

Research Article

Similarity Solutions of MHD Mixed Convection Flow with Variable Reactive Index, Magnetic Field, and Velocity Slip Near a Moving Horizontal Plate: A Group Theory Approach

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The mixed convection of Newtonian fluid flow along a moving horizontal plate with higher-order chemical reaction, variable concentration reactant, and variable wall temperature and concentration is considered. Velocity slip and the thermal convective boundary conditions are applied at the plate surface. The governing partial differential equations are transformed into similarity equations via dimensionless similarity transformations developed by one-parameter continuous group method. The numerical solutions of the transformed ordinary differential equations are constructed for velocity, temperature and concentration functions, the skin friction factor, the rate of heat, and the rate of mass transfer using an implicit finite difference numerical technique. The investigated parameters are buoyancy parameters λ_1, λ_2 , chemical reaction parameter K , suction/injection parameter fw , velocity slip parameter a convective heat transfer parameter γ , magnetic parameter M , Prandtl number Pr and Schmidt number, Sc . Comparison with results from the open literature shows a very good agreement.

1. Introduction

A convection situation involving both free and forced convection is known as mixed convection and has been an important topic because of its application in electronic equipment cooled by a fan and flows in the ocean and in the atmosphere ([1, 2]). In mixed convection flows, the forced and the free convection effects are of comparable magnitude. Thus, mixed convection occurs if the effect of buoyancy forces on a forced flow or vice versa is significant. The laminar mixed convection takes place in various applications in thermal engineering

and science and has drawn attention from researchers in the past decades. Examples of these applications include atmospheric boundary layer flows, solar energy systems, boilers, compact heat exchangers, and cooling of electronic devices [3, 4]. Many studies exist for the mixed convection boundary layer flow about vertical/inclined/horizontal plate and wedge surfaces immersed in a viscous fluid. The details of mixed convection can be found in books by Chen and Armaly [5], Jaluria [6], Gebhart et al. [7], Bejan [8], Pop and Ingham [9] and in the articles in the literatures by Lloyd and Sparrow [10], Schneider [11], Wilks [12], Yao [13], Ramachandran et al. [14], Moulic and Yao, [15], Daskalakis et al. [16], Fan et al. [17], Magyari et al. [18], Hassanien et al. [19], Aydin and Kaya [20], Ishak [21], Moulic and Yao [22], Datta et al. [23], Ishak et al. [24], Magyari [25].

In all these studies, conventional no slip boundary conditions were applied at the surface. However, fluid flows in micro/nanoscale such as micronozzles, micropumps, micro-turbines, microheat exchangers, microvalves, turbines, sensors, and actuators are important for micro and nanoscience and the conventional no slip boundary condition at the solid fluid interface must be replaced with the slip condition [26]. The slip condition states that the tangential component of the velocity at the surface is equal to the velocity gradient normal to the surface ([27, 28]). Slips may occur on a stationary/moving boundary when the fluid is in particulate form such as in the form of emulsions, suspensions, foams, and polymer solutions [29, 30]. An excellent review for the fundamentals, theory, and applications of these microscale systems may be found in the text by Duncan and Peterson [31] Gad-el-Hak [32]. Slips effect on flow field has been studied by various authors; Fang and Lee [33], Martin and Boyd [34], Matthews and Hill [35], Wang [36]. Cao and Baker [37] studied laminar mixed convection over an isothermal vertical plate with first-order momentum and thermal discontinuities at the wall. The hydrodynamic and thermal slip flow boundary layer over a flat plate with constant heat flux boundary condition has been investigated by Hak [27] who concluded that as the slip parameter increases, the slip velocity increases and the wall shear stress decreases.

Due to the nonlinearities of the governing equations, no analytical solution is available and the nonlinear equations usually solved numerically subject to initial/boundary conditions. The solution of differential equations with/without initial/boundary conditions is presented by different authors such as Ames [38], Boyce [39], Na [40], Zauderer [41], Seshadri and Na [42], Kincaid and Cheney [43], Strauss [44], Aziz [45], and White and Subramanian [46].

We study combined convective Newtonian fluid flow adjacent to a moving horizontal permeable flat plate with higher order chemical reaction taking into account velocity slip and convective surface boundary conditions. The governing boundary layer equations are converted into a two point boundary value problem using similarity transformations developed by group method and the transformed equations are solved by means of an implicit finite difference numerical method. Graphs are plotted for velocity, temperature and concentration profiles as well as the skin friction factor, the rate of heat transfer and the rate of mass transfer to exhibits the influences of the parameters involving in our problem.

2. Mathematical Formulations of the Problem

Consider a continuous moving permeable horizontal flat plate which moves with a non-uniform velocity $U(x)$ as shown in Figure 1. A variable magnetic field of strength $B(x)$ is applied to the normal direction of the plate. The bottom surface of the plate is heated by convection from a hot fluid of temperature $T_f(x)$ which provides a heat transfer

where $v_w(x)$: velocity normal to the plate, ν : coefficient of kinematic viscosity, ρ : density of the fluid, σ : electric conductivity, p : pressure, k : thermal conductivity, α : thermal diffusivity, D : mass diffusivity of species in fluid, β_T : volumetric thermal coefficient, β_C : volumetric concentration coefficient, g : acceleration due to gravity, k_0 : reaction rate, n : order of chemical reaction, and $N_1(x)$: velocity slip factor with dimension (velocity)⁻¹.

