Research Article

Difference-Equation-Based Digital Frequency Synthesizer

Lu-Ting Ko,¹ Jwu-E. Chen,¹ Yaw-Shih Shieh,² Hsi-Chin Hsin,³ and Tze-Yun Sung²

¹ Department of Electrical Engineering, National Central University, Chungli 320-01, Taiwan

² Department of Electronics Engineering, Chung Hua University, Hsinchu 300-12, Taiwan

³ Department of Computer Science and Information Enginnering, National United University, Miaoli 360-03, Taiwan

Correspondence should be addressed to Tze-Yun Sung, bobsung@chu.edu.tw

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This paper presents a novel algorithm and architecture for digital frequency synthesis (DFS). It is based on a simple difference equation. Simulation results show that the proposed DFS algorithm is preferable to the conventional phase-locked-loop frequency synthesizer and the direct digital frequency synthesizer in terms of the spurious-free dynamic range (SFDR) and the peak-signal-to-noise ratio (PSNR). Specifically, the results of SFDR and PSNR are more than 186.91 dBc and 127.74 dB, respectively. Moreover, an efficient DFS architecture for VLSI implementation is also proposed, which has the advantage of saving hardware cost and power consumption.

1. Introduction

Many modern devices, for example, radio receivers, ADSL (Asymmetric Digital Subscriber Line), XDSL (X Digital Subscriber Line), 3G/4G mobile phones, walkie-talkies, CB radios, satellite receivers, and GPS systems, require frequency synthesizers with fine resolutions, fast channel switching, and large bandwidths. There are two types of frequency synthesizer available: phase-locked loop (PLL) and direct digital frequency synthesis (DDFS).

PLL is a control system, which generates an output signal with phase matched that of the input reference signal. Figure 1 shows the conventional PLL frequency synthesizer consisting of a phase detector, a charge bump, a lowpass filter, a voltage control oscillator, and a frequency divider [1–8]. The lower frequency signal, F_{div} , obtained by dividing the output signal via the frequency divider, is compared with the reference signal, F_{ref} , in the phase



Figure 1: Block diagram of the conventional PLL system.



Figure 2: Block diagram of the conventional DDFS architecture.

detector to generate an error signal, which is proportional to the phase difference. The charge bump converts the error signal pulse into analog current pulses, which are then integrated by using the lowpass filter, and drives the voltage-controlled oscillator to obtain the desired frequency.

The commonly used architecture of DDFS [9] shown in Figure 2 consists of a phase accumulator, a sine/cosine generator, a digital-to-analog converter (DAC), and a lowpass filter (LPF). It takes two inputs: a reference clock and a *v*-bit frequency control word (FCW). In each clock cycle, the phase accumulator integrates FCW with periodical overflow to produce an angle in the range of $[0, 2\pi)$, the sine/cosine generator computes its sinusoidal value, which in practice is implemented digitally and, therefore, follows by DAC and LPF [10–23]. Various fractional-order ideal filters and fractional oscillators were proposed in [24–29].

Instead of using the conventional methods above, we propose a novel digital frequency synthesis (DFS) algorithm based on a simple difference equation. The rest of the paper is organized as follows. In Section 2, a novel DFS algorithm is proposed. In Section 3, the VLSI (very large-scale integration) digital frequency synthesizer is presented. In Section 4, the FPGA implementation and the performance evaluation are given. Conclusion can be found in Section 5.

2. The Proposed DFS Algorithm

The difference equation of DFS is as follows:

$$y[n-2] - 2a_1y[n-1] + a_2y[n] = x[n].$$
(2.1)

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Thus, we have the following characteristic equation:

$$z^{-2} - 2a_1 z^{-1} + a_2 = 0. (2.2)$$

The eigen-functions of (2.2) are represented as

$$z_{1,2} = r e^{\pm j\theta}.\tag{2.3}$$

The ZIR (zero-input response) of DFS can be written as

$$ZIR = r^{n}(B_{1}\cos(n\theta) + B_{2}\sin(n\theta)), \qquad (2.4)$$

where B_1 and B_2 are determined by initial conditions, and n = 0, 1, 2, ...For DFS with sine wave generator, we have

r = 1, $B_1 = 0,$ $B_2 = 1,$ $ZIR = sin(n\theta).$ (2.5)

The eigen-functions of DFS are therefore as follows:

$$z_{1,2} = e^{\pm j\theta} = \cos\theta \pm j\sin\theta = c \pm jd.$$
(2.6)

Thus, the characteristic equation can be expressed as

$$z^{-2} - 2cz^{-1} + c^2 + d^2 = 0, (2.7)$$

where $c^2 + d^2 = 1$.

