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Research Article

Exponential Synchronization Analysis and Control for Discrete-Time Uncertain Delay Complex Networks with Stochastic Effects

Tianbo Wang,[1,2] Wuneng Zhou,[2] Dejun Zhao,[1] and Shouwei Zhao[1]

[1] *College of Fundamental Studies, Shanghai University of Engineering Science, Shanghai 201620, China*
[2] *College of Information Sciences and Technology, Donghua University, Shanghai 201620, China*

Correspondence should be addressed to Tianbo Wang, tb_wang@163.com

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The exponential synchronization for a class of discrete-time uncertain complex networks with stochastic effects and time delay is investigated by using the Lyapunov stability theory and discrete Halanay inequality. The uncertainty arises from the difference of the nodes' reliability in the complex network. Through constructing an appropriate Lyapunov function and applying inequality technique, some synchronization criteria and two control methods are obtained to ensure the considered complex network being exponential synchronization. Finally, a numerical example is provided to show the effectiveness of our proposed methods.

1. Introduction

Since the discovery of small-world effect [1] and scale-free feature [2] of complex networks, many researchers in the fields of science and engineering have paid more attention to the topic and provided some valuable results which can be found in [3–9] and the references therein. Particularly, the broad application in the fields of ecosystems, the Internet, biological neural networks, and large-scale robotic system (see [10–12]), and so forth, promotes the complex network becoming a more significant topic.

Synchronization, as one of the important dynamical characters of the complex networks, has been studied in many papers. For example, the authors studied the pinning synchronization problem of stochastic impulsive network by using Lyapunov stability theory and provided some sufficient criteria to ensure that the dynamical network is asymptotical synchronization and exponential synchronization in mean square in [13]. Based on the parameter-dependent Lyapunov function, the authors considered the synchronization problem for a network family with different network structure and proposed some synchronization criteria in [14]. Similar with the continuous complex networks, there also exist many control methods to study the synchronization stability for discrete complex networks recently, which can be found in [15–20] and the references therein. For instance, the authors investigated the synchronization problem for the discrete-time complex networks with distributed time delays by using the Lyapunov stability theory, Kronecker product, and the linear matrix inequalities method in [17]. In [18], the authors revisited the synchronization stability problem for discrete complex dynamical networks with a time varying delay and constructed a new Lyapunov-Krasovskii functional by dividing the time-varying delay into a constant part and a variant part. In [20], the authors investigated the synchronization and state estimation problems for discrete-time complex network by utilizing a time varying real-valued function and the Kronecker product and provided a novel concept of bounded H_∞ synchronization.

However, in the real world, some nodes in a complex network usually do not normally work for some reasons. Particularly, this phenomenon easily appears in a complex network composed of many electronic components since that the reliability of every electric component exists the difference in general. The reason resulted in this phenomenon can be found in [21–23]. Therefore, it is necessary to study the synchronization problem for this kind of complex network with uncertain nodes. Motivated by the above discussion, we intend to study the exponential synchronization problem for a discrete-time uncertain complex network with stochastic effects in this paper. Different from some previous papers, the contributions of our paper are as follows. (1) We consider the uncertainty arising from the nodes' reliability in the complex network. (2) We consider the case that all the nodes in the complex network are effected by the working circumstance. (3) Our approach used in the paper is different from the methods in the papers listed.

The rest of this paper is organized as follows. In Section 2, the investigated discrete complex network and some necessary lemmas, assumptions are given. In Section 3, the exponential synchronization criteria and control methods for the complex network are derived. In Section 4, a numerical example is provided to illustrate the effectiveness of our method. Finally, this paper is ended with a conclusion in Section 5.

Notation 1. In this paper, R^n and $R^{n\times m}$, respectively, denote the n-dimensional Euclidean space and the set of all $n \times m$ real matrices. For a vector $x(t) = (x_1(t), x_2(t), \ldots, x_n(t))^T \in R^n$, $\|x(t)\| = \sqrt{\sum_{i=1}^{n} x_i^2(t)}$ denotes its norm. A^T denotes the transpose of matrix A. $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, \mathcal{P})$ denotes the complete probability space with a filtration $\{\mathcal{F}_t\}_{t\geq 0}$ satisfying right continuous and $\mathcal{F}_0$ containing all $\mathcal{P}$-null sets. I_n is the $n \times n$ identical matrix. $\mathbf{1}_n = (1, 1, \ldots, 1)^T$ and $\mathbf{1}_{n\times n} \in R^{n\times n}$ are an n-dimensional vector and an $n \times n$ matrix with all the elements being 1, respectively. $\otimes$ is the Kronecker product. $\lambda_{\max}(H)$ stands for the biggest eigenvalues of matrix H. $E\{\cdot\}$ denotes the mathematical expectation.

