Research Article

Adaptive Control of a Fermentation Bioprocess for Lactic Acid Production

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This paper presents the design and the analysis of an indirect adaptive control strategy for a lactic acid production, which is carried out in continuous stirred tank bioreactors. Firstly, an indirect adaptive control structure based on the nonlinear process model is derived by combining a linearizing control law with a new parameter estimator. This estimator is used for on-line estimation of the bioprocess unknown kinetics, avoiding the introduction of a state observer. Secondly, a tuning procedure of estimator design parameters is achieved by stability analysis of the control scheme. The effectiveness and performance of estimation and control algorithms are illustrated by numerical simulations applied in the case of a lactic fermentation bioprocess for which kinetic dynamics are strongly nonlinear, time varying, and completely unknown, and not all the state variables are measurable.

1. Introduction

The control of biotechnological processes has been and remains an important problem attracting wide attention, the main engineering motivation being the improvement of the operational stability and production efficiency of such living processes.

It is well known that traditional control design involves a complicated mathematical analysis and has many difficulties especially in controlling highly nonlinear and time varying plants. A powerful tool for nonlinear controller design is the feedback linearization technique [1–3], but the use of it requires the complete knowledge of the process. In practice, there are many processes characterized by highly nonlinear dynamics; as a consequence, an accurate model for these processes is difficult to develop. Therefore, in recent years, it has been noticed a great progress in adaptive and robust adaptive control of nonlinear systems, due to their ability to compensate for parametric uncertainties. An important assumption in previous works on nonlinear adaptive control was the linear dependence on the unknown parameters [2, 4, 5].

In the modern industry, the development of advanced control strategies for bioprocesses is hampered by two major difficulties [6–9]. The first one is related to kinetics of these processes, which are strongly nonlinear and furthermore the kinetic and process parameters are often partially or completely unknown. Another difficulty lies in the absence, for many processes, of cheap and reliable instrumentation suited for real-time monitoring of the process variables. In order to overcome these difficulties, several strategies were developed, such as exactly linearizing control [6, 7, 10], adaptive approach proposed by Bastin and Dochain and applied in several works [6, 7, 10–12], optimal control, especially for fed-batch bioprocesses [6, 13], neural control [14], sliding mode control [15], robust and robust adaptive control [12, 16], and model predictive control [17]. Some problems which occur in implementation of numerical control of bioreactors are presented and analyzed in [18].

The difficulties encountered in the measurement of state variables impose the use of so-called "software sensors"—combinations between hardware sensors and software estimators [6, 7]. Note that these software sensors are used not only for the estimation of concentrations of some components but also for the estimation of kinetic parameters or even kinetic reactions. The interest for the development of software sensors for bioreactors is proved by the big number of publications and applications in this area [6–8, 19].

This paper, which is an extended version of Petre et al. [20], presents the design and the analysis of a nonlinear adaptive control strategy, capable to deal with the model uncertainties in an adaptive way, for a lactic fermentation bioprocess that is carried out in two continuous stirred tank bioreactors sequentially connected. In order to avoid the derivation of additional state observers, a new indirect adaptive control algorithm is presented. More exactly, the consumption rates of two limiting substrates are considered completely unknown and summarized in two unknown and time varying parameters, which are estimated by means of an appropriately observer-based estimator. This algorithm could be considered as an extension of the theory proposed by Bastin and Dochain [6] and improved by Ignatova et al. [11], since the substrate consumption rates are considered as completely unknown functions describing the whole kinetics.

The present work is focused in two directions. First, an adaptive control structure based on the nonlinear model of the process is designed as a combination of a linearizing control law and of a parameter estimator, used for the on-line estimation of bioprocess unknown kinetics. Second, by using the stability and convergence analysis of the proposed control scheme, a tuning procedure of the kinetic estimator design parameters is achieved.

The effectiveness and performance of both estimation and control algorithms are illustrated by simulations applied in the case of a lactic fermentation bioprocess for which kinetic dynamics are strongly nonlinear, time varying and completely unknown, and not all the state variables are measurable.

We note that the bioprocess model and the control goal are the same as in the Ben Youssef et al. [21], where an adaptive-multivariable predictive control law was proposed and analyzed. However, in the present work, an indirect multivariable adaptive linearizing controller is developed and analyzed. The control structure is derived by combining a linearizing control law with a new parameter estimator used for on-line estimation of the bioprocess unknown kinetics.

In the work by Ignatova et al. [11], by using the observer-based estimator proposed by Petre [22], an indirect adaptive scheme was derived and analyzed for a gluconic acid production bioprocess. In the present work, the multivariable control strategy presented in Petre [22], and further developed by Ignatova et al. [11], is improved and full stability

and convergence proof are provided. Finally, a tuning procedure of kinetic estimator design parameters for a lactic acid production bioprocess was obtained.

The paper is organized as follows. Section 2 is devoted to a brief description and mathematical modeling of a lactic acid fermentation bioprocess. Some nonlinear and adaptive control strategies are proposed in Section 3. From the stability analysis of the adaptive control scheme, a tuning procedure of kinetic estimator design parameters is achieved. Simulations results presented in Section 4 illustrate the performance of the proposed control algorithms and, finally, Section 5 concludes the paper.

2. Process Modeling and Control Problem

Lactic acid has traditionally been used in the food industry as an acidulating and/or preserving agent, and in the biochemical industry for cosmetic and textile applications [21, 23, 24]. Recently, lactic acid fermentation has received much more attention because of the increasing demand for new biomaterials such as biodegradable and biocompatible polylactic products. Two major factors limit its biosynthesis, affecting growth, and productivity: the nutrient limiting conditions and the inhibitory effect caused by lactic acid accumulation in the culture broth [21]. It was shown that, in addition to the inhibition effect of lactic acid, the richness of the fermentation medium may have a strong influence on growth dynamics and that the nutritional limitations during culture may interfere with inhibition by acid lactic [21]. A reliable model that explicitly integrates nutritional factor effects on both growth and lactic acid production in a batch fermentation process implementing *Lb. casei* was developed by Ben Youssef et al. [21] and it is described by the following basic differential equations:

$$\dot{X} = \mu X - k_d X, \qquad \dot{P} = v_p X, \qquad \dot{S} = -q_s X, \qquad (2.1)$$

where X, S, and P are, respectively, the concentrations of cells, substrate (glucose) and lactic acid. μ, v_p, and q_s correspond, respectively, to specific growth rate of the cells, specific rate of lactic acid production, and to specific rate of glucose consumption. k_d is the rate of cell death.