Equation (2.7) could be rewritten as

$$z^{-2} - 2cz^{-1} + 1 = 0, (2.8)$$

and the transfer function of DFS can be derived as

$$H(z) = \frac{1}{z^{-2} - 2cz^{-1} + 1}.$$
(2.9)

According to (2.9), the corresponding difference equation could be derived as

$$y[n] = x[n] - y[n-2] + 2c \cdot y[n-1], \qquad (2.10)$$

where

$$y[0] = B_1 = 0,$$

$$y[1] = -B_1 \cos \theta + B_2 \sin \theta = d.$$
(2.11)

As one can see, a rotation of angle ϕ in the circular coordinate system can be obtained by performing a sequence of microrotations in an iterative manner. In particular, a vector can be successively rotated through the use of a sequence of predetermined step angles: $\alpha(i) = \tan^{-1}(2^{-i})$. This technique can be applied to generate many elementary functions, in which only simple adders and shifters are required. Thus, the well-known coordinate rotation digital computer (CORDIC) algorithm can be used for the DFS applications. The conventional CORDIC in the circular coordinate system is as follows [36–39]:

$$u(i + 1) = u(i) - \sigma(i)2^{-i}v(i),$$

$$v(i + 1) = v(i) + \sigma(i)2^{-i}u(i),$$

$$w(i + 1) = w(i) - \sigma(i)\alpha(i),$$

$$\alpha(i) = \tan^{-1}2^{-i},$$

(2.12)

where $\sigma(i) \in \{-1, +1\}$ denotes the direction of the *i*th microrotation, $\sigma_i = \operatorname{sign}(w(i))$ with $w(i) \to 0$ in the vector rotation mode, $\sigma_i = -\operatorname{sign}(u(i)) \cdot \operatorname{sign}(v(i))$ with $v(i) \to 0$ in the angle accumulated mode, the corresponding scale factor k(i) is equal to $\sqrt{1 + \sigma^2(i)2^{-2i}}$, and $i = 0, 1, \dots, l - 1$. The product of all scale factors after *l* microrotations is given by

$$K_1 = \prod_{i=0}^{n-1} k(i) = \prod_{i=0}^{n-1} \sqrt{1 + 2^{-2i}}.$$
(2.13)

In the vector rotation mode, $\sin \phi$ and $\cos \phi$ can be obtained, where the initial value (u(0), v(0)) = ($1/K_1$, 0). In principle, u_{out} and v_{out} can be computed from the initial value (u_{in} , v_{in}) = (u(0), v(0)) by using the following equation:

$$\begin{bmatrix} u_{\text{out}} \\ v_{\text{out}} \end{bmatrix} = K_1 \begin{bmatrix} \cos \phi & -\sin \phi \\ \sin \phi & \cos \phi \end{bmatrix} \begin{bmatrix} u_{\text{in}} \\ v_{\text{in}} \end{bmatrix}.$$
 (2.14)

In order to evaluate the sinusoidal parameters: *c* and *d* for the proposed digital frequency synthesizer, the inputs of the CORDIC processor are $u_{in} = 1/K_1$, $v_{in} = 0$, and $w_{in} = \theta$ as shown in Figure 3.

3. Proposed Architecture for Digital Frequency Synthesizer

In this section, the architecture and the terminology associated with the proposed digital frequency synthesizer are presented. Our scheme is based on the proposed DFS algorithm combined with a CORDIC processor. It consists mainly of the radian converter, the CORDIC processor, and the sine generator as shown in Figure 4.

Figure 5 shows the radian converter. It is a constant multiplier, which converts the input signal into radians. Figure 6 shows the CORDIC processor, which evaluates the sinusoidal value and consists of three adders and two shifters.

Figure 7 shows the architecture of sine generator, which is the core of the proposed digital frequency synthesizer. It consists of one multiplier, one adder, and two latches only.



Figure 3: The CORDIC arithmetic for the proposed digital frequency synthesizer.



Figure 4: The proposed digital frequency synthesizer.

The key terminologies associated with the proposed digital frequency synthesizer are as follows.

3.1. Output Frequency

The output frequency of the proposed digital frequency synthesizer is determined by the coefficients d and c, since

$$\theta = \omega T_s = \tan^{-1} \left(\frac{d}{c} \right),$$

$$F_o = \frac{1}{2\pi} \cdot \tan^{-1} \left(\frac{d}{c} \right) \cdot F_s.$$
(3.1)

3.2. Frequency Resolution

For *m*-bit digital frequency synthesizer, the minimum change of the output frequency $\Delta F_{o,\min}$ is expressed as

$$\Delta F_{o,\min} = \frac{1}{2\pi} \cdot \tan^{-1} \left(2^{-(m-1)} \right) \cdot F_s.$$
(3.2)



Figure 5: The radian converter.