2. Preliminaries

In this paper, we consider the following discrete-time complex network consisting of N identical nodes with diffusive couplings. Each node is an n-dimensional dynamical system and the state equation is

$$x_i(k+1) = Ax_i(k) + f(x_i(k), x_i(k-\tau(k))) + c\sum_{j=1, j\neq i}^{N} \xi_i g_{ij}\Gamma\left[x_j(k-\tau(k)) - x_i(k-\tau(k))\right] + u_i(k)$$
$$+ \varphi(x_i(k))w(k), \quad i = 1, 2, \ldots, N, \tag{2.1}$$

where N is the number of coupled nodes. $x_i(k) = (x_{i1}(k), x_{i2}(k), \ldots, x_{in}(k))^T \in R^n$ is the state vector of node i at sampling time kT with sampling period $T > 0$, $A \in R^{n\times n}$ is a constant matrix, $f(\cdot) : R^n \times R^n \to R^n$ is a nonlinear vector function, and scalar $c > 0$ denotes the coupling strength. The working situation of every node in the complex network is described by two random events:

$$\begin{aligned} &\text{Event 1}: \text{ the node is available,} \\ &\text{Event 2}: \text{ the node is unavailable.} \end{aligned} \tag{2.2}$$

Random variables ξ_i $(i = 1, 2, \ldots, N)$ are defined as

$$\xi_i = \begin{cases} 1, & \text{if Event 1 occurs,} \\ 0, & \text{if Event 2 occurs,} \end{cases} \tag{2.3}$$

where $\xi_i (i = 1, 2, \ldots, N)$ are N independent random variables with mathematical expectation $E\{\xi_i\} = p_i$ and the variance $\mathrm{Var}\{\xi_i\} = q_i$. In practice, since the availability of each node in the considered complex network is usually not identical, so it is very reasonable to describe the working situation using different random variables for different nodes. Outer-coupling matrix

$$G = \begin{bmatrix} g_{11} & g_{12} & \cdots & g_{1N} \\ g_{21} & g_{22} & \cdots & g_{2N} \\ \cdots & \cdots & \cdots & \cdots \\ g_{N1} & g_{N2} & \cdots & g_{NN} \end{bmatrix} = \begin{bmatrix} g_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix}, \tag{2.4}$$

where $g_{ii} = -\sum_{j=1, j\neq i}^{N} g_{ij}$, $G_{12} = [\, g_{12} \; g_{13} \; \cdots \; g_{1N} \,]$, and $G_{21} = [\, g_{21} \; g_{31} \; \cdots \; g_{N1} \,]^T$. g_{ij} $(i, j = 1, 2, \ldots, N)$ are defined as follows: if there exists a connection between node i with node j, then $g_{ij} = 1$, or else $g_{ij} = 0$. Inner-coupling matrix $\Gamma \in R^{n\times n}$ is a positive definite diagonal matrix. $\tau(k)$ denotes the transmission time delay and satisfies $0 \leq \tau(k) \leq \tau$ for a positive scalar $\tau > 0$. $w(k)$ is a scalar Wiener process defined on a probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, \mathcal{P})$ with

$$E\{w(k)\} = 0, \qquad E\left\{w^2(k)\right\} = 1, \qquad E\{w(i)w(j)\} = 0, \quad i \neq j. \tag{2.5}$$

The noise strength $\varphi(\cdot) : R^n \to R^n$ is a vector function. $u_i(k) \in R^n$ $(i = 1, 2, \ldots, N)$ are the control input to be designed. The complex network (2.1) can be written as

$$x_i(k+1) = Ax_i(k) + f(x_i(k), x_i(k-\tau(k))) + c\sum_{j=1}^{N} \xi_i g_{ij} \Gamma x_j(k-\tau(k)) + u_i(k) + \varphi(x_i(k))w(k), \quad i = 1, 2, \ldots, N. \tag{2.6}$$

Letting $e_i(k) = x_i(k) - x_1(k)$, we get

$$e_i(k+1) = Ae_i(k) + f(x_i(k), x_i(k-\tau(k))) - f(x_1(k), x_1(k-\tau(k))) + c\sum_{j=1}^{N} \xi_i g_{ij} \Gamma x_j(k-\tau(k)) - c\sum_{j=1}^{N} \xi_1 g_{1j} \Gamma x_j(k-\tau(k)) + u_i(k) - u_1(k) + [\varphi(x_i(k)) - \varphi(x_1(k))]w(k), \quad i = 2, \ldots, N. \tag{2.7}$$