Since in lactic acid fermentation process the main cost of raw material comes from the substrate and nutrient requirements, in [21] some possible continuous-flow control strategies that satisfy the economic aspects of lactic acid production were investigated. The advantage of implementing continuous-flow process is that the main product, which is also an inhibitor, is continuously withdrawn from the system. Much more, according to microbial engineering theory, for a product-inhibited reaction like lactic acid or alcoholic fermentation [21, 25], a multistage system composed of many interconnected continuous stirred tank reactors, where in the different reactors some physiological states of microbial culture (substrate, metabolites) can be kept constant to some optimal values, may be a good idea.

Therefore, the model (2.1) can be extended to a continuous-flow process that is carried out in two continuous stirred tank reactors sequentially connected, as in the schematic view presented in Figure 1.

For this two-stage continuous flow bioreactor, the mathematical model is given by the following set of differential equations, each stage being of same constant volume V.
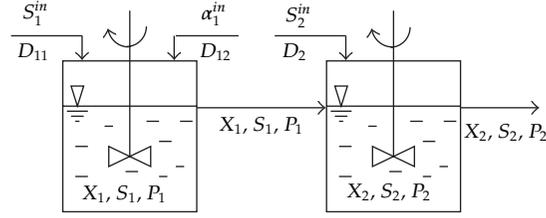
Figure 1: A cascade of two reactors for the lactic acid production.

First Stage:

$$\dot{X}_1 = (\mu_1 - k_d)X_1 - D_1 X_1, \tag{2.2}$$

$$\dot{P}_1 = v_{p1} X_1 - D_1 P_1, \tag{2.3}$$

$$\dot{S}_1 = -q_{s1} X_1 + D_{11} S_1^{in} - D_1 S_1, \tag{2.4}$$

$$\dot{\alpha}_1 = D_{12} \alpha_1^{in} - D_1 \alpha_1, \tag{2.5}$$

where $D_1 = D_{11} + D_{12}$.

Second Stage:

$$\dot{X}_2 = (\mu_2 - k_d)X_2 + D_1 X_1 - (D_1 + D_2)X_2, \tag{2.6}$$

$$\dot{P}_2 = v_{p2} X_2 + D_1 P_1 - (D_1 + D_2)P_2, \tag{2.7}$$

$$\dot{S}_2 = -q_{s2} X_2 + D_1 S_1 + D_2 S_2^{in} - (D_1 + D_2)S_2, \tag{2.8}$$

$$\dot{\alpha}_2 = D_1 \alpha_1 - (D_1 + D_2)\alpha_2, \tag{2.9}$$

where X_i, S_i, P_i, and α_i, $(i = 1, 2)$ are, respectively, the concentrations of biomass (cells), substrate, lactic acid, and enrichment factor in each bioreactor. μ_i, v_{pi}, and q_{si} $(i = 1, 2)$ correspond, respectively, to specific growth rate of the cells, specific rate of lactic acid production, and specific rate of glucose consumption in each bioreactor. D_{11} is the first-stage dilution rate of a feeding solution at an influent glucose concentration S_1^{in}. D_{12} is the first-stage dilution rate of a feeding solution at an influent enrichment factor α_1^{in}. D_2 is the influent dilution rate added at the second stage and S_2^{in} is the corresponding feeding glucose concentration. It can be seen that in the second stage no growth factor feeding tank is included since this was already feeding in the first reactor.

In the model (2.2)–(2.9), the mechanism of cell growth, the specific lactic acid production rate and the specific consumption rate in each bioreactor are given by [21]:

$$\mu_i = \bar{\mu}_{\max\ i} \left(\frac{\overline{K}_{P_i}^{gc}}{\overline{K}_{P_i}^{gc} + P_i} \right) \left(\frac{S_i}{\overline{K}_S^{gc} + S_i} \right) \left(1 - \frac{P_i}{P_C^{gc}} \right),$$

$$v_{pi} = \eta \mu_i + \beta \left(\frac{S_i}{\overline{K}_{S_i}^{rc} + S_i} \right), \quad q_{si} = \frac{v_{pi}}{Y_{PS}}, \quad i = 1, 2, \tag{2.10}$$

with $\overline{\mu}_{\max i}$ the maximum specific growth rate, $\overline{K}_{P_i}^{gc}$ the lactic acid inhibition constant, $\overline{K}_S^{gc}$ the affinity constant of the growing cells for glucose, P_C^{gc} the critical lactic acid concentrations, $\overline{K}_{S_i}^{rc}$ is the affinity constant of the resting cells for glucose, and Y_{PS} is the constant substrate-to-product conversion yield. The superscript gc denotes the parameters related to growing cells and rc to that of the resting cells, and η and β are positive constants.

