Hindawi Publishing Corporation
Mathematical Problems in Engineering
Volume 2012, Article ID 578291, 20 pages
doi:10.1155/2012/578291

Research Article

Structural Analysis of Large-Scale Power Systems

K. F. Zhang and X. Z. Dai

School of Automation, Southeast University, Nanjing, Jiangsu 210096, China

Correspondence should be addressed to K. F. Zhang, kaifengzhang@seu.edu.cn

Received 9 December 2011; Revised 14 February 2012; Accepted 16 February 2012

Academic Editor: Xi-Ming Sun

Some fundamental structural characteristics of large-scale power systems are analyzed in the paper. Firstly, the large-scale power system is decomposed into various hierarchical levels: the main system, subsystems, sub-subsystems, down to its basic components. The proposed decomposition method is suitable for arbitrary system topology, and the relations among various decomposed hierarchical levels are explicitly expressed by introducing the interface concept. Then, the structural models of various hierarchical levels are constructed in a bottom-up manner. The constructed hierarchical model can reveal the self-similarity characteristic of large-scale power systems.

1. Introduction

Analysis of the structure of power systems is a very important task for many problems, such as the time-scale simulation, control strategy design, and so forth. There are some important specialties that should be considered when analyzing the structure of power systems. For example, (1) power system is a large-scale system; (2) the topology of power system is very complex and power systems are normally with multilevel/hierarchical structures [1]; (3) various components of power systems are interconnected with each other via electrical network (not interconnected directly), and without considering electromagnetic effect, power systems' networks follow basic principles of electrical networks.

Aiming at the specialties of power systems, a lot of work has been done to analyze the structure of power systems. In the early time, when constructing the structure preserving model (SPM) [2, 3] and component connection model (CCM) [4], the "planar" structural characteristics or the interconnection relations between components (mainly generators and loads) and AC grid are analyzed. However, there are seldom literatures discussing the hierarchical structural characteristics of modern power systems. In [5], the relations among subsystems of power systems are investigated, but the results have not been extended to the analysis of more hierarchical levels and are less flexible.

In recent years, in order to satisfy the demand of large-scale numerical simulations (such as parallel computation, etc.), the analysis of the complex topology and hierarchical structure of modern power systems has drawn more attention. For example, in [6], under the platform of the AnyLogic Simulation Software, the hierarchical characteristics are investigated; in [7], the concept of multilevel MATE (Multi-Area Thévénin Equivalent) is proposed, and thus power system networks could be partitioned into multilevels; in [8], a power system is decomposed and described by a component tree, which is valuable for the computation parallelism and programming flexibility.

In the area of structural analysis of power systems, there is a prominent phenomenon, which is that the analysis purpose is mainly for system's analysis and simulations, and little attention is paid to the needs of other issues (such as designing control strategy). For example, when analyzing the interconnection relations between components and AC grid, the current and voltage vectors are normally used to describe the relations. However, some local measurable variables that are very meaningful for designing component controllers (as feedback variables) are not used, such as the amplitude of terminal voltage and the active and reactive powers. In the previous work of the authors [9], some immeasurable variables of synchronous generator are transformed into the local measurable variables, but there is still lack of systematic methods.

Moreover, until present there is still a lack of systematic hierarchical decomposition and modeling method of large-scale power systems, and some internal structural characteristics are still not revealed.

In the paper, firstly the large-scale power systems will be decomposed into various hierarchical levels (such as main system, subsystems, sub-subsystems, and components) from top to bottom. Secondly, the relations among various levels will be analyzed, described, and classified by introducing the interface concepts. Thirdly, with a well-defined rule, the structural models of various hierarchical levels will be constructed one by one in an order of component, sub-subsystem, subsystem, and main system. On constructing the hierarchical structural model, a natural characteristic of power systems, or the self-similar characteristic, can be revealed. Finally, the application of the proposed structural model of power systems in designing decentralized controller will be demonstrated briefly.

2. Hierarchical Decomposition of Large-Scale Power Systems

2.1. An Example of Hierarchical Decomposition

In this section, the large-scale power system will be decomposed into various hierarchical levels (such as main system, subsystem, sub-subsystem, . . ., down to its components). In the following, the IEEE 39-bus system will be chosen as an example to illustrate the hierarchical decomposition.

Firstly, one may divide the whole IEEE 39-bus system (named main system) into three subsystems (see Figure 1). In Figure 1, all of the three subsystems are interconnected with each other via corresponding transmission lines. A main grid could then be used to explicitly describe the relation among these three subsystems (see Figure 1). As seen from Figure 1, the main grid only consists of some interconnection lines and would be called the "virtual" main grid.

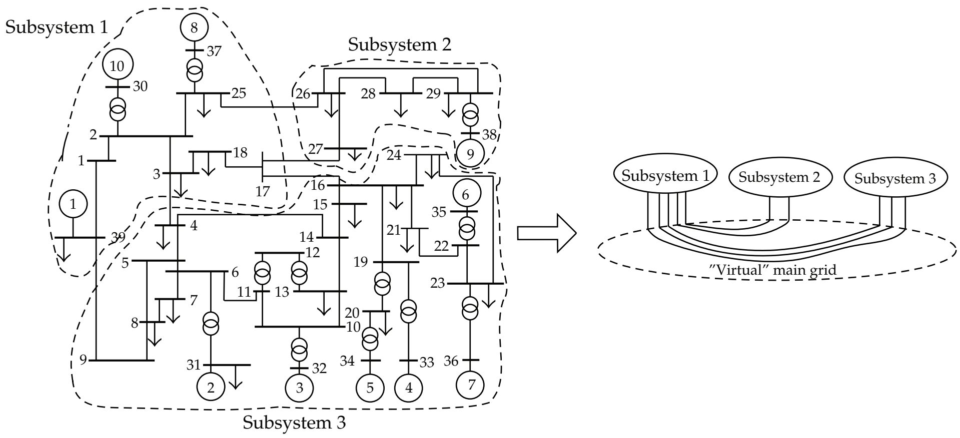

Figure 1: Decomposition of the IEEE 39-bus system into three subsystems.

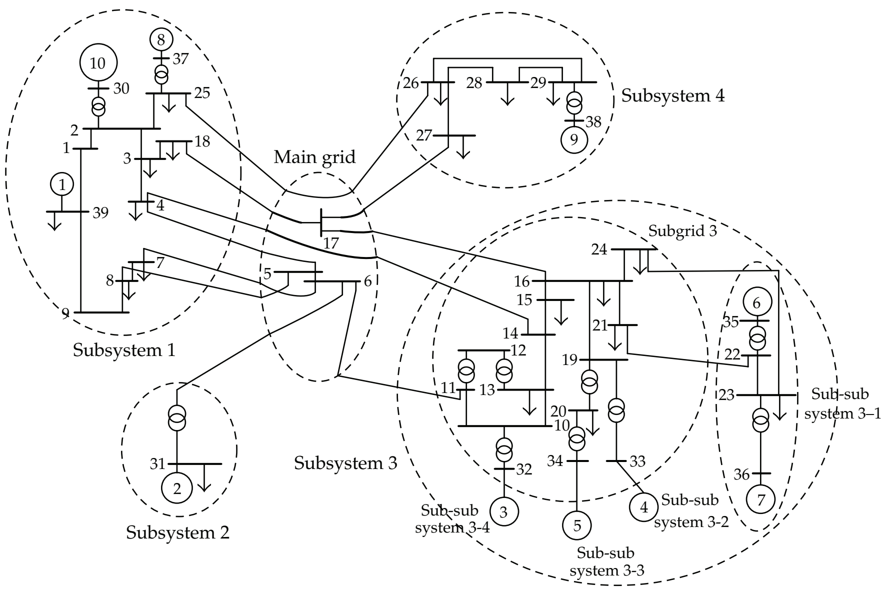

Figure 2: Decomposition of the IEEE 39-bus system into four subsystems.

According to the different demands of real power engineering, the system can also be decomposed in the other forms. For example, the above IEEE 39-bus system can also be divided into four subsystems as shown in Figure 2.

Compared with the virtual main grid as shown in Figure 1, the main grid in Figure 2 includes not only some transmission lines but also some buses (bus 5, 6, and 17). Furthermore, some loads could also be included in the main grid, if they are considered as the invariant impedances (static loads).