Introducing stream function ψ , dimensionless temperature function θ and concentration function ϕ are defined by

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x}, \quad \theta = \frac{T - T_\infty}{T_f - T_\infty}, \quad \phi = \frac{C - C_\infty}{C_w - C_\infty}. \quad (2.3)$$

Using (2.3) we get from (2.1)

$$\frac{\partial \psi}{\partial y} \frac{\partial^2 \psi}{\partial x \partial y} - \frac{\partial \psi}{\partial x} \frac{\partial^2 \psi}{\partial y^2} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \frac{\partial^3 \psi}{\partial y^3} - \frac{\sigma B^2(x)}{\rho} \frac{\partial \psi}{\partial y}, \quad (2.4)$$

$$-\frac{1}{\rho} \frac{\partial p}{\partial y} + g\beta_T \theta \Delta T + \beta_C \phi \Delta C = 0, \quad (2.5)$$

$$\frac{\partial \psi}{\partial y} \frac{\partial}{\partial x} (\theta \Delta T) - \frac{\partial \psi}{\partial x} \frac{\partial}{\partial y} (\theta \Delta T) = \alpha \frac{\partial^2}{\partial y^2} (\theta \Delta T), \quad (2.6)$$

$$\frac{\partial \psi}{\partial y} \frac{\partial}{\partial x} (\phi \Delta C) - \frac{\partial \psi}{\partial x} \frac{\partial}{\partial y} (\phi \Delta C) = D \frac{\partial^2}{\partial y^2} (\phi \Delta C) - k_0 (\phi \Delta C)^n. \quad (2.7)$$

Eliminating the pressure gradient terms from (2.4) and (2.5) by cross-differentiation, we get

$$\begin{aligned} \nu \frac{\partial^4 \psi}{\partial y^4} + \frac{\partial \psi}{\partial x} \frac{\partial^3 \psi}{\partial y^3} - \frac{\partial \psi}{\partial y} \frac{\partial^3 \psi}{\partial x \partial y^2} - g\beta_T (\Delta T \theta)_x - g\beta_T (\Delta C \phi)_x - \frac{\sigma B^2(x)}{\rho} \psi_{yy} &= 0, \\ \alpha \frac{\partial^2}{\partial y^2} (\theta \Delta T) + \frac{\partial \psi}{\partial x} \frac{\partial}{\partial y} (\theta \Delta T) - \frac{\partial \psi}{\partial y} \frac{\partial}{\partial x} (\theta \Delta T) &= 0, \\ D \frac{\partial^2}{\partial y^2} (\phi \Delta C) + \frac{\partial \psi}{\partial x} \frac{\partial}{\partial y} (\phi \Delta C) - \frac{\partial \psi}{\partial y} \frac{\partial}{\partial x} (\phi \Delta C) - k_0 (\phi \Delta C)^n &= 0. \end{aligned} \quad (2.8)$$

The boundary conditions (2.2) become

$$\begin{aligned} \frac{\partial \psi}{\partial y} = U(x) + \nu N_1(x) \frac{\partial^2 \psi}{\partial y^2}, \quad -\frac{\partial \psi}{\partial x} = v_w(x) \frac{\partial \theta}{\partial y} = -\frac{h_f(x)}{k} [1 - \theta(0)], \quad \phi = 1 \quad \text{at } y = 0, \\ \frac{\partial \psi}{\partial y} \longrightarrow 0, \quad \frac{\partial \psi}{\partial x} \longrightarrow 0, \quad \theta \longrightarrow 0, \quad \phi \longrightarrow 0 \quad \text{as } y \longrightarrow \infty. \end{aligned} \quad (2.9)$$

3. Application of Group Theory

We will now search group invariant solutions (similarity solutions) of (2.8) to (2.9) under a particular continuous one-parameter group. We define the following one-parameter group transformations Γ :

$$\begin{aligned} x^* &= e^{\varepsilon\alpha_1} x, & y^* &= e^{\varepsilon\alpha_2} y, & \psi^* &= e^{\varepsilon\alpha_3} \psi, & \theta^* &= e^{\varepsilon\alpha_4} \theta, & \phi^* &= e^{\varepsilon\alpha_5} \phi, & B^* &= e^{\varepsilon\alpha_6} B \\ \Delta T^* &= e^{\varepsilon\alpha_7} \Delta T, & \Delta C^* &= e^{\varepsilon\alpha_8} \Delta C, & U &= e^{\varepsilon\alpha_9} U, & v_w^* &= e^{\varepsilon\alpha_{10}} v_w, & h_f^* &= e^{\varepsilon\alpha_{11}} h_f \end{aligned} \quad (3.1)$$

Here ε is the parameter of the group and α_i ($i = 1, 2, 3, \dots, 11$) are the arbitrary real numbers whose interrelationship will be determined by our investigation. Transformations in (3.1) may be considered as point transformations which transform the coordinates

$$(x, y, \psi, \theta, \phi, B, \Delta T, \Delta C, U, v_w, h_f) \text{ to } (x^*, y^*, \psi^*, \theta^*, \phi^*, B^*, \Delta T^*, \Delta C^*, U^*, v_w^*, h_f^*). \quad (3.2)$$