The kinetic parameters of this model may be readjusted depending on the medium enrichment factor α_i as follows [21]:

$$\overline{\mu}_{\max i} = \frac{\mu_{\max}(\alpha_i - \alpha_0)}{K_{\alpha\mu} + (\alpha_i - \alpha_0)}, \qquad \overline{K}_{P_i}^{gc} = \frac{K_{P\max}^{gc}(\alpha_i - \alpha_0)}{K_{\alpha P} + (\alpha_i - \alpha_0)}, \qquad \overline{K}_{S_i}^{rc} = \frac{K_{S\max}^{rc}(\alpha_i - \alpha_0)}{K_{\alpha S} + (\alpha_i - \alpha_0)}, \qquad (2.11)$$

where α_0 is the minimal nutritional factor necessary for growth, $K_{\alpha\mu}$, $K_{\alpha P}$ and $K_{\alpha S}$ are saturation constants. $\mu_{\max}$, $K_{P\max}^{gc}$ and $K_{S\max}^{rc}$ correspond to the limit value of each kinetic parameter. Note that the hyperbolic-type expressions in (2.11) quantify the nutritional limitations and are based on the growth model proposed by Bibal [26].

For this two-stage reactor configuration, the operating point of the continuous lactic acid fermentation process could be adjusted by acting on at least two control inputs, that is, the glucose feeding flow rates both in the first and in the second reactor. These considerations show the multivariable nature of the control problem.

3. Control Strategies

3.1. Control Objective

As it was formulated in the previous section, the control objective consists in adjusting the plant's load in order to convert the glucose into lactic acid via fermentation, which is directly correlated to the economic aspects of lactic acid production. More exactly, considering that the process model (2.2)–(2.9) is incompletely known, its parameters are time varying and not all the states are available for measurements, the control goal is to maintain the process at some operating points, which correspond to a maximal lactic production rate and a minimal residual glucose concentration. After a detailed process steady-state analysis, in [21] it has been demonstrated that these desiderata can be satisfied if the operating point of lactic acid fermentation is kept around the points $S_1^* = 3\,g/L$ and $S_2^* = 5\,g/L$. This choice is the best option in order to simultaneously satisfy the both objectives: maximal lactic production rate and minimal residual glucose concentration [21]. As control variables we select the glucose feeding flow rates both in the first and in the second reactor denoted by F_1 and F_2, respectively, where $F_1 = (D_1 - D_{12})S_1^{in}$ and $F_2 = D_2 S_2^{in}$. In this way we obtain a multivariable control problem with two inputs: F_1 and F_2 and two outputs: S_1 and S_2.

In the following, in Section 3.2 we will present the design of an exactly linearizing feedback controller. This controller will be used as a benchmark in order to compare its behavior with the behavior of the indirect multivariable adaptive controller developed in Section 3.3. Also, in this subsection, the parameter estimator used in the indirect multivariable adaptive controller will be designed. In Section 3.4 the tuning procedure of estimator design parameters via stability and convergence analysis of the control scheme will be derived.

3.2. Exactly Linearizing Feedback Controller

Firstly, we consider the ideal case where maximum prior knowledge concerning the process (kinetics, yield coefficients, and state variables) is available, and the relative degree of differential equations in process model is equal to 1. Assume that for the two interconnected reactors we wish to have the following first order linear stable closed loop (process + controller) behavior:

$$\begin{bmatrix} \dot{S}_1^* - \dot{S}_1 \\ \dot{S}_2^* - \dot{S}_2 \end{bmatrix} + \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \begin{bmatrix} S_1^* - S_1 \\ S_2^* - S_2 \end{bmatrix} = 0, \quad \lambda_1, \lambda_2 > 0, \tag{3.1}$$

where S_1^* and S_2^* are the desired values of S_1 and S_2.

Since (2.4) and (2.8) in the process model have the relative degree equal to 1, these can be considered as an input-output model and can be rewritten in the following form:

$$\begin{bmatrix} \dot{S}_1 \\ \dot{S}_2 \end{bmatrix} = \begin{bmatrix} -q_{s1} X_1 \\ -q_{s2} X_2 \end{bmatrix} - \begin{bmatrix} D_1 & 0 \\ -D_1 & D_1 + D_2 \end{bmatrix} \begin{bmatrix} S_1 \\ S_2 \end{bmatrix} + \begin{bmatrix} F_1 \\ F_2 \end{bmatrix}. \tag{3.2}$$

Then from (3.1) and (3.2) one obtains the following *exactly multivariable decoupling linearizing feedback control law*:

$$\begin{bmatrix} F_1 \\ F_2 \end{bmatrix} = \begin{bmatrix} \dot{S}_1^* \\ \dot{S}_2^* \end{bmatrix} + \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \begin{bmatrix} S_1^* - S_1 \\ S_2^* - S_2 \end{bmatrix} + \begin{bmatrix} q_{s1} X_1 \\ q_{s2} X_2 \end{bmatrix} + \begin{bmatrix} D_1 & 0 \\ -D_1 & D_1 + D_2 \end{bmatrix} \begin{bmatrix} S_1 \\ S_2 \end{bmatrix}. \tag{3.3}$$

The control law (3.3) applied to the process (3.2) will result in:

$$\begin{bmatrix} \dot{S}_1 \\ \dot{S}_2 \end{bmatrix} = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \begin{bmatrix} S_1^* - S_1 \\ S_2^* - S_2 \end{bmatrix} + \begin{bmatrix} \dot{S}_1^* \\ \dot{S}_2^* \end{bmatrix}, \tag{3.4}$$

and leads to the following linear error models:

$$\dot{e}_1 = -\lambda_1 e_1, \qquad \dot{e}_2 = -\lambda_2 e_2, \tag{3.5}$$

where $e_1 = S_1^* - S_1$ and $e_2 = S_2^* - S_2$ represent the tracking errors. It is clear that for $\lambda_1, \lambda_2 > 0$, the error models (3.5) have an exponential stable point at $e_1 = 0$ and $e_2 = 0$.