The subsystems can also be further divided into some sub-subsystems and one subgrid (see Figure 2). It should be noted that when decomposing, some components can even be

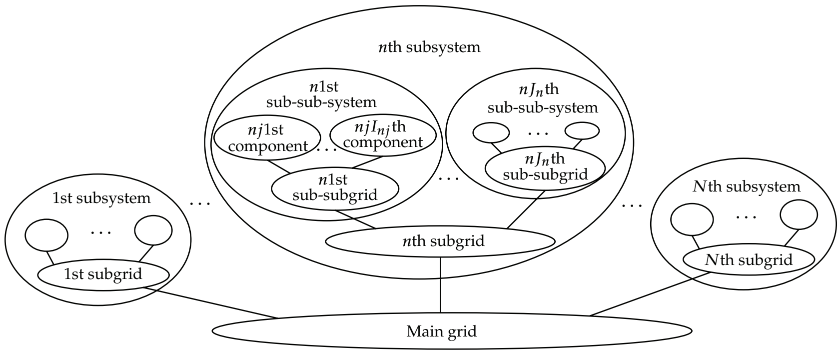

Figure 3: Four-level decomposition of large-scale power systems.

Table 1: Constituent elements of every hierarchical level.

Hierarchical level	Constituent elements	
Main system	One main grid	N subsystems
nth subsystem	One subgrid (the nth subgrid)	J_n sub-subsystems
njth sub-subsystem	One sub-subgrid (the njth sub-subgrid)	I_{nj} components
njith component		njith component

directly connected to the subgrid, or a single component can be chosen as an individual sub-subsystem if necessary. For example, in Figure 2 synchronous generator sets 3, 4, and 5 are directly connected to subgrid 3. Sometimes, some components can even be directly connected to the main grid.

The sub-subsystem can also be further decomposed. For example, the sub-subsystem 3-1 in Figure 2 can be divided into sub-subgrid 3-1 and components 3-1-1 (generator set 6), 3-1-2 (generator set 7), and 3-1-3 (the load in bus 23).

2.2. *General Expressions of Hierarchical Decomposition*

As a whole, the above hierarchical decomposition method is in a top-down manner (from top to bottom) [10] and has the following characters.

(i) There are no special restrictions for the hierarchical decomposition, and thus the proposed decomposition method is suitable for arbitrary power systems.

(ii) The hierarchical levels or "depth" to be decomposed can be arbitrarily chosen.

(iii) For a given hierarchical level, the decomposition form is arbitrary.

In the following of the paper, without loss of generality, the four-level decomposition of large-scale power systems (see Figure 3) will be considered. For the four-level decomposition shown in Figure 3, the constituent elements of every hierarchical level could be sequentially defined (see Figure 3 and Table 1).

Totally there are I_{nj} components in the njth sub-subsystem ($j = 1, \ldots, J_n$); there are J_n sub-subsystems and $\sum_{j=1}^{J_n} I_{nj}$ components in the nth subsystem ($n = 1, \ldots, N$), and so there

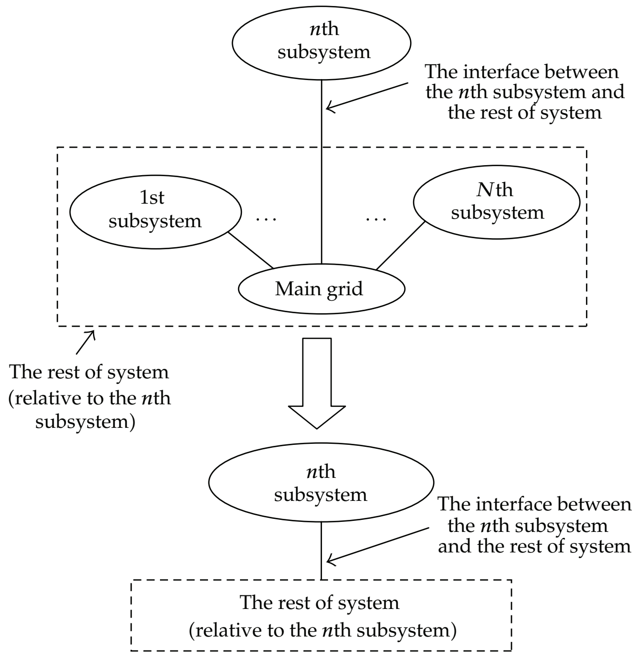

Figure 4: The interface between the nth subsystem and the rest of system (relative to the nth subsystem).

are N subsystems, $\sum_{n=1}^{N} J_n = J$ sub-subsystems, and $\sum_{n=1}^{N}(\sum_{j=1}^{J_n} I_{nj}) = I$ components in the main system.

3. The Relations among Various Hierarchical Levels

3.1. Interface Concepts

In this subsection, the interface concept will be introduced to explicitly describe the relations among various hierarchical levels. Firstly, the interface among subsystems and main grid will be discussed.

The interconnection relations among N subsystems and main grid are shown in Figure 4. With respect to the nth subsystem, all the other subsystems and the main grid could be seen as "the rest of system (relative to the nth subsystem)".

Definition 3.1. The interconnection line between the nth subsystem and the main grid is called the interface between the nth subsystem and the rest of system, or the nth subsystem's interface for simplicity.

For example, in Figure 2 the three interconnection lines between subsystem 3 and the main grid just constitute the interface of subsystem 3.

Similarly, the interface between the sub-subsystem and the rest of system and the interface between the component and the rest of system can also be defined. The interface between the component and the rest of system is as shown in Figure 5.

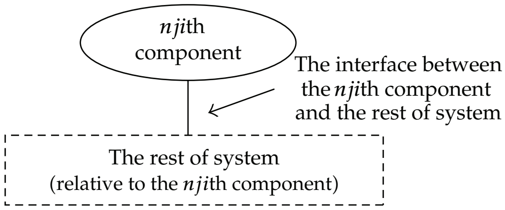

Figure 5: The interface between the njith component and the rest of system (relative to the njith component).

The interfaces or the numbers of the interconnection lines between different components (or sub-subsystems, subsystems) and the rest of system may be different. However, in Figures 3–5, this difference cannot be expressed explicitly, or, in these three figures, all of the interfaces are expressed by a "single line." In the following, the concept of "port" will be introduced to describe this difference.

Firstly, components are chosen as an example. As is well known, if the influence of asymmetry of three phases is not considered, the three-phase circuits are generally shown by single line diagrams in which only one line is shown instead of all the three as shown in Figure 1. Then, according to the number of the interconnection lines between the component and the rest of system, components could be classified as follows.

Definition 3.2. If there is one interconnection line between the component and the rest of system, it is called the one-port component.

Synchronous generator set, motor load, resistance load, and shunt FACTS apparatus are all one-port components.

Definition 3.3. If there are two interconnection lines between the component and the rest of system, it is called the two-port component.

Series FACTS apparatus (TCSC, SSSC, TSSC, etc.) and shunt-series FACTS apparatus are two-port components.

Definition 3.4. If the number of interconnection lines between the component and the rest of system is greater than or equal to three, the component is called the multiport component.

For example, unified series-series apparatus, unified apparatus for multiple lines, multiport HVDC system, and so forth, [11] are multiport components.

Generally, assuming the number of the interconnection lines between the njith component and the rest of system is I_{nji}^0 (the subscript nji denotes the sequence of the component and the superscript 0 denotes that it is a number), the njith component could be classified as I_{nji}^0-port component (see Figure 6(a)).

Similarly, the njth sub-subsystem and the nth subsystem can be classified or named as J_{nj}^0-port sub-subsystem (see Figure 6(b)) or N_n^0-port subsystem. For example, in Figure 2 there are three interconnection lines between subsystem 3 and the rest of system, and thus subsystem 3 is a three-port subsystem.

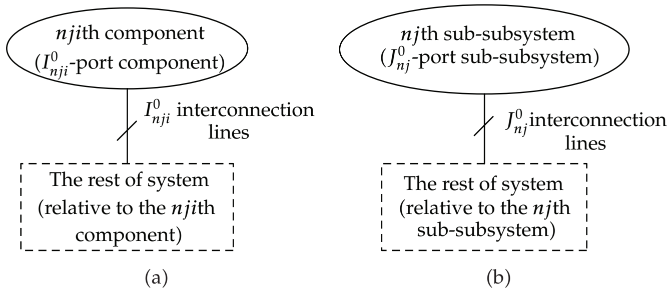

Figure 6: I^0_{nji}-port component (a) and J^0_{nj}-port sub-subsystem (b).

3.2. Interface Variables

Based on the interface concept given above, the interface variables could then be defined to describe the mathematical relations among various hierarchical levels.

If the AC grid of power systems adopts quasi-steady model, the mutual interface relation between any component and the corresponding rest of system can be determined by the current and voltage vectors. For example, the mutual relation between the njith component and the rest of system (see Figure 6(a)) can be described by $\mathbf{I}_{Fnji}$ and $\mathbf{U}_{Fnji}$. Here, $\mathbf{I}_{Fnji} \in \mathbf{R}^{2I^0_{nji}}$ and $\mathbf{U}_{Fnji} \in \mathbf{R}^{2I^0_{nji}}$ are the current and voltage vectors, respectively.