We now find the relationship among the exponents α_i such that (2.8) with the boundary conditions in (2.9) are invariant (the structure of the equations remains same before and after the transformations) under the transformation group in (3.1). Substituting transformations in (3.1) into (2.8) and equating various exponents of e for constant conformally invariant, we get the following algebraic equations:

$$\begin{aligned} 4\alpha_2 - \alpha_3 &= \alpha_1 + 3\alpha_2 - 2\alpha_3 = \alpha_1 - \alpha_4 - \alpha_7 = \alpha_1 - \alpha_5 - \alpha_8 = 2\alpha_2 - \alpha_3 - 2\alpha_6, \\ 2\alpha_2 - \alpha_4 - \alpha_7 &= \alpha_1 + \alpha_2 - \alpha_3 - \alpha_4 - \alpha_7, \\ 2\alpha_2 - \alpha_5 - \alpha_8 &= \alpha_1 + \alpha_2 - \alpha_3 - \alpha_5 - \alpha_8 = n(-\alpha_5 - \alpha_8). \end{aligned} \quad (3.3)$$

Using (3.1) in boundary conditions (2.9) and equating various exponents of e , we get

$$\alpha_2 - \alpha_3 = -\alpha_9, \quad \alpha_1 - \alpha_3 = -\alpha_{10}, \quad \alpha_2 = -\alpha_{11}. \quad (3.4)$$

Without loss of generality, we may put $\alpha_4 = \alpha_5 = 0$.

Solving (3.3) and (3.4) we have the following relationship among the exponents

$$\begin{aligned} \alpha_1 &= \frac{(7-5n)}{4\alpha_8}, & \alpha_2 &= \frac{(1-n)}{2\alpha_8}, & \alpha_3 &= \frac{(5-3n)}{4\alpha_8}, & \alpha_4 &= 0, & \alpha_5 &= 0, \\ \alpha_7 &= \alpha_8, & \alpha_9 &= \frac{(3-n)}{4\alpha_8}, & \alpha_6 &= \alpha_{10} = \alpha_{11} = \frac{(n-1)}{2\alpha_8}. \end{aligned} \quad (3.5)$$

It can be easily verified that (2.8) and the boundary conditions (2.9) are invariant under the transformations in (3.1) subject to the conditions in (3.5).

3.1. Similarity Transformations

Using the relationships in (3.5), we have from (3.1)

$$\begin{aligned}
 x^* &= e^{\varepsilon(7-5n)/4\alpha_8} x, & y^* &= e^{\varepsilon(1-n)/2\alpha_8} y, & \psi^* &= e^{\varepsilon(5-3n)/4\alpha_8} \psi, & \theta^* &= \theta, & \phi &= \phi, \\
 B^* &= e^{\varepsilon(n-1)/2\alpha_8} B, & \Delta T^* &= e^{\varepsilon\alpha_8} \Delta T, & \Delta C^* &= e^{\varepsilon\alpha_8} \Delta C, & U^* &= e^{\varepsilon(3-n)/4\alpha_8} U, \\
 v_w^* &= e^{\varepsilon\alpha_8(n-1)/2} v_w, & h_f^* &= e^{\varepsilon\alpha_8(n-1)/2} h_f.
 \end{aligned} \tag{3.6}$$

Expanding each of the transformations in (3.6) in powers of ε , keeping the first order term and neglecting higher order terms, we get the following characteristic equations:

$$\begin{aligned}
 \frac{4dx}{(7-5n)x} &= \frac{2dy}{(1-n)y} = \frac{4d\psi}{(5-3n)\psi} = \frac{d\theta}{0} = \frac{d\phi}{0} = \frac{2dB}{(n-1)B} \\
 &= \frac{d(\Delta T)}{\Delta T} = \frac{d(\Delta C)}{\Delta C} = \frac{4dU}{(n-3)U} = \frac{2dv_w}{(n-1)v_w} = \frac{2dh_f}{(n-1)h_f}.
 \end{aligned} \tag{3.7}$$

Solving the system of first order linear differential equations in (3.7), we get

$$\begin{aligned}
 \eta^* &= \beta_1 y x^{(2n-2)/(7-5n)}, & \psi^* &= \beta_2 x^{(5-3n)/(7-5n)} f(\eta), \\
 \theta^* &= \theta, & \phi^* &= \phi, & B^* &= \beta_3 x^{(2n-2)/(7-5n)}, \\
 \Delta T^* &= \beta_4 x^{4/(7-5n)}, & \Delta C^* &= \beta_5 x^{4/(7-5n)}, & U^* &= \beta_6 x^{(3-n)/(7-5n)}, \\
 v_w^* &= \beta_7 x^{(2n-2)/(7-5n)}, & h_f^* &= \beta_8 x^{(2n-2)/(7-5n)},
 \end{aligned} \tag{3.8}$$

where β_i ($i = 1, 2, 3, \dots, 8$) are real constants. To get dimensionless form of the transformations, we define the following dimensionless transformations:

$$\begin{aligned}
 \eta &= \sqrt{\text{Re}} \frac{y}{L} \left(\frac{x}{L}\right)^{(2n-2)/(7-5n)}, & \psi &= \nu \sqrt{\text{Re}} \left(\frac{x}{L}\right)^{(5-3n)/(7-5n)} f(\eta), \\
 \theta &= \theta(\eta), & \phi &= \phi(\eta), \\
 B &= B_0 \left(\frac{x}{L}\right)^{(2n-2)/(7-5n)}, & \Delta T &= (\Delta T)_0 \left(\frac{x}{L}\right)^{4/(7-5n)}, & \Delta C &= (\Delta C)_0 \left(\frac{x}{L}\right)^{4/(7-5n)}, \\
 U &= U_0 \left(\frac{x}{L}\right)^{(3-n)/(7-5n)}, & v_w &= v_0 \left(\frac{x}{L}\right)^{(2n-2)/(7-5n)}, & h_f(x) &= h_0 \left(\frac{x}{L}\right)^{(2n-2)/(7-5n)},
 \end{aligned} \tag{3.9}$$

where $f(\eta)$, $\theta(\eta)$ and $\phi(\eta)$, represent dimensionless velocity, temperature, and concentration functions, respectively. B_0 , $(\Delta T)_0$, $(\Delta C)_0$, U_0 , v_0 and h_0 are constants.