3.3. An Indirect Multivariable Adaptive Controller

In the previous subsection, it was assumed that the functional forms of the nonlinearities as well as the process parameters are known. Since such prior knowledge is not realistic, in this subsection we consider that the kinetics functions are completely unknown. So, the substrate consumption rates $-q_{s1} X_1$ and $-q_{s2} X_2$ in (3.2) can be expressed as follows:

$$-q_{s1} X_1 = \rho_1, \qquad -q_{s2} X_2 = \rho_2, \tag{3.6}$$

where ρ_1 and ρ_2 are two unknown time-varying parameters. In this case, the control law (3.3) becomes:

$$\begin{bmatrix} F_1 \\ F_2 \end{bmatrix} = \begin{bmatrix} \dot{S}_1^* \\ \dot{S}_2^* \end{bmatrix} + \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \begin{bmatrix} S_1^* - S_1 \\ S_2^* - S_2 \end{bmatrix} - \begin{bmatrix} \rho_1 \\ \rho_2 \end{bmatrix} + \begin{bmatrix} D_1 & 0 \\ -D_1 & D_1 + D_2 \end{bmatrix} \begin{bmatrix} S_1 \\ S_2 \end{bmatrix}, \tag{3.7}$$

where the unknown parameters ρ_1 and ρ_2 will be substituted by their on-line estimates $\widehat{\rho}_1$ and $\widehat{\rho}_2$ calculated by using an observer-based parameter estimator (OBE) [22] applied only for the dynamics of S_1 and S_2 from (2.4) and (2.8) rewritten as follows:

$$\dot{S}_1 = \rho_1 + F_1 - D_1 S_1,$$
$$\dot{S}_2 = \rho_2 + D_1 S_1 + F_2 - (D_1 + D_2) S_2. \tag{3.8}$$

Under these notations, the algorithm for on-line computation of $\widehat{\rho}_1$ and $\widehat{\rho}_2$ is particularized as follows:

$$\dot{\widehat{S}}_1 = \widehat{\rho}_1 + F_1 - D_1 S_1 + \omega_1 \left(S - \widehat{S}_1 \right), \tag{3.9}$$

$$\dot{\widehat{S}}_2 = \widehat{\rho}_2 + D_1 S_1 + F_2 - (D_1 + D_2) S_2 + \omega_2 \left(S_2 - \widehat{S}_2 \right), \tag{3.10}$$

$$\dot{\widehat{\rho}}_1 = \gamma_1 \left(S - \widehat{S}_1 \right), \tag{3.11}$$

$$\dot{\widehat{\rho}}_2 = \gamma_2 \left(S_2 - \widehat{S}_2 \right), \tag{3.12}$$

where $\gamma_1, \gamma_2 > 0$ are the gains of the updating laws (3.11) and (3.12), and $\omega_1, \omega_2 > 0$ are design parameters to control the stability and the tracking properties of the estimator.

Remark 3.1. The parameter error convergence for the estimator (3.9)–(3.12) is assured if some signals in system satisfy the well-known persistency of excitation condition [4, 5]: A piecewise continuous signal vector $\phi : \mathbb{R}^+ \rightarrow \mathbb{R}^n$ is referred to as persistent excitation with a level of excitation $\kappa_0 > 0$ if there exist constants $\kappa_1, T_0 > 0$ such that $\kappa_1 I \geq (1/T_0) \int_t^{t+T_0} \phi(\tau)\phi^T(\tau)d\tau \geq \kappa_0 I$, for all $t \geq 0$. Although the matrix $\phi(\tau)\,\phi^T(\tau)$ is singular for each τ, this condition requires that $\phi(t)$ varies in such a way with time that the integral of the matrix $\phi(\tau)\phi^T(\tau)$ is uniformly positive definite over any time interval $[t, t + T_0]$. In our particular case it is necessary that, for example, D_1, D_2 or some kinetic parameters to be persistently exciting (which is generically fulfilled).

Finally, *an indirect multivariable adaptive linearizing controller* is obtained by combination of (3.9)–(3.12) and (3.7) rewritten as follows:

$$\begin{bmatrix} F_1 \\ F_2 \end{bmatrix} = \begin{bmatrix} \dot{S}_1^* \\ \dot{S}_2^* \end{bmatrix} + \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \begin{bmatrix} S_1^* - S_1 \\ S_2^* - S_2 \end{bmatrix} - \begin{bmatrix} \widehat{\rho}_1 \\ \widehat{\rho}_2 \end{bmatrix} + \begin{bmatrix} D_1 & 0 \\ -D_1 & D_1 + D_2 \end{bmatrix} \begin{bmatrix} S_1 \\ S_2 \end{bmatrix}. \tag{3.13}$$

A block diagram of the designed indirect multivariable adaptive control system is shown in Figure 2.
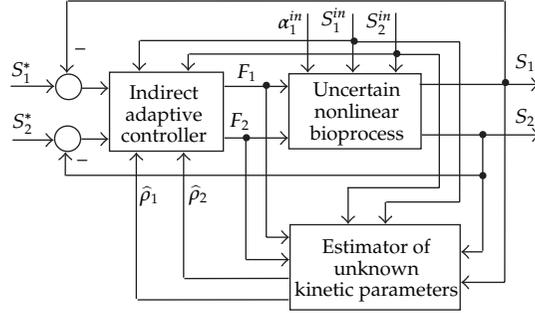
Figure 2: The structure of the indirect adaptive control system.

Remark 3.2. Since the on-line measurement of the lactic acid concentrations is not possible, in order to provide on-line evaluation of the current lactic acid production, an asymptotic state observer is derived as follows.