Definition 3.5. $(\mathbf{I}_{Fnji}, \mathbf{U}_{Fnji})$ are called the basic interface variables of the njith component.

Apart from the basic interface variables, other sets of variables can also be used to describe the interface relations.

Definition 3.6. $\mathbf{v}_{nji} \in \mathbf{R}^{4I^0_{nji}}$ are called the interface variables of the njith component, if there are the following equivalent relations between $(\mathbf{I}_{Fnji}, \mathbf{U}_{Fnji})$ and $\mathbf{v}_{nji}$:

$$\mathbf{v}_{nji} = \mathbf{\Phi}_{nji}\left(\mathbf{I}_{Fnji}, \mathbf{U}_{Fnji}\right), \qquad \left(\mathbf{I}_{Fnji}, \mathbf{U}_{Fnji}\right)^T = \mathbf{\Phi}^{-1}_{nji}\left(\mathbf{v}_{nji}\right). \tag{3.1}$$

In (3.1), $\mathbf{\Phi}^{-1}_{nji}(\cdot)$ is the inverse of $\mathbf{\Phi}_{nji}(\cdot)$.

Apparently, the basic interface variables can be considered as a special group of interface variables, and the selection of interface variables is more flexible than that of the basic interface variables. Thus some locally measurable variables that are very commonly used in real engineering, such as the amplitude of voltage and current V_{tnji}, I_{tnji}, the active and reactive power P_{tnji}, Q_{tnji}, and so forth, can be chosen as the interface variables. Then, these interface variables could be chosen as the feedback variables of component-decentralized controllers [9], and so the introduction of the interface concept and interface variables is valuable for designing component decentralized controllers.

Choosing the synchronous generator set as an example, the basic interface variables are $[I_{xnji}, I_{ynji}, U_{xnji}, U_{ynji}]$, and $[V_{tnji}, I_{tnji}, Q_{tnji}, \theta_{Unji}]$ could be chosen as the interface variables for the following reasons. Here, I_{xnji} and I_{ynji} are the x- and y-axis components of the terminal currents, respectively (pu); U_{xnji} and U_{ynji} are the x- and y-axis components of the terminal voltages, respectively (pu); θ_{Unji} is the phasor between the terminal voltage and an arbitrary reference (degree). Firstly, there are

$$\begin{aligned}
V_{tnji} &= \sqrt{U_{xnji}^2 + U_{ynji}^2},\\
I_{tnji} &= \sqrt{I_{xnji}^2 + I_{ynji}^2},\\
Q_{tnji} &= U_{ynji}I_{xnji} - U_{xnji}I_{ynji},\\
\theta_{Unji} &= \operatorname{arcctg}\left(\frac{U_{xnji}}{U_{ynji}}\right).
\end{aligned} \tag{3.2}$$

Equation (3.2) is the relation of $\mathbf{v}_{nji} = \boldsymbol{\Phi}_{nji}(\mathbf{I}_{Fnji}, \mathbf{U}_{Fnji})$. Defining (3.2) as $\mathbf{F}_{nji}(\mathbf{v}_{nji}, \mathbf{I}_{Fnji}, \mathbf{U}_{Fnji}) = \mathbf{v}_{nji} - \boldsymbol{\Phi}(\mathbf{I}_{Fnji}, \mathbf{U}_{Fnji}) = 0$, there is

$$\det\left(\frac{\partial(\mathbf{F}_{nji}(\mathbf{v}_{nji}, \mathbf{I}_{Fnji}, \mathbf{U}_{Fnji}))}{\partial(I_{xnji}, I_{ynji}, U_{xnji}, U_{ynji})^T}\right) = -\frac{P_{tnji}}{V_{tnji}I_{tnji}}. \tag{3.3}$$

In the normal operating area of generator, there is $\det(\partial(\mathbf{F}_{nji}(\mathbf{v}_{nji}, \mathbf{I}_{Fnji}, \mathbf{U}_{Fnji}))/\partial(I_{xnji}, I_{ynji}, U_{xnji}, U_{ynji})^T) = -P_{tnji}/V_{tnji}I_{tnji} \neq 0$. Thus according to the Theorem of Implicit Function [12], theoretically there is

$$\begin{aligned}
I_{xnji} &= f_{Ixnji}(V_{tnji}, I_{tnji}, Q_{tnji}, \theta_{Unji}),\\
I_{ynji} &= f_{Iynji}(V_{tnji}, I_{tnji}, Q_{tnji}, \theta_{Unji}),\\
U_{xnji} &= f_{Uxnji}(V_{tnji}, I_{tnji}, Q_{tnji}, \theta_{Unji}),\\
U_{ynji} &= f_{Uynji}(V_{tnji}, I_{tnji}, Q_{tnji}, \theta_{Unji}).
\end{aligned} \tag{3.4}$$

Equation (3.4) is just the relation of $(\mathbf{I}_{Fnji}, \mathbf{U}_{Fnji})^T = \boldsymbol{\Phi}^{-1}(\mathbf{v}_{nji})$, and thus $[V_{tnji}, I_{tnji}, Q_{tnji}, \theta_{Unji}]$ can be chosen as the interface variables.

Similarly, the above concept of interface variables could also be extended to other hierarchical levels. For the njth sub-subsystem, $\boldsymbol{\xi}_{nj} \in \mathbf{R}^{4J_{nj}^0}$ can be defined as its interface variables. For the nth subsystem, $\boldsymbol{\eta}_n \in \mathbf{R}^{4N_n^0}$ can be defined as its interface variables.

Other variables, including the input variables and state variables of all hierarchical levels in Figure 3, can also be defined as follows.

$$\begin{aligned}
\mathbf{u}_{nj} &= \left[\mathbf{u}_{nj1}, \ldots, \mathbf{u}_{nji}, \ldots, \mathbf{u}_{njI_{nj}}\right],\\
\mathbf{x}_{nj} &= \left[\mathbf{x}_{nj}, \ldots, \mathbf{x}_{nji}, \ldots, \mathbf{x}_{njI_{nj}}\right],\\
\mathbf{u}_{n} &= \left[\mathbf{u}_{n1}, \ldots, \mathbf{u}_{nj}, \ldots, \mathbf{u}_{nJ_n}\right],\\
\mathbf{x}_{n} &= \left[\mathbf{x}_{n1}, \ldots, \mathbf{x}_{nj}, \ldots, \mathbf{x}_{nJ_n}\right],\\
\mathbf{u} &= [\mathbf{u}_1, \ldots, \mathbf{u}_n, \ldots, \mathbf{u}_N],\\
\mathbf{x} &= [\mathbf{x}_1, \ldots, \mathbf{x}_n, \ldots, \mathbf{x}_N].
\end{aligned} \tag{3.5}$$

4. Hierarchical Structural Model and the Self-Similar Characteristic of Large-Scale Power Systems

Based on the proposed hierarchical decomposition method, the hierarchical structural model of large-scale power systems will be constructed in this section in a bottom-up manner (from the components level up to the main system). Meanwhile, by constructing the hierarchical structural model, some internal structural characteristics (such as the self-similarity) of large-scale power systems can be revealed.

4.1. Component Structural Model

4.1.1. Structural Model of Synchronous Generator Set

The discussion will begin with a model of synchronous generator set [13]. When the 3rd order one-axis generator model without ignoring the transient saliency is used and both excitation and governor parts are represented by the 1st order models, the model of the synchronous generator set (as the njith component of the njth sub-subsystem) includes the following two kinds of equations.

(1) The nonlinear differential equations describing the dynamics of the synchronous generator set:

$$\begin{aligned}
\dot{\delta}_{nji} &= \omega_{nji} - \omega_0, \\
\dot{\omega}_{nji} &= \left(\frac{\omega_0}{H_{nji}}\right)\left[P_{Hnji} + C_{MLnji}P_{mnji0} - \frac{D_{nji}(\omega_{nji} - \omega_0)}{\omega_0}\right] \\
&\quad -\left(\frac{\omega_0}{H_{nji}}\right)\left[E'_{qnji} + \left(x_{qnji} - x'_{dnji}\right)I_{dnji}\right]I_{qnji}, \\
\dot{E}'_{qnji} &= \left(\frac{1}{T'_{dnji0}}\right)\left[E_{fdnji} - E'_{qnji} - \left(x_{dnji} - x'_{dnji}\right)I_{dnji}\right], \\
\dot{P}_{Hnji} &= \left(\frac{1}{T_{H\sum nji}}\right)\left(-P_{Hnji} + C_{Hnji}P_{mnji0} + C_{Hnji}U_{cnji}\right), \\
\dot{E}_{fdnji} &= \left(\frac{1}{T_{Rnji}}\right)\left(-E_{fdnji} + K_{Anji}E_{ftnji}\right),
\end{aligned} \tag{4.1}$$

where δ_{nji} is the power angle of synchronous generator (degree); ω_{nji} is the rotor speed of synchronous generator (rad/s); ω_0 is the synchronous speed (rad/s); H_{nji} is the inertia constant of synchronous generator (s); P_{Hnji} is the mechanic power of HP (high pressure) (pu); C_{MLnji} is the distribution coefficient of IP (intermediate pressure) and LP (low pressure); P_{mnji0} is the initial static value of total mechanic power (pu); D_{nji} is the damping constant of synchronous generator (pu); E'_{qnji} is the q axis transient electromagnetic fields of synchronous generator (pu); x_{qnji} and x_{dnji} are the q and d axis synchronous reactances of synchronous generator, respectively (pu); x'_{dnji} is the d axis transient reactances of synchronous generator (pu); I_{dnji} and I_{qnji} are the d and q axis stator circuit currents, respectively (pu); T'_{dnji0} is the d axis transient open-circuit time constants of synchronous generator (s); E_{fdnji} is the excitation control input of synchronous generator (pu); $T_{H\sum nji}$ is the time constant of the turbine (s); C_{Hnji} is the distribution coefficient of HP; U_{cnji} is the valve control input of generator set (pu); T_{Rnji} is the time constant of the terminal voltage transducer of exciter system (s); K_{Anji} is the amplification of exciter (pu).