Using (3.9), (2.8) become

$$\begin{aligned}
 f'''' + \frac{5-3n}{7-5n} f f'''' - \frac{n+1}{7-5n} f' f'' - M f' - \lambda_1 \left[\frac{2n-2}{7-5n} \eta \theta' + \frac{4}{7-5n} \theta \right] \\
 - \lambda_2 \left[\frac{2n-2}{7-5n} \eta \phi' + \frac{4}{7-5n} \phi \right] = 0, \\
 \text{Pr}^{-1} \theta'' + \frac{5-3n}{7-5n} f \theta' - \frac{4}{7-5n} f' \theta = 0, \\
 \text{Sc}^{-1} \phi'' + \frac{5-3n}{7-5n} f \phi' - \frac{4}{7-5n} f' \phi - K \phi^n = 0,
 \end{aligned} \tag{3.10}$$

subject to the boundary conditions

$$\begin{aligned}
 f(0) = \frac{7-5n}{5-3n} f w, \quad f'(0) = 1 + a f''(0), \quad \theta'(0) = -\gamma [1 - \theta(0)], \quad \phi(0) = 1, \\
 f(\infty) = f'(\infty) = \theta(\infty) = \phi(\infty) = 0,
 \end{aligned} \tag{3.11}$$

where $f w = v_0 L / \nu \sqrt{\text{Re}}$ is the suction/injection parameter, $\gamma = L h_0 / \text{Re} k$ is the convective heat transfer parameter, $\lambda_1 = \text{Gr} / \text{Re}^{2.5}$ is the thermal buoyancy parameter, $\lambda_2 = \text{Gc} / \text{Re}^{2.5}$ is the concentration buoyancy parameter, $M = \sigma B_0^2 L^2 / \rho \nu$ is the magnetic parameter, $K = L^2 k_0 (\Delta C)_0^{n-1} / \nu$ is the reaction parameter, $\text{Gr} = g \beta_c (\Delta T)_0 L^3 / \nu^2$ is the thermal Grashof number, $\text{Gc} = g \beta_c (\Delta T)_0 L^3 / \nu^2$ is the solutal Grashof number, and $a = \nu \sqrt{\text{Re}} N_0 / L$ is the velocity slip parameter. Note that similarity solutions will exist if $N_1(x) = N_0 x^{(2-2n)/(7-5n)}$, N_0 is a constant.

The physical quantities of interest are the local skin friction coefficient C_{fx} , local Nusselt number Nu_x (rate of heat transfer), and local Sherwood number Sh_x (rate of mass transfer), which are defined as

$$C_{fx} = \frac{-\tau_w}{\rho U^2}, \quad \text{Nu}_x = \frac{-x q_w}{k \Delta T}, \quad \text{Sh}_x = \frac{-m_w x}{D \Delta C}, \tag{3.12}$$

where τ_w is the wall shear stress, q_w is the wall heat flux, m_w is the quantity of mass transfer through the unit area of the surface, which are given by

$$\tau_w = \mu \left(\frac{\partial u}{\partial y} \right)_{y=0}, \quad q_w = k \left(\frac{\partial T}{\partial y} \right)_{y=0}, \quad m_w = D \left(\frac{\partial C}{\partial y} \right)_{y=0}. \tag{3.13}$$

It can be shown that physical quantities are proportional to $-f''(0)$, $-\theta'(0)$ and $-\phi'(0)$.

4. Results and Discussion

Equations (3.10) with boundary conditions (3.11) were solved using the *dsolve* command in MAPLE 14 with *numeric* option. Depending upon the nature of the ordinary differential

Table 1: Comparison of results for $f''(0)$ and $-\phi'(0)$ for different values of parameters.

Pr = 0.72, Pr = 0.72, $\lambda_1 = 1, \lambda_2 = 1, K = 1$				
Sc	Fan et al. [17]		Present results	
	$f''(0)$	$\phi'(0)$	$f''(0)$	$-\phi'(0)$
0.1	0.9098	0.13678	0.9099	0.1368
0.72	0.47511	0.28697	0.4751	0.287
5	0.23157	0.77288	0.2316	0.7729
Pr = 0.72, Pr = 0.72, Sc = 0.72, $\lambda_2 = 1, K = 1$				
λ_1	Fan et al. [17]		Present results	
	$f''(0)$	$\phi'(0)$	$f''(0)$	$-\phi'(0)$
0	0.06194	0.29757	0.0619	0.2976
0.4	0.24155	0.29252	0.2415	0.2925
1	0.47511	0.28697	0.4751	0.287
Pr = 0.72, Pr = 0.72, $\lambda_1 = 1, Sc = 0.72, K = 1$				
λ_2	Fan et al. [17]		Present results	
	$f''(0)$	$\phi'(0)$	$f''(0)$	$-\phi'(0)$
0	0.14425	0.29487	0.1442	0.2949
0.4	0.2835	0.29133	0.2835	0.2913
1	0.47511	0.28697	0.4751	0.287
Pr = 0.72, Pr = 0.72, $\lambda_1 = 1, \lambda_2 = 1, Sc = 0.72$				
K	Fan et al. [17]		Present results	
	$f''(0)$	$\phi'(0)$	$f''(0)$	$-\phi'(0)$
0	0.53159	0.00015	0.5316	0.00015
1	0.47511	0.28697	0.4751	0.287
3	0.42378	0.61989	0.4238	0.6199