Let us define the auxiliary variables z_1 and z_2 as:

$$z_1 = P_1 + Y_{PS}S_1, \qquad z_2 = P_2 + Y_{PS}S_2, \tag{3.14}$$

which use only the measurements of the outputs S_1 and S_2 that are available on-line. The dynamics of z_i, $i = 1, 2$, deduced from model (2.2)–(2.9), are expressed by the following linear stable equations:

$$\dot{\hat{z}}_1 = -D_1\hat{z}_1 + Y_{PS}F_1, \qquad \dot{\hat{z}}_2 = D_1\hat{z}_1 - (D_1 + D_2)\hat{z}_2 + Y_{PS}F_2, \tag{3.15}$$

that are independent of the process kinetics. Then, from (3.14) and (3.15), the on-line estimation of P_1 and P_2 are given by the following relations:

$$\widehat{P}_1 = \hat{z}_1 - Y_{PS}S_1, \qquad \widehat{P}_2 = \hat{z}_2 - Y_{PS}S_2. \tag{3.16}$$

3.4. Tuning Procedure of Estimator Design Parameters via Stability and Convergence Analysis

Let us denote by $y = [S_1 \ \ S_2]^T$ the vector containing the concentrations of controlled feeding substrates and by $u = [F_1 \ \ F_2]^T$ the vector of the input control variables, that is, the vector of the mass feeds of substrates S_1 and S_2, respectively. Then (3.8) can be written as

$$\dot{y} = \rho - Dy + u, \tag{3.17}$$

where $\rho = [\rho_1 \ \rho_2]^T$ is the unknown time-varying parameter vector and D is the dilution rate matrix, whose structure is as follows:

$$D = \begin{bmatrix} D_1 & 0 \\ -D_1 & D_1 + D_2 \end{bmatrix}. \tag{3.18}$$

The OBE (3.9)–(3.12) can be represented in a matrix form as follows:

$$\dot{\hat{y}} = \hat{\rho} - Dy + u + \Omega(y - \hat{y}),$$

$$\dot{\hat{\rho}} = \Gamma(y - \hat{y}),$$

(3.19)

where $\hat{\rho}$ is the estimation of unknown parameter ρ, and $\Omega, \Gamma \in \Re^{2 \times 2}$ are positive definite diagonal matrices whose entries $\omega_1, \omega_2 > 0$ and $\gamma_1, \gamma_2 > 0$, respectively, represent the estimator design parameters.

Under the previous notations, the exactly multivariable decoupling linearizing feedback control law (3.3), respectively, (3.7) can be written as:

$$u = \dot{y}^* + \Lambda(y^* - y) - \rho + Dy,$$

(3.20)

where $y^* = [S_1^* \; S_2^*]^T$, and the indirect adaptive linearizing control law (3.13) can be written as:

$$u = \dot{y}^* + \Lambda(y^* - y) - \hat{\rho} + Dy,$$

(3.21)

where $\Lambda = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} > 0$.

Case 1. By defining the errors $\tilde{y} = y - \hat{y}$ and $\tilde{\rho} = \rho - \hat{\rho}$, in the case when the unknown parameter ρ is constant, the following error system can be derived from (3.17) and (3.19):

$$\dot{\tilde{y}} = -\Omega \tilde{y} + \tilde{\rho}, \qquad \dot{\tilde{\rho}} = -\Gamma \tilde{y}.$$

(3.22)

It can be seen that the error system in (3.22) is a linear time-invariant system that can be rewritten as:

$$\begin{bmatrix} \dot{\tilde{y}} \\ \dot{\tilde{\rho}} \end{bmatrix} = \begin{bmatrix} -\Omega & I_2 \\ -\Gamma & 0 \end{bmatrix} \begin{bmatrix} \tilde{y} \\ \tilde{\rho} \end{bmatrix},$$

(3.23)

where I_2 is the identity matrix of order two. The stability of the equilibrium ($\tilde{y} = 0, \tilde{\rho} = 0$) of (3.22) can be proved by using the well-known Hurwitz criterion. So, by using the coefficients of the characteristic polynomial associated to (3.23), given by

$$P(\lambda) = \lambda^4 + (\omega_1 + \omega_2)\lambda^3 + (\omega_1\omega_2 + \gamma_1 + \gamma_2)\lambda^2 + (\gamma_2\omega_1 + \gamma_1\omega_2)\lambda + \gamma_1\gamma_2$$

$$= a_4\lambda^4 + a_3\lambda^3 + a_2\lambda^2 + a_1\lambda + a_0,$$

(3.24)

after straightforward calculation we obtain that all Hurwitz determinants are positive for any $\omega_1, \omega_2 > 0$ and $\gamma_1, \gamma_2 > 0$. Indeed, by using the coefficients of the above characteristic

polynomial we can construct the following Hurwitz determinant:

$$
\Delta_4 = \begin{bmatrix} a_3 & a_1 & 0 & 0 \\ a_4 & a_2 & a_0 & 0 \\ 0 & a_3 & a_1 & 0 \\ 0 & a_4 & a_2 & a_0 \end{bmatrix} = \begin{bmatrix} \omega_1 + \omega_2 & \gamma_2\omega_1 + \gamma_1\omega_2 & 0 & 0 \\ 1 & \omega_1\omega_2 + \gamma_1 + \gamma_2 & \gamma_1\gamma_2 & 0 \\ 0 & \omega_1 + \omega_2 & \gamma_2\omega_1 + \gamma_1\omega_2 & 0 \\ 0 & 1 & \omega_1\omega_2 + \gamma_1 + \gamma_2 & \gamma_1\gamma_2 \end{bmatrix}. \tag{3.25}
$$

Now we construct and calculate all Hurwitz determinants as follows:

$$
\Delta_1 = [a_3] = \omega_1 + \omega_2 > 0,
$$

$$
\Delta_2 = \begin{bmatrix} a_3 & a_1 \\ a_4 & a_2 \end{bmatrix} = \begin{bmatrix} \omega_1 + \omega_2 & \gamma_2\omega_1 + \gamma_1\omega_2 \\ 1 & \omega_1\omega_2 + \gamma_1 + \gamma_2 \end{bmatrix}
$$

$$
= \omega_1\omega_2(\omega_1 + \omega_2) + \gamma_2\omega_1 + \gamma_1\omega_2 > 0,
$$