In (4.1), the inputs are $\mathbf{u}_{nji} = [E_{tnji}, U_{cnji}]$; the state variables are $\mathbf{x}_{nji} = [\delta_{nji}, \omega_{nji}, E'_{qnji}, P_{Hnji}, E_{fdnji}]$. As I_{dnji} and I_{qnji} are neither inputs nor state variables, they are named as the "*affiliate variables*" in this paper, or $\mathbf{w}_{nji} = [I_{dnji}, I_{qnji}]$.

(2) The nonlinear algebraic equations describing the interface relation between the generator set and the rest of system, or between $[I_{xnji}, I_{ynji}, U_{xnji}, U_{ynji}]$ and $[\delta_{nji}, E'_{qnji}, I_{dnji}, I_{qnji}]$.

This kind of equations include the stator voltage equations:

$$\begin{aligned} U_{dnji} &= x_{qnji} I_{qnji} - r_{anji} I_{dnji}, \\ U_{qnji} &= E'_{qnji} - x'_{dnji} I_{dnji} - r_{anji} I_{qnji}, \end{aligned} \tag{4.2}$$

and the *dq-xy* transformation equations:

$$\begin{aligned} \begin{bmatrix} U_{xnji} \\ U_{ynji} \end{bmatrix} &= \begin{bmatrix} \cos\delta_{nji} & \sin\delta_{nji} \\ \sin\delta_{nji} & -\cos\delta_{nji} \end{bmatrix} \begin{bmatrix} U_{qnji} \\ U_{dnji} \end{bmatrix}, \\ \begin{bmatrix} I_{xnji} \\ I_{ynji} \end{bmatrix} &= \begin{bmatrix} \cos\delta_{nji} & \sin\delta_{nji} \\ \sin\delta_{nji} & -\cos\delta_{nji} \end{bmatrix} \begin{bmatrix} I_{qnji} \\ I_{dnji} \end{bmatrix}, \end{aligned} \tag{4.3}$$

where U_{dnji} and U_{qnji} are d and q axis components of the terminal voltages, respectively, (pu); r_{anji} is the resistance of the armature of synchronous generator (pu).

If we choose $[V_{tnji}, I_{tnji}, Q_{tnji}, \theta_{Unji}]$ as the interface variables $\mathbf{v}_{nji}$ and substitute (4.2) and (4.3) into (3.2) in order to eliminate the basic interface variables $[I_{xnji}, I_{ynji}, U_{xnji}, U_{ynji}]$ and U_{dnji}, U_{qnji}, there are

$$\begin{gathered} V_{tnji} = \sqrt{\left(x_{qnji} I_{qnji} - r_{anji} I_{dnji}\right)^2 + \left(E'_{qnji} - x'_{dnji} I_{dnji} - r_{anji} I_{qnji}\right)^2}, \\ I_{tnji} = \sqrt{I_{dnji}^2 + I_{qnji}^2}, \\ Q_{tnji} = E'_{qnji} I_{dnji} - x_{qnji} I_{qnji}^2 - x'_{dnji} I_{dnji}^2, \\ \theta_{Unji} = \delta_{nji} - \arctan \frac{x_{qnji} I_{qnji} - r_{anji} I_{dnji}}{E'_{qnji} - x'_{dnji} I_{dnji} - r_{anji} I_{qnji}}. \end{gathered} \tag{4.4}$$

Equations (4.4) are named as the "*interface equations*" between generator set and AC grid in this paper.

Thus, the generator set could be described by the dynamic (differential) equations (or (4.1)) and the algebraic (interface) equations (or (4.4)), and this kind of model is a class of

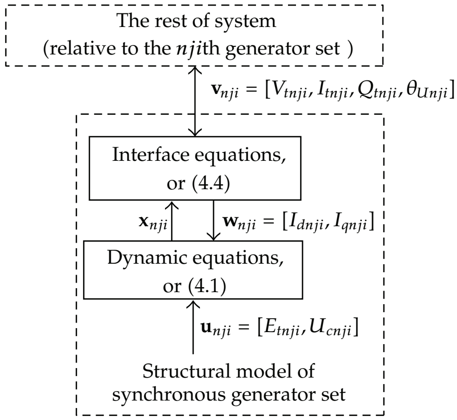

Figure 7: The structural model of synchronous generator set.

nonlinear DAE (differential-algebraic equation) models and is named as "*structural model*" of synchronous generator set in this paper (see Figure 7).

4.1.2. General Expression of Component Structural Model

Similar to the structural model of synchronous generator set as shown in Figure 7, the general expression of the structural model of the njith (I_{nji}^0-port) component could be defined as follows:

$$\begin{aligned} \dot{\mathbf{x}}_{nji} &= \mathbf{f}_{nji}^{w}\left(\mathbf{x}_{nji}, \mathbf{w}_{nji}, \mathbf{u}_{nji}\right), \\ \mathbf{g}_{nji}^{w}\left(\mathbf{x}_{nji}, \mathbf{w}_{nji}, \mathbf{v}_{nji}\right) &= \mathbf{0}, \end{aligned} \tag{4.5}$$

where $\mathbf{u}_{nji} \in \mathbf{R}^{U_{nji}}$ are the input variables; $\mathbf{v}_{nji} \in \mathbf{R}^{4I_{nji}^0}$ are the interface variables; $\mathbf{x}_{nji} \in \mathbf{R}^{X_{nji}}$ are the state variables; $\mathbf{w}_{nji} \in \mathbf{R}^{W_{nji}}$ are the affiliate variables; $\mathbf{f}_{nji}^{w}$ and $\mathbf{g}_{nji}^{w}$ are the corresponding mappings.

In (4.5), there are two kinds of nonlinear equations:

(1) dynamic equations or differential equations $\dot{\mathbf{x}}_{nji} = \mathbf{f}_{nji}^{w}(\mathbf{x}_{nji}, \mathbf{w}_{nji}, \mathbf{u}_{nji})$, which are used to describe the internal complex dynamics of components,

(2) interface equations or algebraic equations $\mathbf{g}_{nji}^{w}(\mathbf{x}_{nji}, \mathbf{w}_{nji}, \mathbf{v}_{nji}) = \mathbf{0}$, which are used to describe the algebraic relations among $\mathbf{x}_{nji}$, $\mathbf{w}_{nji}$ and $\mathbf{v}_{nji}$.

To sum up, in the structural model of the njith (I_{nji}^0-port) component, totally there are $U_{nji}+X_{nji}+W_{nji}+4I_{nji}^0$ variables in $X_{nji}+W_{nji}+2I_{nji}^0$ equations. This means that the component structural model is constrained by U_{nji} input variables and $2I_{nji}^0$ interface variables (These $2I_{nji}^0$ interface variables could be considered as the disturbance variables from the rest of system). When these U_{nji} input variables and $2I_{nji}^0$ interface variables are known, the model is solvable and all of variables could therefore be determined.