equations, MAPLE uses a suitable scheme based on trapezoid or midpoint rule with Richardson extrapolation or deferred correction enhancement. In order to justify the accuracy of our numerical method, we compared our results with available data in Table 1 and found a good agreement. The influence of different parameters on the dimensionless velocity, temperature, and concentration is shown in Figures 2, 3, and 4, respectively. Figure 2(a) exhibits the influence of thermal buoyancy parameter λ_1 and suction/injection parameter $f\omega$, whilst Figure 2(b) shows the effects of solutal buoyancy parameter λ_2 and suction/injection parameter $f\omega$, on the dimensionless fluid velocity $f'(\eta)$ profiles. We further observed that velocity increases with buoyancy parameters λ_1 and λ_2 . Figure 2(c) describes the influence of magnetic field parameter M on velocity $f'(\eta)$ profiles. It is found that as M increases, the fluid velocity decreases. The effect of velocity slip parameter on the dimensionless velocity is depicted in Figure 2(d). It is observed that an increase in the velocity slip reduces the dimensionless velocity at the surface. In all cases suction increases the velocity whilst injection decreases the velocity, as expected.

Figures 3(a), 3(b), 3(c), and 3(d) display effects of various parameters on the dimensionless temperature function. The dimensionless temperature decreases with suction and increases with injection, as expected. Note that as we increase the values of λ_1 and λ_2 , the values of $\theta(\eta)$ decrease (Figures 3(a) and 3(b)). It is found that magnetic field increases temperature whilst Prandtl number decreases the temperature. Figures 4(a), 4(b), 4(c), and 4(d) exhibit influence of suction/injection $f\omega$, buoyancies λ_1, λ_2 , the magnetic field M and the Schmidt number Sc on the dimensionless concentration $\phi(\eta)$ function. The dimensionless concentration decreases with the increase of $f\omega$. We also notice from Figures 4(a) and 4(b) as we increase the values of λ_1 and λ_2 , values of $\phi(\eta)$ decrease. Figure 4(c) shows that

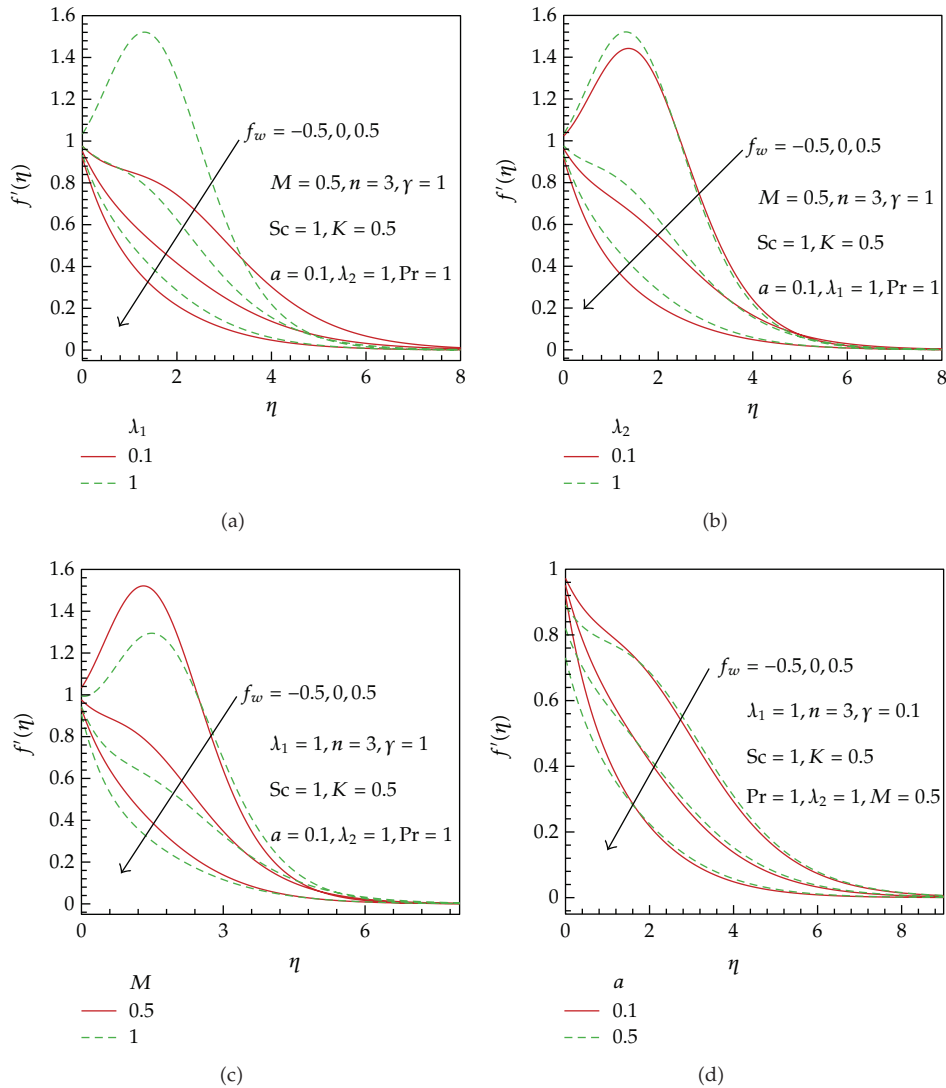


Figure 2: Effects of different parameters on the dimensionless velocity.

the increasing values of magnetic field parameter lead to increase the concentration whilst Figure 4(d) shows that increasing values of Schmidt number Sc leads to decrease the concentration, as expected.