$$
\Delta_3 = \begin{bmatrix} a_3 & a_1 & 0 \\ a_4 & a_2 & a_0 \\ 0 & a_3 & a_1 \end{bmatrix} = \begin{bmatrix} \omega_1 + \omega_2 & \gamma_2\omega_1 + \gamma_1\omega_2 & 0 \\ 1 & \omega_1\omega_2 + \gamma_1 + \gamma_2 & \gamma_1\gamma_2 \\ 0 & \omega_1 + \omega_3 & \gamma_2\omega_1 + \gamma_1\omega_2 \end{bmatrix} \tag{3.26}
$$

$$
= \omega_1\omega_2\Big((\omega_1 + \omega_2)(\gamma_2\omega_1 + \gamma_1\omega_2) + (\gamma_1 - \gamma_2)^2\Big) > 0,
$$

$$
\Delta_4 = \gamma_1\gamma_2 \cdot \Delta_3 > 0.
$$

According to Hurwitz criterion [27, 28], the system (3.23) is stable since all Hurwitz determinants are positive if $\omega_1, \omega_2 > 0$ and $\gamma_1, \gamma_2 > 0$. Then, the matrix associated to (3.23) is a Hurwitz matrix and the equilibrium ($\tilde{y} = 0$, $\tilde{\rho} = 0$) of (3.22) is uniformly asymptotically stable.

Case 2. In the case when the unknown parameter ρ is time-varying, the stability analysis of the proposed indirect multivariable adaptive linearizing controller is carried out under the following realistic *assumptions*.

(A1) The substrate concentrations S_1 and S_2, the components of the dilution rate matrix D, and the substrate feed rates are on-line measured.

(A2) The substrate consumption rates ρ_1 and ρ_2 are fully unknown.

(A3) The reference signal vector y^* and its time derivative are piecewise continuous bounded functions of time, and the measured substrate feed rates are continuous bounded functions of time.

(A4) The measurements of substrate vector, denoted $y_m(t)$, are corrupted by an additive noise vector $w(t) = [w_1 \ w_2]^T$: $y_m(t) = y(t) + w(t)$, for all t.

(A5) The components of time varying parameter vector $\rho(t)$ are bounded as: $0 > \rho_i(t) \geq \rho_i^*$, $\rho_i^* < 0$, $i = 1, 2$, for all t.

(A6) The time derivatives of the components of $\rho(t)$ are bounded as: $|\dot{\rho}_i(t)| \leq m_{1i}$, $m_{1i} > 0$, $i = 1, 2$, for all t.

(A7) The measurement noise vector is bounded as: $|w(t)| \leq m_{2i}$, $m_{2i} > 0$, $i = 1, 2$, for all t.

(A8) The components of dilution rates matrix D are known and bounded: $0 \leq D_i(t) \leq D_i^{\max}$, $i = 1, 2$, $0 \leq D_{12}(t) \leq D_{12}^{\max}$, for all t.

Under these assumptions, the parameter estimator (3.19) and the adaptive control law (3.21) are rewritten as follows:

$$\dot{\hat{y}} = \hat{\rho} - Dy_m + u + \Omega(y_m - \hat{y}), \qquad (3.27)$$

$$\dot{\hat{\rho}} = \Gamma(y_m - \hat{y}), \qquad (3.28)$$

$$u = \dot{y}^* + \Lambda(y^* - y_m) - \hat{\rho} + Dy_m. \qquad (3.29)$$

From (3.17), (3.27), and (3.28) under the condition (A4), the following error system can be derived as follows:

$$\dot{\tilde{y}} = \dot{y} - \dot{\hat{y}} = \rho - Dy + u - \hat{\rho} + Dy_m - u - \Omega(y_m - \hat{y})$$

$$= \tilde{\rho} + D(-y + y + w) - \Omega(y + w - \hat{y}) = -\Omega\tilde{y} + \tilde{\rho} + (D - \Omega)w, \qquad (3.30)$$

$$\dot{\tilde{\rho}} = \dot{\rho} - \dot{\hat{\rho}} = \dot{\rho} - \Gamma(y + w - \hat{y}) = -\Gamma\tilde{y} - \Gamma w + \dot{\rho},$$

which can be rewritten in the following form:

$$\dot{\tilde{y}}_1 = -\omega_1\tilde{y}_1 + \tilde{\rho}_1 + (D_1 - \omega_1)w_1, \qquad \dot{\tilde{\rho}}_1 = -\gamma_1\tilde{y}_1 - \gamma_1 w_1 + \dot{\rho}_1,$$

$$\dot{\tilde{y}}_2 = -\omega_2\tilde{y}_2 + \tilde{\rho}_2 + (D_1 + D_2 - \omega_2)w_2 - D_1 w_1, \qquad \dot{\tilde{\rho}}_2 = -\gamma_2\tilde{y}_2 - \gamma_2 w_2 + \dot{\rho}_2. \qquad (3.31)$$

Defining the state vector x as $x = [x_1^T \ x_2^T]^T$ with $x_1 = [\tilde{y}_1 \ \tilde{\rho}_1]^T$ and $x_2 = [\tilde{y}_2 \ \tilde{\rho}_2]^T$, then (3.31) can be rewritten as:

$$\dot{x} = Ax + b, \qquad (3.32)$$

where $A = \text{diag}\{A_i\}$, $i = 1, 2$ and:

$$A_i = \begin{bmatrix} -\omega_i & 1 \\ -\gamma_i & 0 \end{bmatrix}, \qquad b = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}, \qquad b_1 = \begin{bmatrix} (D_1 - \omega_1)w_1 \\ -\gamma_1 w_1 + \dot{\rho}_1 \end{bmatrix}, \qquad b_2 = \begin{bmatrix} (D_1 + D_2 - \omega_2)w_2 - D_1 w_1 \\ -\gamma_2 w_2 + \dot{\rho}_2 \end{bmatrix}. \qquad (3.33)$$

The two eigenvalues of A_i denoted by v_{i1} and v_{i2} are related to ω_i and γ_i as follows: $v_{i1} + v_{i2} = -\omega_i < 0$, $v_{i1}v_{i2} = \gamma_i > 0$, $i = 1, 2$. It can be seen that $v_{1i}, v_{2i} < 0$. The next assumption is considered for the estimator design parameters.