3.3. Bandwidth

The bandwidth of digital frequency synthesizer is defined as the difference between the highest and lowest attainable output frequencies, which are expressed as follows:

$$F_{o,\max} = \frac{1}{2\pi} \cdot \tan^{-1}(1) \cdot F_s,$$

$$F_{o,\min} = \frac{1}{2\pi} \cdot \tan^{-1}(2^{-(m-1)}) \cdot F_s.$$
(3.3)

3.4. Peak Signal-to-Noise Ratio (PSNR)

A good direct digital frequency synthesizer should have an output signal with low noise, which can be evaluated by using the following signal-to-noise-ratio (PSNR) measured in dB:

$$PSNR = 20 \log\left(\frac{255}{\sqrt{MSE}}\right),\tag{3.4}$$

where MSE is the mean square error.

3.5. Spurious-Free Dynamic Range (SFDR)

The spurious-free dynamic range (SFDR) is defined as the ratio of the amplitude of the desired frequency component to that of the largest undesired frequency component at the output of a DDFS. It is expressed in decibels (dBc) as follows:

$$SFDR = 20 \log\left(\frac{A_p}{A_s}\right) = 20 \log(A_p) - 20 \log(A_s), \qquad (3.5)$$



Figure 6: The CORDIC processor (LUT: Lookup table).



Figure 7: The sine generator.

where A_p is the amplitude of the desired frequency component, A_s is the amplitude of the largest undesired frequency component, and the higher the better.

4. FPGA Implementation of Digital Frequency Synthesizer

In this section, the proposed high-performance architecture of digital frequency synthesizer is presented. Figure 8 depicts the system block diagram. The PSNR and SFDR of the proposed



Figure 8: The proposed digital frequency synthesizer.



Figure 9: The PSNR of the proposed digital frequency synthesizer at various word lengths (100 MHz sampling rate and the maximum output frequency 12.5 MHz).

digital frequency synthesizer at various word lengths at 100 MHz sampling rate and the maximum output frequency 12.5 MHz are shown in Figures 9 and 10, respectively.

The platform for architecture development and verification has also been designed and implemented to evaluate the development cost. The proposed architecture of digital frequency synthesizer has been implemented on the field programmable gate array (FPGA) emulation board [40]. The FPGA has been integrated with the microcontroller (MCU) and I/O interface circuit (USB 2.0) to form the architecture development and verification platform.

Figure 11 depicts the block diagram and circuit board of the architecture development and evaluation platform. In which, the microcontroller reads data and commands from PC and writes the results back to PC via USB 2.0 bus; the FPGA implements the proposed



Figure 10: The SFDR of the proposed digital frequency synthesizer at various word lengths (100 MHz sampling rate and the maximum output frequency 12.5 MHz).



Figure 11: Block diagram and circuit board of the architecture development and verification platform.

architecture of digital frequency synthesizer. The hardware code in portable hardware description language runs on PC with the logic circuit simulator [41] and FPGA compiler [42]. It is noted that the throughput can be improved by using the proposed pipelined architecture while the computation accuracy is the same as that obtained by using the conventional architecture with the same word length. Thus, the proposed digital frequency synthesizer improves the power consumption and performance significantly. Moreover, all the control signals are internally generated onchip. The proposed digital frequency synthesizer provides a high-performance sinusoid waveform.

5. Conclusion

In this paper, we present a novel digital frequency synthesizer based on a simple difference equation with pipelined data path. Circuit emulation shows that the proposed high-performance architecture has the advantages of high precision, high data rate, and simple hardware. For 16-bit digital frequency synthesizer, the PSNR and SFDR obtained by using the proposed architecture at the maximum output frequency are 127.74 dB and 186.91 dBc, respectively. As shown in Table 1, the proposed digital frequency synthesizer is superior to the previous works in terms of SFDR, PSNR, and hardware [18, 30–35]. The proposed digital

Authors	Items					
	Output resolution (bits)	SFDR (dBc)	PSNR (dB)	ROM (words)	Adders	Multipliers
Strollo et al. [18]	13	90.2	—	1344	8	0
Song and Kim [30]	16	100		270	6	6
Langlois and Al-Khalili [31]	14	96.2	—	1152	8	0
De Caro et al. [32]	12	80		0	2	2
De Caro and Strollo [33]	12	83.6		896	2	2
Yang et al. [34]	12	80		2176	6	0
Curticapean and Niittylahti [35]	14	85	—	832	2	2
Ko et al. (This work)	14	133	113	14	8	1
	16	187	128	16	8	1

Table 1: Comparisons between the proposed DFS and other related works.

frequency synthesizer designed by portable hardware description language is a reusable IP, which can be implemented in various VLSI processes with trade-offs of performance, area and power consumption.

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