Figures 5(a) and 5(b) depict the variation of the buoyancies λ_1, λ_2 , the slip a and the suction/injection fw parameters on the dimensionless shear stress. It is observed that suction enhanced the values of the dimensionless shear stress. We also see that the velocity slip a and the parameters λ_1, λ_2 increase the values of the dimensionless shear stress fall. The effect of Prandtl number Pr , convective heat transfer parameter γ , order of chemical reaction n , and the suction/injection parameter fw on the dimensionless rate of heat transfer is shown in Figures 6(a) and 6(b). From Figure 6(a) we can conclude that as Pr, γ and fw increase, the values of the dimensionless rate of heat transfer increase. An increase in the order of reaction

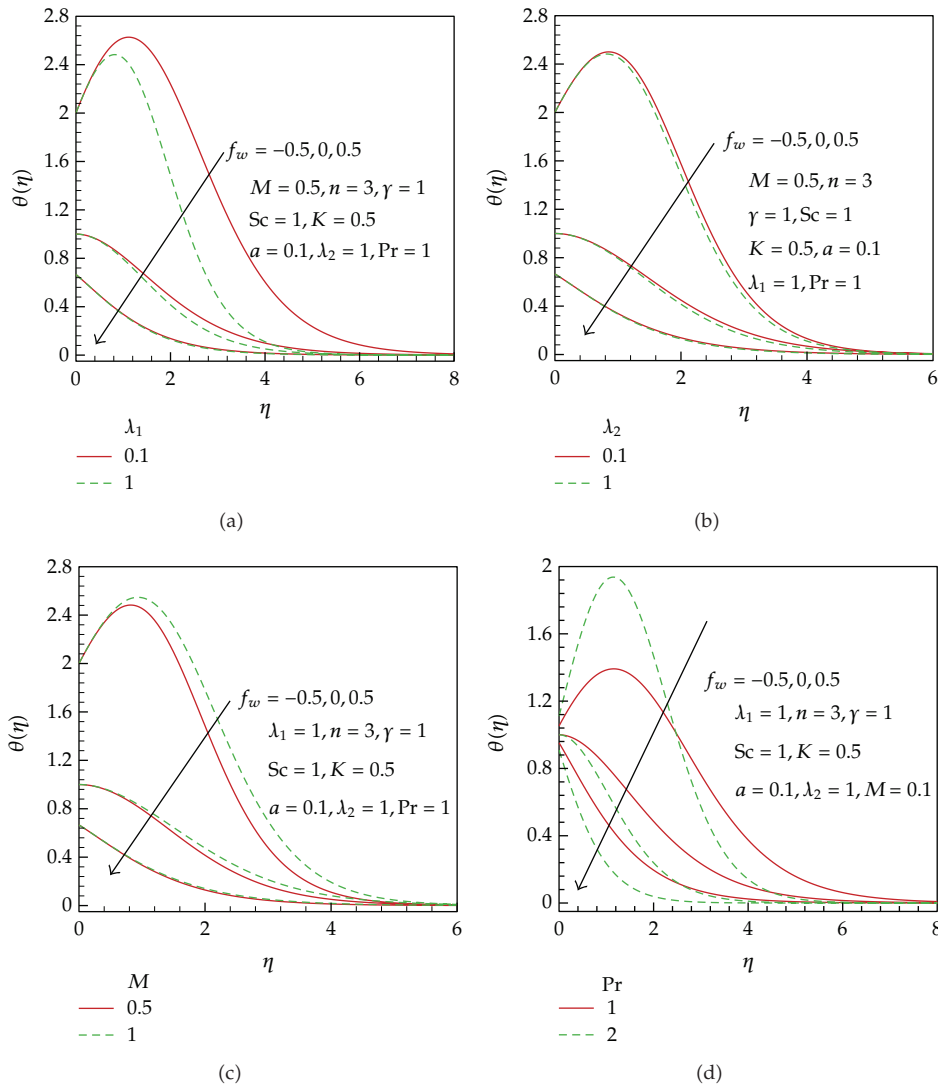


Figure 3: Effects of different parameters on the dimensionless temperature.

causes decrease in the dimensionless rate of heat transfer (Figure 6(b)). The variation of the dimensionless rate of mass transfer with different parameters is shown in Figures 7(a) and 7(b). Like the dimensionless rate of heat transfer, the dimensionless rate of mass transfer increases with an increase in Sc , K , and f_w (Figure 7(a)) and decreases with an increase in the order of reaction n .