(A9) The design parameters ω_i and γ_i are chosen such that A_i has real distinct eigenvalues [6, 11], for example, $v_{i2} < v_{i1} < 0$, where $v_{i1,2} = (-\omega_i \pm \sqrt{\omega_i^2 - 4\gamma_i})/2$, that implies $0 < \gamma_i < \omega_i^2/4$.

The system (3.32) describes the dynamics of the estimation errors of the observer-based estimator including the observations of the measured substrates and the estimations of their unknown consumption rates ρ_i.

We can consider that the estimator (3.27)-(3.28) is a set of quasi-independent second order linear systems because one controlled substrate concentration is associated to one unknown parameter of the vector ρ only. Since the dynamics of estimation errors (3.32) is described also by a linear differential equation with a block diagonal matrix with two 2×2 stable blocks, then, we can have the following stability result, which is an extension of the result presented in [6, 11]:

Theorem 3.3. *Under assumptions (A1)–(A9), the estimation errors $\tilde{y}_i$ and $\tilde{\rho}_i$ are asymptotically bounded for all t as follows:*

(i)

$$\limsup_{t \to \infty} |\tilde{y}_i(t)| \le \frac{2\delta B_{i1}}{\sqrt{\omega_i^2 - 4\gamma_i}} + \frac{B_{i2}}{\gamma_i}, \quad i = 1,\ 2, \tag{3.34}$$

(ii)

$$\limsup_{t \to \infty} |\tilde{\rho}_i(t)| \le B_{i1} + \omega_i \frac{B_{i2}}{\gamma_i}, \quad i = 1,\ 2 \tag{3.35}$$

and then the tracking error $y^* - y$ is bounded as follows:

(iii)

$$\lim_{t \to \infty} |y_i^* - y_i| \le \frac{M_{ii}}{\lambda_i}, \quad i = 1,\ 2, \tag{3.36}$$

where $B_{11} = m_{21}(D_1^{\max} + \omega_1)$, $B_{21} = (D_1^{\max} + D_2^{\max} + \omega_2)m_{22} + D_1^{\max} m_{21}$, $B_{i2} = m_{2i}\gamma_i + m_{1i}$, $\delta = (v_{1i}/v_{2i})^{(v_{1i}/(v_{1i}-v_{2i}))} - (v_{1i}/v_{2i})^{(v_{2i}/(v_{1i}-v_{2i}))}$, and $M_{ii} > 0$, $i = 1, 2$ are constant bounds which will be defined.

Proof. From assumptions (A6)–(A8), it follows that the input vector $b = [b_1^T, b_2^T]^T$ of the system (3.32) is bounded. Therefore, (3.32) behaves as a linear time-invariant differential system with a bounded input, whose time response is:

$$x_i(t) = \Phi_i(t)x_i(0) + \int_0^t \Phi_i(t - \tau)b_i(\tau)d\tau, \quad i = 1, 2, \tag{3.37}$$

where $\Phi_i(t)$ is the state transition matrix associated to A_i defined as:

$$\Phi_i(t) = e^{A_i t} = \begin{bmatrix} \varphi_{11}^i(t) & \varphi_{12}^i(t) \\ \varphi_{21}^i(t) & \varphi_{22}^i(t) \end{bmatrix}, \tag{3.38}$$

with $\varphi_{11}^i(t) = (1/(v_{i1} - v_{i2}))(v_{i1}e^{v_{i1}t} - v_{i2}e^{v_{i2}t})$, $\varphi_{12}^i(t) = (1/(v_{i1} - v_{i2}))(e^{v_{i1}t} - e^{v_{i2}t})$, $\varphi_{21}^i(t) = -\gamma_i\varphi_{12}^i(t)$, $\varphi_{22}^i(t) = \omega_i\varphi_{12}^i(t) + \varphi_{11}^i(t)$. Since A_i is a stable matrix, then the state x_i is bounded

(see [5, Lemma 3.3.5] and [6, Theorem A2.6]). A bound of each component of $x_i(t)$ in (3.37) is given by:

$$|x_{i1}(t)| \leq \left|\varphi_{11}^i(t)x_{i1}(0) + \varphi_{12}^i(t)x_{i2}(0)\right| + \int_0^t \left|\varphi_{11}^i(t-\tau)\right| |b_{i1}(t)| d\tau + \int_0^t \left|\varphi_{12}^i(t-\tau)\right| |b_{i2}(t)| d\tau,$$

$$|x_{i2}(t)| \leq \left|\varphi_{21}^i(t)x_{i1}(0) + \varphi_{22}^i(t)x_{i2}(0)\right| + \int_0^t \left|\varphi_{21}^i(t-\tau)\right| |b_{i1}(t)| d\tau + \int_0^t \left|\varphi_{22}^i(t-\tau)\right| |b_{i2}(t)| d\tau.$$

(3.39)

Then, taking the limit of the above expressions for $t \to \infty$ and using Technical Lemma A2.2 from [6], we obtain the following inequalities:

$$\lim_{t \to \infty} \sup|x_{i1}(t)| \leq \frac{2B_{i1}\delta}{\sqrt{\omega_i^2 - 4\gamma_i}} + \frac{B_{i2}}{\gamma_i}, \qquad \lim_{t \to \infty} \sup|x_{i2}(t)| \leq B_{i1} + \frac{\omega_i}{\gamma_i}B_{i2},$$

(3.40)

where $B_{i1} > 0$ and $B_{i2} > 0$ are the bounds of b_{i1} and b_{i2}, that is, $|b_{i1}| \leq B_{i1}$ and $|b_{i2}| \leq B_{i2}$, $i = 1, 2$, with B_{i1} and B_{i2} defined in Theorem 3.3. Thus, the boundedness of $\tilde{y}_i$ and $\tilde{\rho}_i$ given in (i) and (ii), respectively, is proven.