Define

$$e(k) = \left(e_2^T(k), e_3^T(k), \ldots, e_N^T(k)\right)^T, \quad \widehat{\xi} = \operatorname{diag}(\xi_2, \xi_3, \ldots, \xi_N), \quad \overline{\xi} = \xi_1 \cdot I_{N-1},$$
$$\widehat{P} = \operatorname{diag}(p_2, p_3, \ldots, p_N), \quad \overline{P} = p_1 \cdot I_{N-1}, \quad \widehat{Q} = \operatorname{diag}(q_2, q_3, \ldots, q_N), \quad \overline{Q} = q_1 \cdot I_{N-1},$$
$$F_i(e_i(k)) = f(x_i(k), x_i(k-\tau(k))) - f(x_1(k), x_1(k-\tau(k))),$$
$$F(e(k)) = \left(F_2^T(e_2(k)), F_3^T(e_3(k)), \ldots, F_N^T(e_N(k))\right)^T,$$
$$u(k) = \left(u_2^T(k), u_3^T(k), \ldots, u_N^T(k)\right)^T,$$
$$\Psi_i(e_i(k)) = \varphi(x_i(k)) - \varphi(x_1(k)), \quad \Psi(e(k)) = \left(\Psi_2^T(e_2(k)), \Psi_3^T(e_3(k)), \ldots, \Psi_N^T(e_N(k))\right)^T,$$
$$G_1 = \begin{bmatrix} g_{12} & g_{13} & \cdots & g_{1N} \\ g_{12} & g_{13} & \cdots & g_{1N} \\ \cdots & \cdots & \cdots & \cdots \\ g_{12} & g_{13} & \cdots & g_{1N} \end{bmatrix} \in R^{(N-1)\times(N-1)}, \tag{2.8}$$

then the error system (2.7) can be written as the following form

$$e(k+1) = (I_{N-1} \otimes A)e(k) + F(e(k)) + c\left(\widehat{\xi} G_{22}\right) \otimes \Gamma e(k-\tau(k)) - c\left(\overline{\xi} G_1\right) \otimes \Gamma e(k-\tau(k)) + u(k) - \mathbf{1}_{N-1} \otimes u_1(k) + \Psi(e(k))w(k). \tag{2.9}$$

Note that (2.9) is equivalent to

$$\begin{aligned}e(k+1) = {} & (I_{N-1}\otimes A)e(k) + F(e(k)) + c\left(\widehat{P}G_{22}\right)\otimes\Gamma e(k-\tau(k)) - c\left(\overline{P}G_1\right)\otimes\Gamma e(k-\tau(k)) \\ & + c\left[\left(\widehat{\xi}-\widehat{P}\right)G_{22}\right]\otimes\Gamma e(k-\tau(k)) - c\left[\left(\overline{\xi}-\overline{P}\right)G_1\right]\otimes\Gamma e(k-\tau(k)) \\ & + u(k) - \mathbf{1}_{N-1}\otimes u_1(k) + \Psi(e(k))w(k).\end{aligned} \tag{2.10}$$

Letting $\Theta_1 = c\widehat{P}G_{22} - c\overline{P}G_1$, $\Theta_2 = c(\widehat{\xi}-\widehat{P})G_{22} - c(\overline{\xi}-\overline{P})G_1$, then we have

$$\begin{aligned}e(k+1) = {} & (I_{N-1}\otimes A)e(k) + F(e(k)) + \Theta_1\otimes\Gamma e(k-\tau(k)) + \Theta_2\otimes\Gamma e(k-\tau(k)) + u(k) \\ & - \mathbf{1}_{N-1}\otimes u_1(k) + \Psi(e(k))w(k).\end{aligned} \tag{2.11}$$

Throughout this paper, the following assumptions are needed.

(A1) The nonlinear vector function $f(\cdot)$ in the system (2.1) satisfies

$$\begin{aligned}&\left\| f(x(k), x(k-\tau(k))) - f(y(k), y(k-\tau(k)))\right\|^2 \\ &\quad \le L_1\left\|x(k)-y(k)\right\|^2 + L_2\left\|x(k-\tau(k)) - y(k-\tau(k))\right\|^2\end{aligned} \tag{2.12}$$

for any $x(k)\in R^n$ and $y(k)\in R^n$, where $L_1 \ge 0$ and $L_2 \ge 0$ are positive constants.

From (2.12), it can be verified that

$$\|F_i(e_i(k))\|^2 \le L_1\|e_i(k)\|^2 + L_2\|e_i(k-\tau(k))\|^2 \tag{2.13}$$

for $i = 2, 3, \ldots, N$.

(A2) There exists a positive constant $M > 0$ such that the nonlinear vector function $\varphi(\cdot)$ in the system (2.1) satisfies

$$\left\|\varphi(x(k)) - \varphi(y(k))\right\| \le M\left\|x(k) - y(k)\right\| \tag{2.14}$$

for any $x(k)\in R^n$ and $y(k)\in R^n$.