5. Conclusions

MHD boundary layer equations for mixed convection of Newtonian fluids along a moving horizontal plate with velocity slip and thermal convective boundary condition are

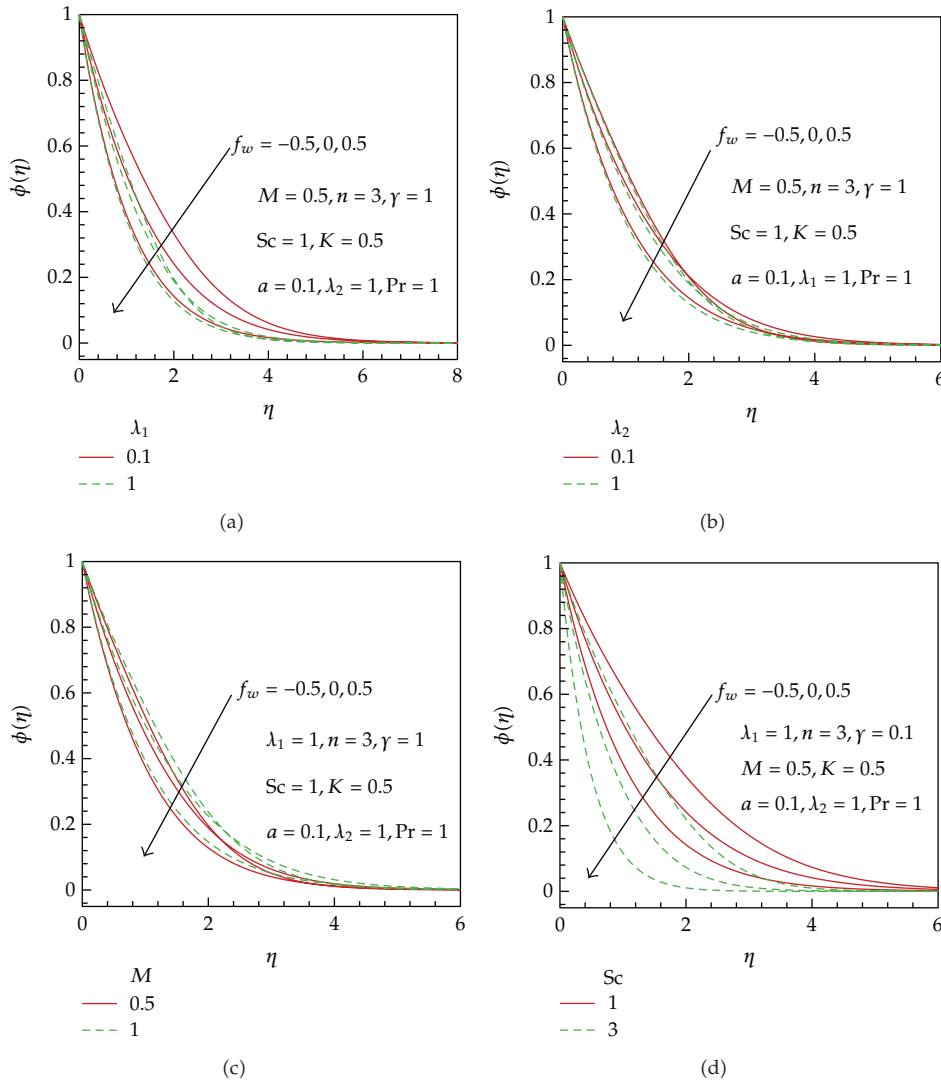


Figure 4: Effects of different parameters on the dimensionless concentration.

transformed into similarity equations using one-parameter continuous group method and then solved numerically using an implicit finite difference numerical method. Our analysis revealed that $-f''(0)$ increases with suction f_w and decreases with a , λ_1 , and λ_2 whilst the values of $-\theta'(0)$ enhance with f_w , Pr , and γ . The values of $-\phi'(0)$ enhance with f_w , Sc and k .

Nomenclature

- $a, B(x)$: Velocity slip parameter, variable magnetic field
- C, c_p : Concentration, specific heat at constant pressure
- c_{fx} : Skin friction factor
- D, f : Mass diffusivity, dimensionless stream function

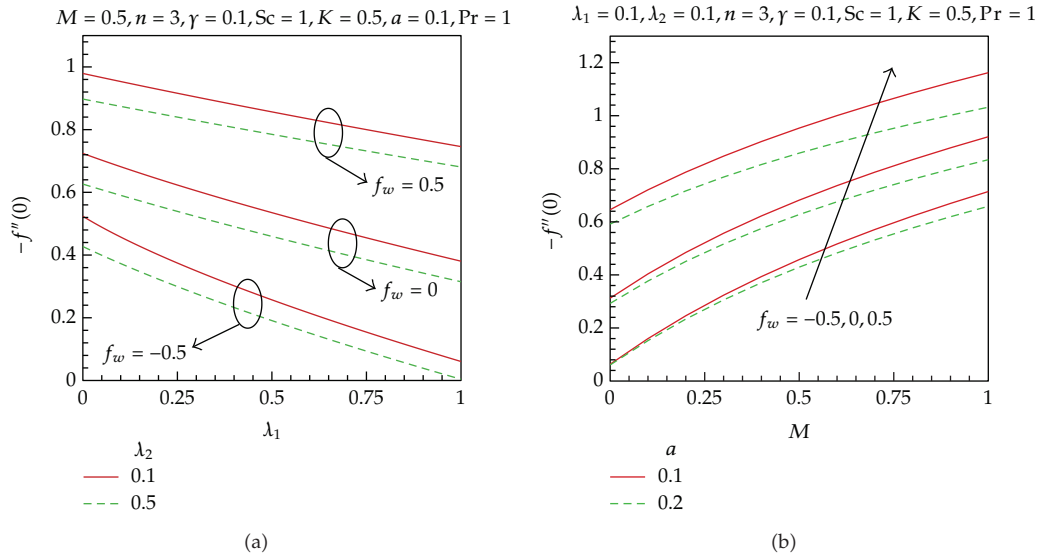


Figure 5: Effects of different parameters on the dimensionless shear stress.

