\left(s, v_{k-1}(s), v_{k-1}'(s)\right)\right] ds \\
&\le \frac{1}{2}\int_0^1 s\left[f\left(s, v_k(s), v_k'(s)\right) - f\left(s, v_{k-1}(s), v_{k-1}'(s)\right)\right] ds, \quad t \in [0,1], \\
v_{k+1}(t) - v_k(t) &= \int_0^1 G(t,s)\left[f\left(s, v_k(s), v_k'(s)\right) - f\left(s, v_{k-1}(s), v_{k-1}'(s)\right)\right] ds \\
&\ge \frac{1}{3}t^2\int_0^1 s\left[f\left(s, v_k(s), v_k'(s)\right) - f\left(s, v_{k-1}(s), v_{k-1}'(s)\right)\right] ds, \quad t \in [0,1],
\end{aligned} \tag{3.9}$$

which implies that

$$v_{k+1}(t) - v_k(t) \ge \frac{2}{3}t^2 \|v_{k+1} - v_k\|_\infty, \quad t \in [0,1]. \tag{3.10}$$

At the same time, by Lemma 2.2 and (3.1), we also have

$$v_{k+1}'(t) - v_k'(t) = \int_0^1 \frac{\partial G(t,s)}{\partial t}\left[f\left(s, v_k(s), v_k'(s)\right) - f\left(s, v_{k-1}(s), v_{k-1}'(s)\right)\right] ds \ge 0, \quad t \in [0,1]. \tag{3.11}$$

It follows from (3.10) and (3.11) that $v_{k+1} - v_k \in K$, which indicates that $v_k \lesssim v_{k+1}$. Thus, we have shown that $v_n \lesssim v_{n+1}$, $n = 0, 1, 2 \ldots$.

Since $\{v_n\}_{n=1}^{\infty}$ is relatively compact and monotone, there exists a $v^* \in K_a$ such that $\|v_n - v^*\| \to 0$ $(n \to \infty)$, which together with the continuity of T and the fact that $v_{n+1} = Tv_n$ implies that $v^* = Tv^*$. Moreover, in view of $f(t,0,0) \not\equiv 0$ for $t \in (0,1)$, we know that the zero function is not a solution of the BVP (1.3). Thus, $\|v^*\|_\infty > 0$. So, it follows from $v^* \in K_a$ that

$$0 < v^*(t) \le a \quad \text{for } t \in (0,1], \qquad 0 \le (v^*)'(t) \le a \quad \text{for } t \in [0,1]. \tag{3.12}$$

□

4. An Example

Example 4.1. Consider the BVP

$$\begin{gathered}
u^{(4)}(t) = \frac{1}{2}tu(t) + \frac{1}{8}\left(u'(t)\right)^2 + 1, \quad t \in (0,1), \\
u(0) = u'(1) = u''(0) = u'''(1) = 0.
\end{gathered} \tag{4.1}$$

If we let $f(t,u,v) = (1/2)tu + (1/8)v^2 + 1$ for $(t,u,v) \in [0,1] \times [0,+\infty) \times [0,+\infty)$, then all the hypotheses of Theorem 3.1 are fulfilled with $a = 1$. It follows from Theorem 3.1 that the BVP (4.1) has a monotone positive solution v^* satisfying

$$0 < v^*(t) \le 1 \quad \text{for } t \in (0,1], \qquad 0 \le (v^*)'(t) \le 1 \quad \text{for } t \in [0,1]. \tag{4.2}$$

Moreover, the iterative scheme is

$$\begin{aligned} v_0(t) &\equiv 0, \quad t \in [0,1], \\ v_{n+1}(t) &= \frac{1}{6}\int_0^t s\left(6t - 3t^2 - s^2\right)\left(\frac{1}{2}sv_n(s) + \frac{1}{8}\left(v_n'(s)\right)^2 + 1\right)ds \\ &\quad + \frac{1}{6}\int_t^1 t\left(6s - 3s^2 - t^2\right)\left(\frac{1}{2}sv_n(s) + \frac{1}{8}\left(v_n'(s)\right)^2 + 1\right)ds, \quad t \in [0,1],\ n = 0,1,2,\ldots. \end{aligned} \tag{4.3}$$

The first, second, third, and fourth terms of the scheme are as follows:

$$\begin{aligned} v_0(t) &\equiv 0, \\ v_1(t) &= \frac{1}{3}t - \frac{1}{6}t^3 + \frac{1}{24}t^4, \\ v_2(t) &= \frac{73}{216}t + \frac{5}{144}t^2 - \frac{79}{432}t^3 + \frac{1}{64}t^4 + \frac{25}{1152}t^5 - \frac{23}{8640}t^6 - \frac{1}{192}t^7 + \frac{47}{11520}t^8 - \frac{11}{8640}t^9 + \frac{1}{6912}t^{10}, \\ v_3(t) &= \frac{378577}{1119744}t + \frac{6935}{186624}t^2 - \frac{404629}{2239488}t^3 + \frac{60361}{8957952}t^4 + \frac{71689}{2985984}t^5 + \frac{110633}{14929920}t^6 - \frac{22181}{1492992}t^7 \\ &\quad + \frac{373759}{119439360}t^8 + \frac{2467627}{477757440}t^9 - \frac{2449943}{716636160}t^{10} - \frac{800341}{1592524800}t^{11} + \frac{16256149}{11466178560}t^{12} \\ &\quad - \frac{3556357}{7166361600}t^{13} - \frac{655267}{4777574400}t^{14} + \frac{70313}{477757440}t^{15} - \frac{25471}{2388787200}t^{16} - \frac{363911}{9555148800}t^{17} \\ &\quad + \frac{7781}{298598400}t^{18} - \frac{25973}{2866544640}t^{19} + \frac{8917}{4777574400}t^{20} - \frac{31}{143327232}t^{21} + \frac{25}{2293235712}t^{22}. \end{aligned} \tag{4.4}$$

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