From (2.14), one can conclude that

$$\begin{aligned}\Psi^T(e(k))\Psi(e(k)) &= \sum_{i=2}^{N}\Psi_i^T(e_i(k))\Psi_i(e_i(k))\\ &\le \sum_{i=2}^{N}M^2e_i^T(k)e_i(k)\\ &= M^2e^T(k)e(k).\end{aligned} \tag{2.15}$$

Definition 2.1. The complex network (2.1) is said to be exponential synchronization in mean square if there exist positive constants $h > 0$ and $\gamma \in (0,1)$ such that

$$E\left\{\|x_i(k) - x_j(k)\|^2\right\} \le h\gamma^k, \quad i,j = 1,2,\ldots,N,\ k = 1,2,\ldots \tag{2.16}$$

for any initial values $x(s), s = -\tau,\ldots,0$, where γ is called the exponential convergence rate.

Remark 2.2. From Definition 2.1, it is easy to see that the complex network (2.1) is exponential synchronization in mean square only if there exist positive constants $h > 0$ and $\gamma \in (0,1)$ such that

$$E\left\{\|x_i(k) - x_1(k)\|^2\right\} \le h\gamma^k, \quad i = 2,\ldots,N,\ k = 1,2,\ldots \tag{2.17}$$

for any initial values $x(s), s = -\tau,\ldots,0$.

Remark 2.3. The complex network model (2.1) not only includes time delay and stochastic disturbances, but also considers the uncertainty of nodes' working situation. To date, there have existed many literatures [13, 15, 19] to study the synchronization control problem for discrete-time complex networks. However, for this case, there exist less results. Moreover, different from [13, 17], we are not necessary to use the information of target node given beforehand in the paper.

Lemma 2.4 (see [24]). *Let $d > 0$ be a natural number and $\{U(k)\}_{k\ge -d}$ a sequence of real numbers satisfying the inequality*

$$\Delta U(k) \le -aU(k) + b\cdot\max\{U(k), U(k-1),\ldots,U(k-d)\}, \quad k \ge 0, \tag{2.18}$$

where $\Delta U(k) = U(k+1) - U(k)$. If $0 < b < a \le 1$, then there exists a constant $\eta_0 \in (0,1)$ such that

$$U(k) \le \max\{0, U(0), U(-1),\ldots,U(-d)\}\eta_0^k, \quad k \ge 0. \tag{2.19}$$

Moreover, η_0 *can be chosen as the root of the equation*

$$\eta^{d+1} + (a-1)\eta^d - b = 0 \tag{2.20}$$

in the interval $(0, 1)$.

Lemma 2.5 (see [25]). *The Kronecker product* $\otimes$ *has the following properties:*

(1) $(A + B) \otimes C = A \otimes C + B \otimes C, C \otimes (A + B) = C \otimes A + C \otimes B,$

(2) $(A \otimes B)^T = A^T \otimes B^T,$

(3) $(A \otimes B)^{-1} = A^{-1} \otimes B^{-1},$

(4) $(A \otimes C)(B \otimes D) = AB \otimes CD.$

where, $A, B, C,$ *and* D *are real matrices with appropriate dimensions.*

3. Synchronization Analysis and Control

In this section, we will derive some synchronization criteria for the complex network (2.1) without input and two different synchronization control methods, respectively.

Theorem 3.1. *Under assumptions (A1)~(A2), if there exist positive constants* $\delta_1 > 0, \delta_2 > 0, \alpha > 0,$ *and* $\beta > 0$ *such that*

$$\begin{gathered}
\beta < \alpha \leq 1, \\
\Pi_1 = \begin{bmatrix} I_{N-1} \otimes \left[\left(M^2 + \alpha\right) \cdot I_n + (1 + \delta_1 + \delta_2) L_1 \cdot I_n + A^T A - I_n\right] & I_{N-1} \otimes A^T \\ I_{N-1} \otimes A & -\delta_1 \cdot I_{n(N-1)} \end{bmatrix} < 0, \\
\Pi_2 = \begin{bmatrix} (1 + \delta_1 + \delta_2) L_2 \cdot I_{n(N-1)} - \beta I_{n(N-1)} + \left(\Theta_1^T \Theta_1 + \Theta_3\right) \otimes \Gamma^T \Gamma & \Theta_1^T \otimes \Gamma^T \\ \Theta_1 \otimes \Gamma & -\delta_2 \cdot I_{n(N-1)} \end{bmatrix} < 0,
\end{gathered} \tag{3.1}$$

where $\Theta_3 = c^2 G_{22}^T \widehat{Q} G_{22} + c^2 G_1^T \overline{Q} G_1$, *then the complex network* (2.1) *without input is exponential synchronization in mean square.*