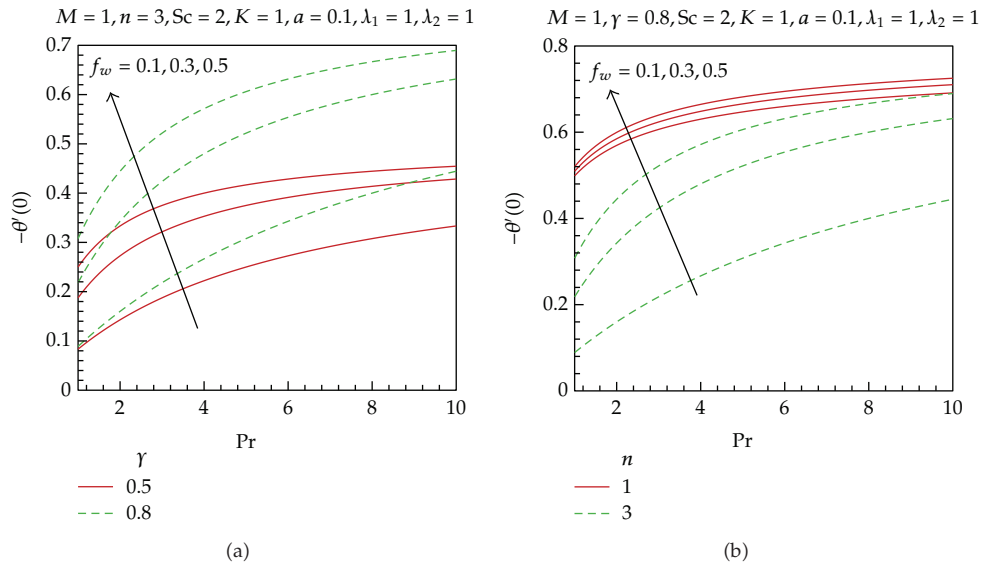


Figure 6: Effects of different parameters on the dimensionless rate of heat transfer.

- f_w, g : Suction/injection parameter, gravitational acceleration
- Gr, Gc : Thermal and solutal Grashof number
- $h_f(x)$: Heat transfer coefficient
- k, k_0 : Thermal conductivity, reaction rate constant
- K, L : Chemical reaction parameter, characteristic length of the plate

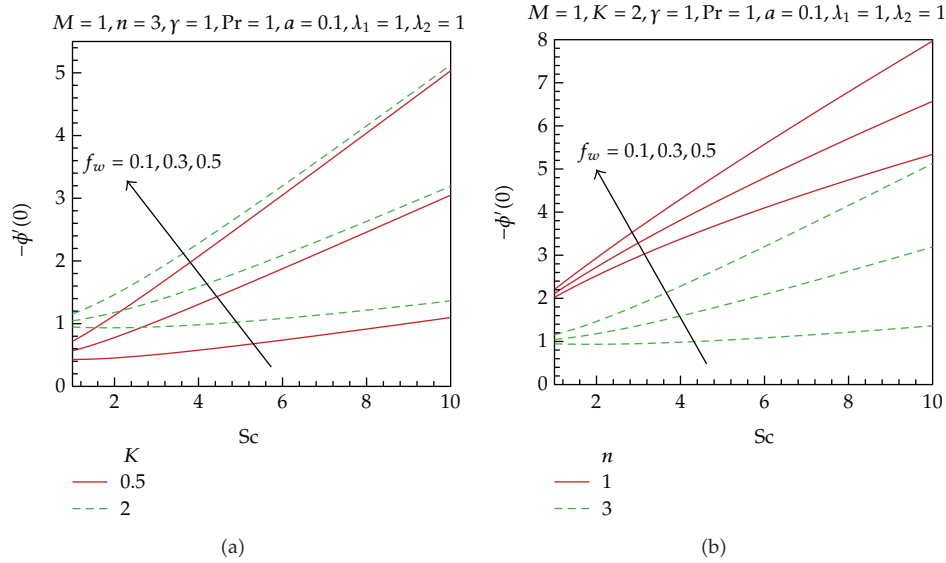


Figure 7: Effects of different parameters on the dimensionless rate of mass transfer.

- M, n, Nu_x : Magnetic parameter, order of chemical reaction, Nusselt number
- $N_1(x), p$: Variable slip factor
- Pr, Re, Sc : Prandtl, Reynolds, and Schmidt number
- $Sh_x T$: Sherwood number, dimensional temperature
- U, U_0 : Velocity of the plate and reference velocity
- u, v : Velocity components in x -direction and y -direction
- x, y : Coordinate along and normal to plate.

Greek Symbols

- α_i, β_i : Real numbers
- β_T, β_C : Coefficient of thermal and mass expansion
- γ, Γ : Convective heat transfer parameter, group
- μ, ν : Dynamic and kinematic coefficient of viscosity
- ε, σ : Parameter of the group transformations, electric conductivity
- λ_1, λ_2 : Thermal, concentration buoyancy parameter
- η, ψ : Similarity independent variable, stream function
- θ, ϕ : Dimensionless temperature and concentration function
- ρ : Density of the fluid.

Subscript

- ∞ : Condition at infinity
- w : Condition at the wall.

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