Further, the interface equations $\mathbf{g}^{w}_{nji}(\mathbf{v}_{nji}, \mathbf{w}_{nji}, \mathbf{x}_{nji}) = \mathbf{0}$ could be decomposed into two parts (Note that such decomposition does exist for all of the present components to the best of our knowledge.):

$$\begin{aligned} \mathbf{w}_{nji} &= \mathbf{h}_{nji}\left(\mathbf{x}_{nji}, \mathbf{v}_{nji}\right), \\ \mathbf{g}_{nji}\left(\mathbf{x}_{nji}, \mathbf{v}_{nji}\right) &= \mathbf{0}. \end{aligned} \tag{4.6}$$

By substituting $\mathbf{w}_{nji} = \mathbf{h}_{nji}(\mathbf{x}_{nji}, \mathbf{v}_{nji})$ into $\dot{\mathbf{x}}_{nji} = \mathbf{f}^{w}_{nji}(\mathbf{x}_{nji}, \mathbf{w}_{nji}, \mathbf{u}_{nji})$, the affiliate variables $\mathbf{w}_{nji}$ in (4.5) could be eliminated. Thus, the component structural model without affiliate variables is

$$\begin{aligned} \dot{\mathbf{x}}_{nji} &= \mathbf{f}^{v}_{nji}\left(\mathbf{x}_{nji}, \mathbf{v}_{nji}, \mathbf{u}_{nji}\right), \\ \mathbf{g}_{nji}\left(\mathbf{x}_{nji}, \mathbf{v}_{nji}\right) &= \mathbf{0}. \end{aligned} \tag{4.7}$$

The component structural model given in (4.5) or (4.7) is suitable for not only the existing components of today's power systems but also new components in the future.

4.2. Structural Model of Sub-Subsystem

The model of sub-subsystem could be obtained by combining the structural models of all of the components in this sub-subsystem and the model of the sub-subgrid.

As shown in Figure 2, there may be some loads in the AC grid. When constructing the model of the sub-subsystem, all of the loads in the sub-subgrid must be expressed by invariant impedances. Thus, the model of the njth sub-subgrid can be expressed as

$$\mathbf{z}^{b}_{nj}\left(\mathbf{I}_{Fnj1}, \mathbf{U}_{Fnj1}, \ldots, \mathbf{I}_{Fnji}, \mathbf{U}_{Fnji}, \ldots, \mathbf{I}_{FnjI_{nj}}, \mathbf{U}_{FnjI_{nj}}, \mathbf{I}_{Fnj}, \mathbf{U}_{Fnj}\right) = \mathbf{0}, \tag{4.8}$$

where $(\mathbf{I}_{Fnji}, \mathbf{U}_{Fnji})$ $(i = 1, 2, \ldots, I_{nj})$ are the basic interface variables of the njith component; $(\mathbf{I}_{Fnj}, \mathbf{U}_{Fnj})$ are the basic interface variables of the njth sub-subsystem.

Meanwhile, the equivalent relation between $(\mathbf{I}_{Fnji}, \mathbf{U}_{Fnji})$ $(i = 1, 2, \ldots, I_{nj})$ and the corresponding interface variables $\mathbf{v}_{nji}$ $(i = 1, 2, \ldots, I_{nj})$ for each component could be set up easily, and so do the equivalent relation between $(\mathbf{I}_{Fnj}, \mathbf{U}_{Fnj})$ and $\boldsymbol{\xi}_{nj}$. Or there are

$$\begin{aligned} \left(\mathbf{I}_{Fnji}, \mathbf{U}_{Fnji}\right)^{T} &= \mathbf{\Phi}^{-1}_{nji}\left(\mathbf{v}_{nji}\right) \quad \left(i = 1, 2, \ldots, I_{nj}\right), \\ \left(\mathbf{I}_{Fnj}, \mathbf{U}_{Fnj}\right)^{T} &= \mathbf{\Phi}^{-1}_{nj}\left(\boldsymbol{\xi}_{nj}\right). \end{aligned} \tag{4.9}$$

Substituting (4.9) into (4.8), the model of the njth sub-subgrid can be purely expressed by the interface variables, or

$$\mathbf{z}_{nj}\left(\mathbf{v}_{nj1}, \ldots, \mathbf{v}_{nji}, \ldots, \mathbf{v}_{njI_{nj}}, \boldsymbol{\xi}_{nj}\right) = \mathbf{0}. \tag{4.10}$$

Thus, combining the model of the njth sub-subgrid (4.10) with the component structural model (4.5) or (4.7), the mathematical model of the njth sub-subsystem can be derived as follow:

$$\begin{cases} \dot{\mathbf{x}}_{nji} = \mathbf{f}^{w}_{nji}(\mathbf{x}_{nji}, \mathbf{w}_{nji}, \mathbf{u}_{nji}) \\ \mathbf{g}^{w}_{nji}(\mathbf{x}_{nji}, \mathbf{w}_{nji}, \mathbf{v}_{nji}) = \mathbf{0} \end{cases} \quad (i = 1, 2, \ \ldots, I_{nj}), \\ \mathbf{z}_{nj}\left(\mathbf{v}_{nj1}, \ldots, \mathbf{v}_{nji}, \ldots, \mathbf{v}_{njI_{nj}}, \boldsymbol{\xi}_{nj}\right) = \mathbf{0}, \tag{4.11}$$

$$\begin{cases} \dot{\mathbf{x}}_{nji} = \mathbf{f}^{v}_{nji}(\mathbf{x}_{nji}, \mathbf{v}_{nji}, \mathbf{u}_{nji}) \\ \mathbf{g}_{nji}(\mathbf{x}_{nji}, \mathbf{v}_{nji}) = \mathbf{0} \end{cases} \quad (i = 1, 2, \ \ldots, I_{nj}), \\ \mathbf{z}_{nj}\left(\mathbf{v}_{nj1}, \ldots, \mathbf{v}_{nji}, \ldots, \mathbf{v}_{njI_{nj}}, \boldsymbol{\xi}_{nj}\right) = \mathbf{0}. \tag{4.12}$$

Further, the algebraic equations in (4.12), or $\mathbf{g}_{nji}(\mathbf{x}_{nji}, \mathbf{v}_{nji}) = \mathbf{0}$ $(i = 1, 2, \ldots, I_{nj})$ and $\mathbf{z}_{nj}(\mathbf{v}_{nj1}, \ldots, \mathbf{v}_{nji}, \ldots, \mathbf{v}_{njI_{nj}}, \boldsymbol{\xi}_{nj}) = \mathbf{0}$, could be described as

$$\mathbf{g}^{v}_{nj}\left(\mathbf{x}_{nj}, \mathbf{v}_{nj}, \boldsymbol{\xi}_{nj}\right) = \mathbf{0}. \tag{4.13}$$

Meanwhile, the X_{nj} differential equations $\dot{\mathbf{x}}_{nji} = \mathbf{f}^{v}_{nji}(\mathbf{x}_{nji}, \mathbf{v}_{nji}, \mathbf{u}_{nji})$ $(i = 1, 2, \ldots, I_{nj})$ in (4.12) could be described as

$$\dot{\mathbf{x}}_{nj} = \mathbf{f}^{v}_{nj}(\mathbf{x}_{nj}, \mathbf{v}_{nj}, \mathbf{u}_{nj}), \tag{4.14}$$

where $\mathbf{x}_{nj} = [\mathbf{x}_{nj1}, \ldots, \mathbf{x}_{nji}, \ldots, \mathbf{x}_{njI_{nj}}]$, $\mathbf{v}_{nj} = [\mathbf{v}_{nj1}, \ldots, \mathbf{v}_{nji}, \ldots, \mathbf{v}_{njI_{nj}}]$, and $\mathbf{u}_{nj} = [\mathbf{u}_{nj1}, \ldots, \mathbf{u}_{nji}, \ldots, \mathbf{u}_{njI_{nj}}]$.

Thus, the njth sub-subsystem could be expressed by dynamic equations and algebraic equations, or a class of nonlinear DAE models, just like that of a single component, as shown in (4.5)

$$\begin{aligned} \dot{\mathbf{x}}_{nj} &= \mathbf{f}^{v}_{nj}(\mathbf{x}_{nj}, \mathbf{v}_{nj}, \mathbf{u}_{nj}), \\ \mathbf{g}^{v}_{nj}&\left(\mathbf{x}_{nj}, \mathbf{v}_{nj}, \boldsymbol{\xi}_{nj}\right) = \mathbf{0}. \end{aligned} \tag{4.15}$$

This kind of model is named as the "*structural model*" of sub-subsystem in this paper.

It should be mentioned that the variables $\mathbf{v}_{nj} = [\mathbf{v}_{nj1}, \ldots, \mathbf{v}_{nji}, \ldots, \mathbf{v}_{njI_{nj}}]$ in (4.15) act as the sub-subsystem's *affiliate variables* like $\mathbf{w}_{nji}$ in component structural model, although they are the component's interface variables between components and the sub-subgrid. Similarly, the affiliate variables of other hierarchical levels can also be defined.