Proof. Choosing the following Lyapunov function:

$$V(e(k)) = e^T(k) e(k), \tag{3.2}$$

and calculating the difference of $V(e(k))$ along the trajectories of the system (2.11) without the input, we get

$$\begin{aligned}
E\{\Delta V(e(k))\} &= E\{V(e(k+1)) - V(e(k))\} \\
&= E\Big\{[(I_{N-1}\otimes A)e(k) + F(e(k)) + \Theta_1\otimes\Gamma e(k-\tau(k)) + \Theta_2\otimes\Gamma e(k-\tau(k))]^T \\
&\qquad \times [(I_{N-1}\otimes A)e(k) + F(e(k)) + \Theta_1\otimes\Gamma e(k-\tau(k)) + \Theta_2\otimes\Gamma e(k-\tau(k))] \\
&\qquad - e^T(k)e(k) + \Psi^T(e(k))\Psi(e(k))\Big\} \\
&= E\Big\{e^T(k)\Big[I_{N-1}\otimes\Big(A^TA\Big)\Big]e(k) + 2e^T(k)\Big(I_{N-1}\otimes A^T\Big)F(e(k)) \\
&\qquad + 2e^T(k)\Big(\Theta_1\otimes A^T\Gamma\Big)e(k-\tau(k)) + 2e^T(k)\Big(\Theta_2\otimes A^T\Gamma\Big)e(k-\tau(k)) \\
&\qquad + F^T(e(k))F(e(k)) + 2F^T(e(k))(\Theta_1\otimes\Gamma)e(k-\tau(k)) \\
&\qquad + 2F^T(e(k))(\Theta_2\otimes\Gamma)e(k-\tau(k)) \\
&\qquad + e^T(k-\tau(k))\Big[\Big(\Theta_1^T\Theta_1 + 2\Theta_1^T\Theta_2 + \Theta_2^T\Theta_2\Big)\otimes\Gamma^T\Gamma\Big]e(k-\tau(k)) \\
&\qquad - e^T(k)e(k) + \Psi^T(e(k))\Psi(e(k))\Big\} \\
&= E\Big\{e^T(k)\Big[I_{N-1}\otimes\Big(A^TA - I_n\Big)\Big]e(k) + 2e^T(k)\Big(I_{N-1}\otimes A^T\Big)F(e(k)) \\
&\qquad + 2e^T(k)\Big(\Theta_1\otimes A^T\Gamma\Big)e(k-\tau(k)) + F^T(e(k))F(e(k)) \\
&\qquad + 2F^T(e(k))(\Theta_1\otimes\Gamma)e(k-\tau(k)) + \Psi^T(e(k))\Psi(e(k)) \\
&\qquad + e^T(k-\tau(k))\Big[\Big(\Theta_1^T\Theta_1 + \Theta_3\Big)\otimes\Gamma^T\Gamma\Big]e(k-\tau(k))\Big\}.
\end{aligned} \tag{3.3}$$

It is noted that

$$\begin{aligned}
&E\Big\{2e^T(k)\Big(I_{N-1}\otimes A^T\Big)F(e(k))\Big\} \\
&\quad\le E\Big\{\delta_1^{-1}e^T(k)\Big(I_{N-1}\otimes A^T\Big)(I_{N-1}\otimes A)e(k) + \delta_1F^T(e(k))F(e(k))\Big\} \\
&\quad\le E\Big\{\delta_1^{-1}e^T(k)\Big(I_{N-1}\otimes A^T\Big)(I_{N-1}\otimes A)e(k) \\
&\qquad + L_1\delta_1e^T(k)e(k) + L_2\delta_1e^T(k-\tau(k))e(k-\tau(k))\Big\},
\end{aligned}$$

$$\begin{aligned}
&E\left\{2F^T(e(k))(\Theta_1\otimes\Gamma)e(k-\tau(k))\right\}\\
&\quad\leq E\left\{\delta_2F^T(e(k))F(e(k))+\delta_2^{-1}e^T(k-\tau(k))\left(\Theta_1^T\otimes\Gamma^T\right)(\Theta_1\otimes\Gamma)e(k-\tau(k))\right\}\\
&\quad\leq E\left\{\delta_2L_1e^T(k)e(k)+\delta_2L_2e^T(k-\tau(k))e(k-\tau(k))\right.\\
&\qquad\left.+\ \delta_2^{-1}e^T(k-\tau(k))\left(\Theta_1^T\otimes\Gamma^T\right)(\Theta_1\otimes\Gamma)e(k-\tau(k))\right\},\\
&E\left\{F^T(e(k))F(e(k))\right\}\leq E\left\{L_1e^T(k)e(k)+L_2e^T(k-\tau(k))e(k-\tau(k))\right\},\\
&\Psi^T(e(k))\Psi(e(k))\leq M^2e^T(k)e(k).
\end{aligned}\tag{3.4}$$

From (3.4), one can get

$$\begin{aligned}
E\{\Delta V(e(k))\}&\leq E\left\{e^T(k)\left[I_{N-1}\otimes\left(A^TA-I_n\right)+\delta_1^{-1}\left(I_{N-1}\otimes A^T\right)(I_{N-1}\otimes A)\right.\right.\\
&\qquad\left.+\ (1+\delta_1+\delta_2)L_1\cdot I_{n(N-1)}+M^2\cdot I_{n(N-1)}\right]e(k)\\
&\qquad+e^T(k-\tau(k))\left[\left(\Theta_1^T\Theta_1+\Theta_3\right)\otimes\Gamma^T\Gamma+(1+\delta_1+\delta_2)L_2\cdot I_{n(N-1)}\right.\\
&\qquad\left.\left.+\ \delta_2^{-1}\left(\Theta_1^T\otimes\Gamma^T\right)(\Theta_1\otimes\Gamma)\right]e(k-\tau(k))\right\}\\
&\leq E\left\{e^T(k)\Omega_1e(k)+e^T(k-\tau(k))\Omega_2e(k-\tau(k))\right\},
\end{aligned}\tag{3.5}$$

where

$$\begin{aligned}
\Omega_1&=M^2\cdot I_{n(N-1)}+(1+\delta_1+\delta_2)L_1\cdot I_{n(N-1)}+I_{N-1}\otimes\left(A^TA-I_n\right)+\delta_1^{-1}\left(I_{N-1}\otimes A^T\right)(I_{N-1}\otimes A),\\
\Omega_2&=(1+\delta_1+\delta_2)L_2\cdot I_{n(N-1)}+\left(\Theta_1^T\Theta_1+\Theta_3\right)\otimes\Gamma^T\Gamma+\delta_2^{-1}\left(\Theta_1^T\otimes\Gamma^T\right)(\Theta_1\otimes\Gamma).
\end{aligned}\tag{3.6}$$

By the Schur complement lemma, we know that (3.1) is equivalent to $\Omega_1<-\alpha I_{n(N-1)}$ and $\Omega_2<\beta I_{n(N-1)}$. So, we have

$$E\{\Delta V(e(k))\}\leq E\{-\alpha V(e(k))+\beta\cdot\max\{V(e(k)),V(e(k-1)),\ldots,V(e(k-\tau))\}\}.\tag{3.7}$$

By Lemma 2.4, there exists a constant $\eta_0\in(0,1)$ such that

$$E\{V(e(k))\}\leq\max\{V(e(0)),V(e(-1)),\ldots,V(e(-\tau))\}\eta_0^k,\quad k\geq 0.\tag{3.8}$$

In particular, η_0 is the root of the equation

$$\eta^{d+1} + (\alpha - 1)\eta^{d} - \beta = 0 \tag{3.9}$$

in the interval $(0, 1)$. Therefore, the complex network (2.1) is exponential synchronization in mean square. This completes the proof of Theorem 3.1. □

While using the following state feedback controller:

$$u_i(k) = -kx_i(k), \quad i = 1, 2, \ldots, N, \tag{3.10}$$

to control every node in the complex network (2.1), we can obtain the error system

$$\begin{aligned} e(k+1) = {} & [I_{N-1} \otimes (A - kI_n)]e(k) + F(e(k)) + \Theta_1 \otimes \Gamma e(k - \tau(k)) \\ & + \Theta_2 \otimes \Gamma e(k - \tau(k)) + \Psi(e(k))w(k), \end{aligned} \tag{3.11}$$

where $k > 0$ is the control gain to be determined. So, by Theorem 3.1, we can obtain the following result.

Theorem 3.2. *Under assumptions (A1)~(A2), if there exist positive constants $k > 0$, $\delta_1 > 0, \delta_2 > 0, \alpha > 0$, and $\beta > 0$ such that*

$$\begin{gathered} \beta < \alpha \leq 1, \\ \widehat{\Pi}_1 = \begin{bmatrix} \widehat{\Pi}_{1,11} & I_{N-1} \otimes (A - kI_n)^T & I_{N-1} \otimes (A - kI_n)^T \\ I_{N-1} \otimes (A - kI_n) & -\delta_1 \cdot I_{n(N-1)} & 0 \\ I_{N-1} \otimes (A - kI_n) & 0 & -I_{n(N-1)} \end{bmatrix} < 0, \\ \Pi_2 = \begin{bmatrix} (1 + \delta_1 + \delta_2)L_2 \cdot I_{n(N-1)} - \beta I_{n(N-1)} + \left(\Theta_1^T \Theta_1 + \Theta_3\right) \otimes \Gamma^T \Gamma & \Theta_1^T \otimes \Gamma^T \\ \Theta_1 \otimes \Gamma & -\delta_2 \cdot I_{n(N-1)} \end{bmatrix} < 0, \end{gathered} \tag{3.12}$$

where

$$\widehat{\Pi}_{1,11} = I_{N-1} \otimes \left\{ \left[M^2 + \alpha + (1 + \delta_1 + \delta_2)L_1 - 1 \right] \cdot I_n \right\}, \tag{3.13}$$

then the complex network (2.1) *is exponential synchronization in mean square under the action of the controller* (3.10).