The total number of $\mathbf{v}_{nj}$ is $4I^{0}_{nj}$. If choosing $4I^{0}_{nj}$ of appropriate equations from the $4I^{0}_{nj} + 2J^{0}_{nj}$ interface equations in (4.15), one could derive

$$\mathbf{v}^{T}_{nj} = \left[\mathbf{v}_{nj1}, \ldots, \mathbf{v}_{nji}, \ldots, \mathbf{v}_{njI_{nj}}\right]^{T} = \boldsymbol{\beta}_{nj}\left(\boldsymbol{\xi}_{nj}, \mathbf{x}_{nj1}, \ldots, \mathbf{x}_{nji}, \ldots, \mathbf{x}_{njI_{nj}}\right). \tag{4.16}$$

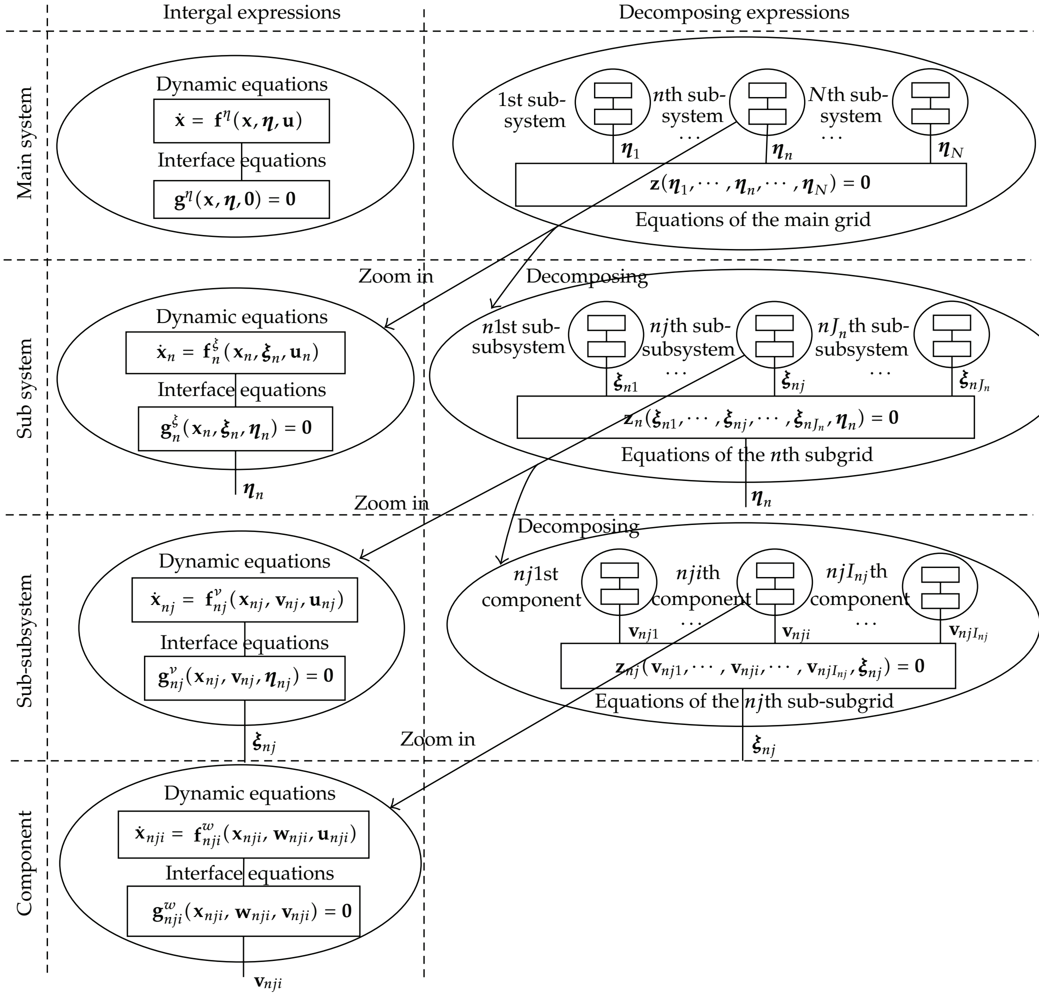

Figure 8: Two kinds of expressions of hierarchical structural models and their relations.

And by substituting (4.16) into both the dynamic equations and interface equations of (4.15) to eliminate the affiliate variables $\mathbf{v}_{nj}$, one can get another kind of model of sub-subsystem as follows (just like that of component shown in (4.7)):

$$\begin{gathered}\dot{\mathbf{x}}_{nj} = \mathbf{f}^{\xi}_{nj}\left(\mathbf{x}_{nj}, \boldsymbol{\xi}_{nj}, \mathbf{u}_{nj}\right), \\ \mathbf{g}_{nj}\left(\mathbf{x}_{nj}, \boldsymbol{\xi}_{nj}\right) = \mathbf{0}.\end{gathered} \tag{4.17}$$

Apparently, there are two kinds of structural models of sub-subsystem. In the paper, the model having the same form as in (4.11) is named as the "*decomposing expressions*" of the sub-subsystem's structural model and the model in a form like (4.15) is named as its "*integral expressions*" (see Figure 8). From this viewpoint, the component's structural model (4.5) can also be considered as the integral expression (see Figure 8). From (4.11) and Figure 8, it can be clearly seen that the integral expression of component structural model is just a part of the decomposing expression of sub-subsystem's model.

In the same way, the structural model of subsystem and main system can also be constructed as shown in the following.

4.3. Structural Model of Subsystem

The "decomposing expressions" of the nth subsystem's structural model are

$$\begin{cases} \dot{\mathbf{x}}_{nj} = \mathbf{f}_{nj}^{v}(\mathbf{x}_{nj}, \mathbf{v}_{nj}, \mathbf{u}_{nj}) \\ \mathbf{g}_{nj}^{v}\left(\mathbf{x}_{nj}, \mathbf{v}_{nj}, \boldsymbol{\xi}_{nj}\right) = \mathbf{0} \end{cases} \quad (j = 1, \dots, J_n),$$
$$\mathbf{z}_n\left(\boldsymbol{\xi}_{n1}, \dots, \boldsymbol{\xi}_{nj}, \dots, \boldsymbol{\xi}_{nJ_n}, \boldsymbol{\eta}_n\right) = \mathbf{0}. \tag{4.18}$$

The "integral expressions" of the nth subsystem's structural model are

$$\begin{aligned} \dot{\mathbf{x}}_n &= \mathbf{f}_n^{\xi}(\mathbf{x}_n, \boldsymbol{\xi}_n, \mathbf{u}_n), \\ \mathbf{g}_n^{\xi}(\mathbf{x}_n, \boldsymbol{\xi}_n, \boldsymbol{\eta}_n) &= \mathbf{0}. \end{aligned} \tag{4.19}$$

Here, $\boldsymbol{\xi}_n$ act as the subsystem's affiliate variables.

4.4. Structural Model of Main System

The "decomposing expressions" of the main system's structural model are

$$\begin{cases} \dot{\mathbf{x}}_n = \mathbf{f}_n^{\xi}(\mathbf{x}_n, \boldsymbol{\xi}_n, \mathbf{u}_n) \\ \mathbf{g}_n^{\xi}(\mathbf{x}_n, \boldsymbol{\xi}_n, \boldsymbol{\eta}_n) = \mathbf{0} \end{cases} \quad (n = 1, \dots, N),$$
$$\mathbf{z}(\boldsymbol{\eta}_1, \dots, \boldsymbol{\eta}_n, \dots, \boldsymbol{\eta}_N) = \mathbf{0}. \tag{4.20}$$

The "integral expressions" of the main system's structural model are

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{f}^{\eta}(\mathbf{x}, \boldsymbol{\eta}, \mathbf{u}), \\ \mathbf{g}^{\eta}(\mathbf{x}, \boldsymbol{\eta}, \mathbf{0}) &= \mathbf{0}. \end{aligned} \tag{4.21}$$

Here, $\boldsymbol{\eta}_n (n = 1, \dots, N)$ are the main system's affiliate variables, and $\mathbf{0}$ would be considered as the main system's interface variables (In fact, the main system is an isolated system, not a subsystem.).

Furthermore, by eliminating $\boldsymbol{\eta}$, the algebraic equations of (4.21) would be identical, and then the main system's model of (4.21) would be simplified as follows:

$$\dot{\mathbf{x}} = \mathbf{f}^{0}(\mathbf{x}, \mathbf{u}). \tag{4.22}$$

This is not a DAE system but a traditional ODE (ordinary differential equation) system.

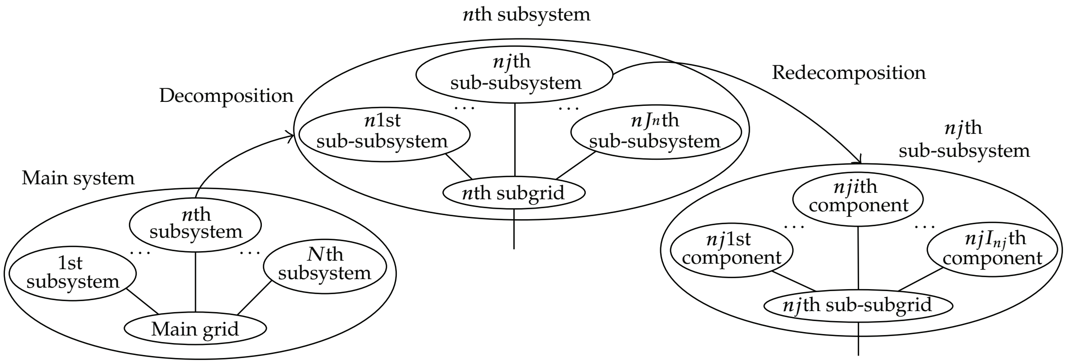

Figure 9: The structural self-similarity of hierarchical power systems.

4.5. The Self-Similar Characteristic of Hierarchical Power Systems

The logical relations among various hierarchical levels (main system, subsystem, sub-subsystem, down to components) can be more explicitly shown in Figure 9. Apart from the components, each hierarchical level of power systems can be divided into several lower levels and a corresponding AC grid. From this point of view, there is an obvious structural self-similarity of hierarchical power systems.

Furthermore, there is also a high self-similarity among the structural models of hierarchical power systems. Comparing (4.5), (4.15), (4.19), and (4.21), one can find out that there is a surprising similarity among the integral expression models of hierarchical power systems. Meanwhile, with reference to (4.11), (4.18), and (4.20), there is also a surprising similarity among the decomposing expression models of hierarchical power systems.

5. Application of Structural Model in Decentralized Control

As an application example, the value of the proposed structural model in decentralized control of power systems will be viewed in this section. Here, it should be noted that in this section only the brief ideas are given, and the detailed discussion about this issue will be given in our next paper.

5.1. The Standard DAE Expression of Component Structural Model

As mentioned above, the component structural model of (4.5) is a class of nonlinear DAE (differential-algebraic equation) model. In control theory, the DAE systems are also called singular systems [14, 15]. Yet, from the viewpoint of the DAE theory, the component structural model of (4.5) is still not the standard DAE model (the number of variables $\mathbf{v}_i$, $\mathbf{w}_i$ is large than the number of algebraic equations), and so it would be convenient to translate it into the standard DAE at first. As the variable's subscript (or nji) in (4.5) is very complex, it will be simplified as i below for the concision of the form.

The interface variables $\mathbf{v}_i$ in (4.5) can be decomposed into two parts, or $\mathbf{v}_i = (\mathbf{v}_i^{\text{out}}, \mathbf{v}_i^{\text{in}})^T$ with the following characteristics:

(i) $\mathbf{v}_i^{\text{in}}$ could "fully" describe the influence of the rest of power systems to the component, or $\mathbf{v}_i^{\text{in}}$ are the interconnection inputs (disturbances) of the component;

(ii) $\mathbf{v}_i^{\text{out}}$ could "fully" describe the influence of the component to the rest of power systems.

Under this decomposition, (4.5) would be

$$\begin{aligned} \dot{\mathbf{x}}_i &= \mathbf{f}_i^{w}(\mathbf{x}_i, \mathbf{w}_i, \mathbf{u}_i), \\ \mathbf{g}_i^{w}\left(\mathbf{x}_i, \mathbf{w}_i, \mathbf{v}_i^{\text{out}}, \mathbf{v}_i^{\text{in}}\right) &= \mathbf{0} \end{aligned} \tag{5.1}$$

Defining $\mathbf{z}_i = (\mathbf{w}_i, \mathbf{v}_i^{\text{out}})^T$, (5.1) would be

$$\begin{aligned} \dot{\mathbf{x}}_i &= \mathbf{f}_i^{z}(\mathbf{x}_i, \mathbf{z}_i, \mathbf{u}_i), \\ \mathbf{g}_i^{z}\left(\mathbf{x}_i, \mathbf{z}_i, \mathbf{v}_i^{\text{in}}\right) &= \mathbf{0}, \end{aligned} \tag{5.2}$$

where $\mathbf{z}_i \in R^{2m_i+W_i}$ are the algebraic variables [14] of the DAE systems; $\mathbf{v}_i^{in} \in R^{2m_i}$ are the interconnection inputs.

When discussing the control problem, the output equations should also be defined. The general expression of the output equation of component would be

$$\mathbf{y}_i = \mathbf{h}_i^{z}\left(\mathbf{x}_i, \mathbf{z}_i, \mathbf{v}_i^{\text{in}}\right). \tag{5.3}$$

Here, it should be noted that the interconnections of (5.2), or $\mathbf{v}_i^{\text{in}}$, are local measurable. This characteristic of (5.2) will be very helpful for designing decentralized controller of component.

5.2. Design of Decentralized Nonlinear Controller of Components

For the control problem of DAE model, one general approach is transforming the DAE to traditional ODE (ordinary differential equation) [14]. In this subsection, the method of transforming the nonlinear DAE as shown in (5.2) to nonlinear ODE will be discussed firstly.

"Index" is an important and basic concept in the theory of DAE. For a DAE, index is the minimum derivative times of the algebraic equations that need to get the differential equation of the algebraic variables [14]. For a DAE system, if its index is higher than 1, the control problem will be very complex. Fortunately, for power systems, the component structural model of (5.2) is just a DAE with index 1. Then, it is relatively easy to transform the nonlinear DAE as shown in (5.2) to nonlinear ODE.

The interface equations in (5.2) can be decomposed into two parts:

$$\begin{aligned} \mathbf{g}_i^{z_1}\left(\mathbf{x}_i, \mathbf{w}_i, \mathbf{v}_i^{\text{out}}, \mathbf{v}_i^{\text{in}}\right) &= \mathbf{w}_i - \mathbf{h}_i\left(\mathbf{x}_i, \mathbf{v}_i^{\text{out}}, \mathbf{v}_i^{\text{in}}\right) = \mathbf{0}, \\ \mathbf{g}_i^{z_2}\left(\mathbf{x}_i, \mathbf{v}_i^{\text{out}}, \mathbf{v}_i^{\text{in}}\right) &= \mathbf{g}_i\left(\mathbf{x}_i, \mathbf{v}_i^{\text{out}}, \mathbf{v}_i^{\text{in}}\right) = \mathbf{0}. \end{aligned} \tag{5.4}$$

For the first group of (5.4) or the first $\mathbf{w}_i$ equations of (5.4), there is $\operatorname{rank}(\partial \mathbf{g}_i^{z_1}(\mathbf{x}_i, \mathbf{w}_i, \mathbf{v}_i^{\text{out}}, \mathbf{v}_i^{\text{in}})/\partial \mathbf{w}_i) = W_i$. As there is no $\mathbf{w}_i$ in the second group of (5.4) and $\mathbf{z}_i = (\mathbf{w}_i, \mathbf{v}_i^{\text{out}})^T$, there must exist $\operatorname{rank}(\partial \mathbf{g}_i(\mathbf{x}_i, \mathbf{v}_i^{\text{out}}, \mathbf{v}_i^{\text{in}})/\partial \mathbf{v}_i^{\text{out}}) = 2m_i$. Therefore, according to the implicit function theorem [12], the interface equations of (5.4) can be theoretically expressed as

$$\begin{aligned} \mathbf{w}_i &= \mathbf{h}_i\left(\mathbf{x}_i, \mathbf{v}_i^{\text{out}}, \mathbf{v}_i^{\text{in}}\right), \\ \mathbf{v}_i^{\text{out}} &= \mathbf{g}_i^{\text{out}}\left(\mathbf{x}_i, \mathbf{v}_i^{\text{in}}\right). \end{aligned} \tag{5.5}$$

Furthermore, substituting the second equation of (5.5) into the first equation of (5.5), and considering the definition of $\mathbf{z}_i = (\mathbf{w}_i, \mathbf{v}_i^{\text{out}})^T$, there theoretically exist

$$\mathbf{z}_i = \mathbf{p}_i\left(\mathbf{x}_i, \mathbf{v}_i^{\text{in}}\right). \tag{5.6}$$

Substituting (5.6) into (5.2) and (5.3), there is

$$\begin{aligned} \dot{\mathbf{x}}_i &= \mathbf{f}_i^p\left(\mathbf{x}_i, \mathbf{p}_i\left(\mathbf{x}_i, \mathbf{v}_i^{\text{in}}\right), \mathbf{u}_i\right) = \mathbf{f}_i^v\left(\mathbf{x}_i, \mathbf{v}_i^{\text{in}}, \mathbf{u}_i\right), \\ \mathbf{y}_i &= \mathbf{h}_i^p\left(\mathbf{x}_i, \mathbf{p}_i\left(\mathbf{x}_i, \mathbf{v}_i^{\text{in}}\right), \mathbf{v}_i^{\text{in}}\right) = \mathbf{h}_i^{\overline{v}}\left(\mathbf{x}_i, \mathbf{v}_i^{\text{in}}\right). \end{aligned} \tag{5.7}$$

Apparently, (5.7) is a standard nonlinear ODE model.