While using the pinning controller to control arbitrary l nodes in the complex network (2.1), we suppose that the number of the controlled nodes are $2, 3, \ldots, l + 1$, respectively. Substituting the following control law:

$$u_i(k) = -k_i x_i(k), \quad i = 2, 3, \ldots, l + 1, \tag{3.14}$$

into the error system (2.11), we get

$$e(k+1) = [I_{N-1} \otimes A - K \otimes I_n]e(k) + F(e(k)) + \Theta_1 \otimes \Gamma e(k-\tau(k)) + \Theta_2 \otimes \Gamma e(k-\tau(k)) + \Psi(e(k))w(k), \tag{3.15}$$

where $k_i > 0$ $(i = 2, 3, .., l+1)$ are the control gains to be determined, $K = \mathrm{diag}(k_2, k_3, \dots, k_{l+1}, \underbrace{0, \dots, 0}_{N-1-l})$. By Theorem 3.1, we can obtain the following result.

Theorem 3.3. *Under assumptions (A1)~(A2), if there exist positive constants $k_i > 0$ (i=2,3,...,l+1), $\delta_1 > 0, \delta_2 > 0, \alpha > 0$, and $\beta > 0$ such that*

$$\beta < \alpha \leq 1,$$
$$\widetilde{\Pi}_1 = \begin{bmatrix} \widetilde{\Pi}_{1,11} & I_{N-1} \otimes A^T - K \otimes I_n & I_{N-1} \otimes A^T - K \otimes I_n \\ I_{N-1} \otimes A - K \otimes I_n & -\delta_1 \cdot I_{n(N-1)} & 0 \\ I_{N-1} \otimes A - K \otimes I_n & 0 & -I_{n(N-1)} \end{bmatrix} < 0,$$
$$\Pi_2 = \begin{bmatrix} (1+\delta_1+\delta_2)L_2 \cdot I_{n(N-1)} - \beta I_{n(N-1)} + \left(\Theta_1^T \Theta_1 + \Theta_3\right) \otimes \Gamma^T \Gamma & \Theta_1^T \otimes \Gamma^T \\ \Theta_1 \otimes \Gamma & -\delta_2 \cdot I_{n(N-1)} \end{bmatrix} < 0, \tag{3.16}$$

where

$$\widetilde{\Pi}_{1,11} = I_{N-1} \otimes \left\{ \left[M^2 + \alpha + (1+\delta_1+\delta_2)L_1 - 1 \right] \cdot I_n \right\}, \tag{3.17}$$

then the complex network (2.1) *is exponential synchronization in mean square under the action of the pinning controller* (3.14).

Remark 3.4. If the time delay $\tau(k) = 0$ in the complex network (2.1), applying the same method in the paper, we can also obtain the synchronization criteria and synchronization controllers for the following complex network:

$$x_i(k+1) = Ax_i(k) + f(x_i(k)) + c \sum_{j=1, j \neq i}^{N} \xi_i g_{ij} \Gamma \left[x_j(k) - x_i(k) \right] + u_i(k) + \varphi(x_i(k))w(k) \tag{3.18}$$

for $i = 1, 2, \dots, N$.

Remark 3.5. Similar with [21–23], we will investigate the H_∞ synchronization for the uncertain complex network (2.1) in our future work.

4. A Numerical Example

Example 4.1. Consider the complex network (2.1) with ten nodes, and let each node be a three-dimensional dynamical subsystem whose parameters are as follows: $A = \operatorname{diag}\{0.3, 0.5, 0.4\}$,

$$f(x_i(k), x_i(k-\tau(k))) = \begin{bmatrix} \tanh(0.2x_{i1}(k)) + \tanh[-0.4x_{i1}(k-\tau(k))] \\ \tanh(0.3x_{i2}(k)) + \tanh[-0.2x_{i2}(k-\tau(k))] \\ \tanh(0.4x_{i3}(k)) + \tanh[0.1x_{i3}(k-\tau(k))] \end{bmatrix},$$

$$c = 0.1, \qquad \Gamma = \operatorname{diag}\{0.1, 0.2, 0.3\}, \qquad \tau(k) = 2 + \frac{1}{k}, \qquad \varphi(x_i(k)) = 0.4x_i(k),$$

$$G = \begin{bmatrix} -6 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 0 & 1 \\ 1 & -5 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & -6 & 0 & 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & -5 & 1 & 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & -7 & 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 & -6 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & -5 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 & 0 & -7 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & -5 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 0 & -7 \end{bmatrix}, \tag{4.1}$$

$$E\{\xi_1\} = 0.6, \qquad E\{\xi_2\} = 0.7, \qquad E\{\xi_3\} = 1, \qquad E\{\xi_4\} = 0.9, \qquad E\{\xi_5\} = 0.7,$$

$$E\{\xi_6\} = 1, \qquad E\{\xi_7\} = 0.5, \qquad E\{\xi_8\} = 0.8, \qquad E\{\xi_9\} = 0.6, \qquad E\{\xi_{10}\} = 0.9.$$