It should be noted that only when the analytic expressions of $\mathbf{p}_i(\mathbf{x}_i, \mathbf{v}_i^{\text{in}})$ in (5.6) exist can one get the analytic expression of (5.7). Fortunately, for most kinds of components in power systems, we can all get the analytical expressions of (5.6). However, in some special circumstances, the interface equations $\mathbf{g}_i^z(\mathbf{x}_i, \mathbf{z}_i, \mathbf{v}_i^{\text{in}}) = \mathbf{0}$ may be very complex, and thus there may be very difficult or impossible to acquire the analytic expressions of $\mathbf{p}_i(\mathbf{x}_i, \mathbf{v}_i^{\text{in}})$. For example, when the generator adopts the 3rd order one-axis model without ignoring the transient saliency, it is very difficult to acquire the analytic expression of $\mathbf{p}_i(\mathbf{x}_i, \mathbf{v}_i^{\text{in}})$. In this case, based on the characteristic of index 1, the nonlinear DAE as shown in (5.2) and (5.3) could be transformed to

$$\begin{gathered} \dot{\mathbf{x}}_i = \mathbf{f}_i^z(\mathbf{x}_i, \mathbf{z}_i, \mathbf{u}_i), \\ \dot{\mathbf{z}}_i = -\left(\frac{\partial \mathbf{g}_i^z}{\partial \mathbf{z}_i}\right)^{-1}\left(\frac{\partial \mathbf{g}_i^z}{\partial \mathbf{x}_i}\right)\mathbf{f}_i^z(\mathbf{x}_i, \mathbf{z}_i,\ \mathbf{u}_i) - \left(\frac{\partial \mathbf{g}_i^z}{\partial \mathbf{z}_i}\right)^{-1}\left(\frac{\partial \mathbf{g}_i^z}{\partial \mathbf{v}_i^{\text{in}}}\right)\dot{\mathbf{v}}_i^{\text{in}}, \\ \mathbf{g}_i^z\left(\mathbf{x}_i, \mathbf{z}_i, \mathbf{v}_i^{\text{in}}\right) = \mathbf{0}, \\ \mathbf{y}_i = \mathbf{h}_i^z\left(\mathbf{x}_i, \mathbf{z}_i, \mathbf{v}_i^{\text{in}}\right). \end{gathered} \tag{5.8}$$

In (5.8), the state variables have been expanded to $(\mathbf{x}_i, \mathbf{z}_i)^T$, and $\mathbf{x}_i$ and $\mathbf{z}_i$ are constrained with each other by the algebraic equation $\mathbf{g}_i^z(\mathbf{x}_i, \mathbf{z}_i, \mathbf{v}_i^{\text{in}}) = \mathbf{0}$. Thus, the model as shown in (5.8) is a constrained nonlinear ODE, or it is not the minimum state space realization of the original nonlinear DAE.

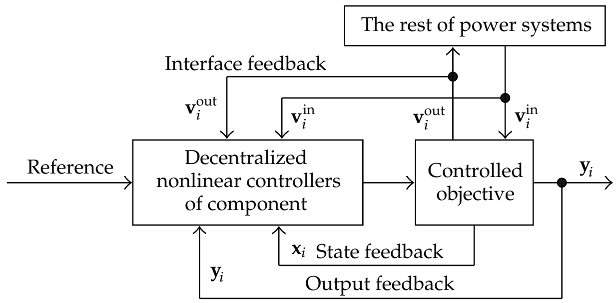

Figure 10: Diagram of component nonlinear decentralized controller.

For the decentralized control of the constrained nonlinear ODE model with measurable interconnection variables as shown in (5.7) or (5.8), traditional nonlinear control methods (e.g., differential geometric theory, direct feedback linearization (DFL) method, inversion control method, etc.) which are suitable for normal nonlinear ODE could be developed and expanded to be suitable. The detailed discussion about these will be also given in our following paper.

The general decentralized control diagram is shown in Figure 10. In Figure 10, there are three kinds of feedback or state feedback, output feedback and interface feedback in, which interface feedback is not a traditional feedback kind. Here, one important reason that makes the interface feedback possible is that the interconnections (interface variables) of the component's structural model are local measurable. By feed-backing interface variables, the component controller can "apperceive" the interconnections between the component and the rest of power systems.

6. Conclusions

The structural characteristics of large-scale power systems are analyzed in the paper. Corresponding results would be valuable for some problems of power systems, such as the time-scale simulation, the control strategy design, and so forth. Compared with the methods in previous literatures [7, 8], there are the following features of the proposed methods and results.

(1) The proposed analysis method is structural. Wholly, the hierarchical levels of power systems are decomposed in a manner of top-down, and the structural models of each hierarchical level are constructed in a manner of bottom-up (from bottom to top).

(2) Some internal structural characteristics of power systems are revealed, including the interface characteristic and the self-similar characteristic.

(3) The proposed models are structural. Or the equations, variables, and number of equations and variables of the models of various hierarchical levels can be defined and derived in a standard manner. Thus, the proposed model is named as "hierarchical structural model" of power systems.

Acknowledgments

This work was supported by the National Natural Science Foundation of China under Grants 60974036 and 51177019, the National High Technology Research and Development Program of China (863 Program) under Grants 2011AA05A105. K. F. Zhang and X. Z. Dai are with the Key Laboratory of Measurement and Control of CSE (School of Automation, Southeast University), Ministry of Education, Nanjing, China.

References

[1] D. Dotta, A. S. e Silva, and I. C. Decker, "Wide-area measurements-based two-level control design considering signal transmission delay," *IEEE Transactions on Power Systems*, vol. 24, no. 1, pp. 208–216, 2009.

[2] A. R. Bergen and D. J. Hill, "A structure preserving model for power system stability analysis," *IEEE Transactions on Power Apparatus and Systems*, vol. 100, no. 1, pp. 25–35, 1981.

[3] M. L. Ourari, L. A. Dessaint, and V. Q. Do, "Dynamic equivalent modeling of large power systems using structure preservation technique," *IEEE Transactions on Power Systems*, vol. 21, no. 3, pp. 1284–1295, 2006.

[4] O. Wasynczuk and R. A. DeCarlo, "The component connection model and structure preserving model order reduction," *Automatica*, vol. 17, no. 4, pp. 619–626, 1981.

[5] S. Sastry and P. Varaiya, "Hierarchical stability and alert state steering control of interconnected power systems," *IEEE Transactions on Circuits and systems*, vol. 27, no. 11, pp. 1102–1112, 1980.

[6] Y. G. Karpov, R. I. Ivanovski, N. I. Voropai, and D. B. Popov, "Hierarchical modeling of electric power system expansion by anyLogic simulation software," in *Proceedings of the IEEE Russia Power Tech, PowerTech*, pp. 27–30, St. Petersburg, Russia, June 2005.

[7] M. Armstrong, J. R. Martí, L. R. Linares, and P. Kundur, "Multilevel MATE for efficient simultaneous solution of control systems and nonlinearities in the OVNI simulator," *IEEE Transactions on Power Systems*, vol. 21, no. 3, pp. 1250–1259, 2006.

[8] D. Z. Fang and Y. Xiaodong, "A new method for fast dynamic simulation of power systems," *IEEE Transactions on Power Systems*, vol. 21, no. 2, pp. 619–628, 2006.

[9] K. Zhang and X. Dai, "Strict decentralized nonlinear controller for synchronous generator set," in *Proceedings of the IEEE Power India Conference*, pp. 529–536, New Delhi, India, April 2006.

[10] F. A. Felder, "Top-down composite modeling of bulk power systems," *IEEE Transactions on Power Systems*, vol. 20, no. 3, pp. 1655–1656, 2005.

[11] M. G. Hingorani and L. Gyugyi, *Understanding FACTS: Concepts and Technology of Flexible AC Transmission Systems*, IEEE Press, New York, NY, USA, 2000.

[12] H. Nijmeijer and A. van der Schaft, *Nonlinear Dynamical Control Systems*, Springer, New York, NY, USA, 1990.

[13] P. Kundur, *Power System Stability and Control*, McGraw-Hill , New York, NY, USA, 1994.

[14] L. Dai, *Singular Control Systems*, vol. 118, Springer, Berlin, Germany, 1989.

[15] M.-N. Contou-Carrere and P. Daoutidis, "An output feedback precompensator for nonlinear DAE systems with control-dependent state-space," *IEEE Transactions on Automatic Control*, vol. 50, no. 11, pp. 1831–1835, 2005.