It is easy to verify that assumptions (A1)~(A2) hold while $L_1 = L_2 = M = 0.2$. By the LMI toolbox in the Matlab, we can obtain a feasible solution of inequalities (3.12) as follows:

$$\delta_1 = 0.2748, \qquad \delta_2 = 0.3624, \qquad \alpha = 0.5126, \qquad \beta = 0.5047, \qquad k = 0.4003. \tag{4.2}$$

Therefore, according to Theorem 3.2, we know that all the nodes in the complex network can exponentially synchronize each other. The state error curves are shown in Figure 1, and these figures show that all the nodes synchronize well. However, for this example, inequalities (3.16) are infeasible. So, from Theorem 3.3, we know that all the nodes in the complex network cannot achieve exponential synchronization by using the pinning controller (3.14).

5. Conclusions

This paper has investigated the exponential synchronization problem for a class of discrete-time uncertain delay complex network with stochastic effects based on the Lyapunov stability theory and discrete Halanay inequality and provided some synchronization criteria and two different control schemes. Different from some existing results, this paper has considered the uncertainty arising from the nodes' working situation. Moreover, we do not need the state information of the target node given beforehand. The numerical illustration has shown that our proposed methods are effective.

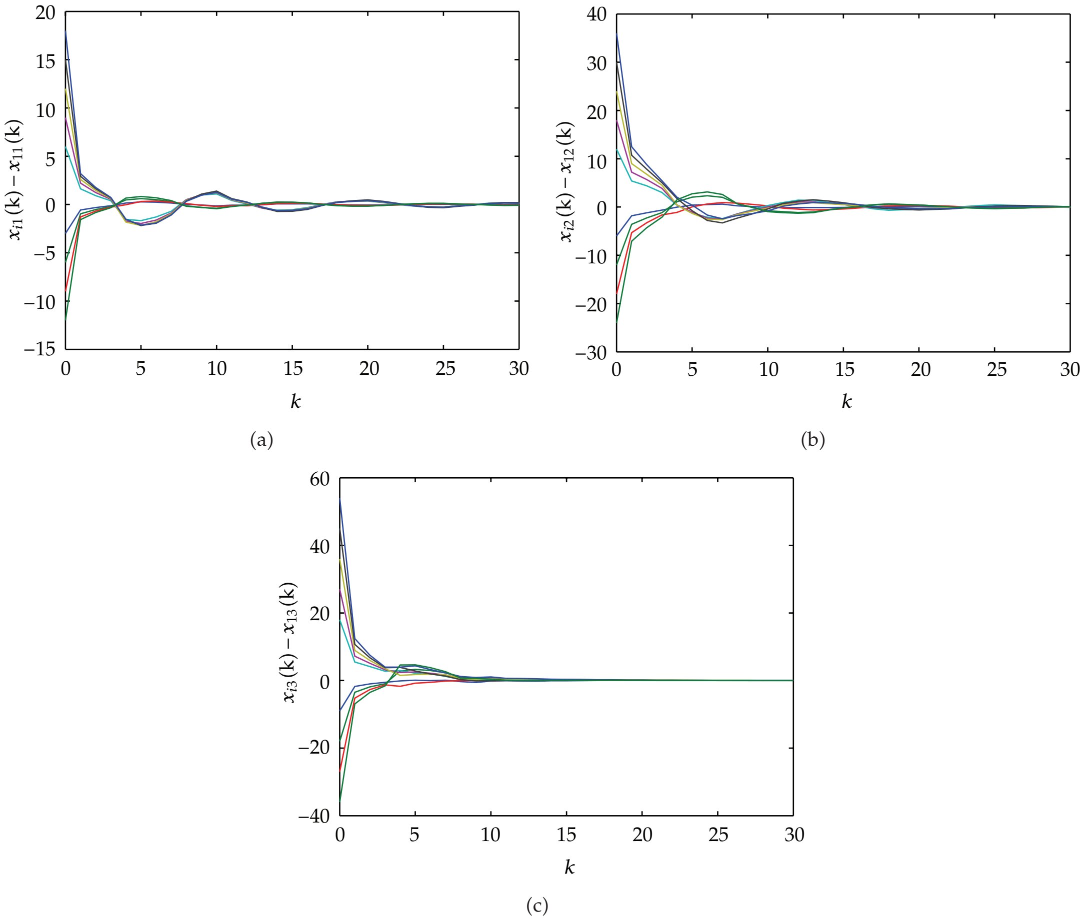

Figure 1: The state error curves of the complex network (2.1) with the given parameters in Example 4.1 ($i = 2, \ldots, 10$).

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