To prove (iii) we proceed as follows. We can see that the adaptive control law (3.29) with parameter estimator (3.27) and (3.28) applied to system (3.17) leads to the following dynamics of the closed-loop system:

$$(\dot{y} - \dot{y}^*) + \Lambda(y - y^*) + \Lambda w = \dot{\tilde{y}} + \Omega(\tilde{y} + w).$$

(3.41)

Since Ω is a diagonal stable matrix, then from (3.41) it can be seen that the tracking error $y^* - y$ is the output of a linear stable filter driven by the observer error $\tilde{y}$. From the first equation in (3.30) we can write: $\dot{\tilde{y}} + \Omega(\tilde{y} + w) = \tilde{\rho} + Dw$. Now, by using (3.35), and assumptions (A7) and (A8), the following inequalities can be written:

$$\dot{\tilde{y}}_1 + \omega_1(\tilde{y}_1 + w_1) = \tilde{\rho}_1 + D_1 w_1 \leq |\tilde{\rho}_1| + |D_1 w_1| \leq B_{11} + m_{21}D_1^{\max} + \omega_1\frac{B_{12}}{\gamma_1} = M_1,$$

$$\dot{\tilde{y}}_2 + \omega_2(\tilde{y}_2 + w_2) = \tilde{\rho}_2 - D_1 w_1 + (D_1 + D_2)w_2 \leq |\tilde{\rho}_2| + |D_1 w_1| + |(D_1 + D_2)w_2|$$

(3.42)

$$\leq B_{21} + \omega_2\frac{B_{22}}{\gamma_2} + m_{22}(D_1^{\max} + D_2^{\max}) + m_{21}D_1^{\max} = M_2.$$

From (3.41) and (3.42) one finds that:

$$(\dot{y}_i - \dot{y}_i^*) + \lambda_i(y_i - y_i^*) \leq |-\lambda_i w_i| + M_i = \lambda_i m_{2i} + M_i = M_{ii}.$$

(3.43)

From (3.43), after some straightforward calculation it can be obtained that $\lim_{t\to\infty} |y_i^* - y_i| \leq M_{ii}/\lambda_i$ and the proof is complete. □

Table 1: Parameter values.

Parameter	Value	Parameter	Value
$\overline{K}_S^{\mathrm{gc}}$	0.5 g/L	$\mu_{\max 0}$	0.45 h^{-1}
$K_{S\max}^{\mathrm{rc}}$	12 g/L	η	3.5
$K_{S\max}^{\mathrm{rc}}$	12 g/L	β	0.9 h^{-1}
$K_{P\max}^{\mathrm{gc}}$	15 g/L	α_0	0.02 g/L
$K_{\alpha\mu}$	0.2 g/L	P_C^{gc}	95 g/L
$K_{\alpha P}$	1.1 g/L	Y_{PS}	0.98 g/g
$K_{\alpha S}$	4 g/L	k_d	0.02 h^{-1}

Table 2: Initial conditions.

Variable	Initial value	Variable	Initial value
X_{10}	0.02 g/L	S_{10}^{in}	50 g/L
S_{10}	0.5 g/L	S_{20}^{in}	200 g/L
P_{10}	0.01 g/L	α_{10}^{in}	6 g/L
α_{10}	3.5 g/L	D_{10}	0.058 h^{-1}
X_{20}	0.02 g/L	D_{20}	0.01 h^{-1}
S_{20}	3.5 g/L	D_{120}	0.025 h^{-1}
P_{20}	0.01 g/L	$\widehat{\rho}_{10}$	0.1 h^{-1}
α_{20}	2.5 g/L	$\widehat{\rho}_{20}$	0.015 h^{-1}

4. Simulation Results and Comments

The performance of the designed multivariable adaptive controller (3.9)–(3.13), by comparison to the exactly linearizing controller (3.3) (which yields the best response and can be used as benchmark), has been tested by performing extensive simulation experiments.

For a proper comparison, the simulations were carried out by using the process model (2.2)–(2.11) under identical and realistic conditions. The simulations were designed so that several set point changes on the controlled variables S_1 and S_2 occurred, nearby the operational point $S_1^* = 3$ g/L, $S_2^* = 5$ g/L.

The kinetic parameter values used in simulations are presented in Table 1 [21], and the initial values of variables considered in simulations in Table 2.

The system's behavior was analyzed assuming that the dilution rates of the two feeding substrates and the influent enrichment factor act as perturbations of the form:

$$D_1(t) = D_{10} \cdot \left(1 - 0.15\sin\left(\frac{\pi t}{25}\right)\right),$$

$$D_2(t) = D_{20} \cdot \left(1 + 0.15\cos\left(\frac{\pi t}{50}\right)\right), \tag{4.1}$$

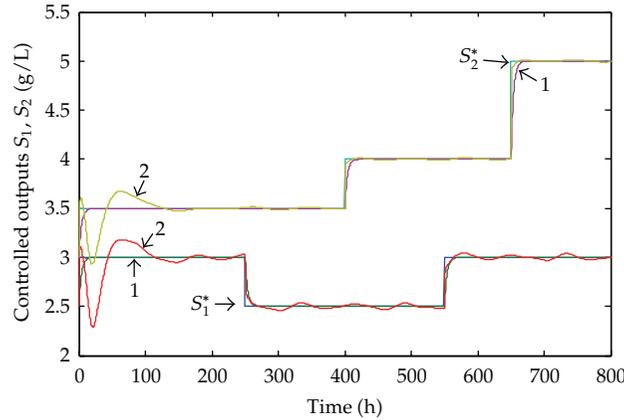$$\alpha_1^{in}(t) = \alpha_{10}^{in} \cdot \left(1 + 0.25\sin\left(\frac{\pi t}{20}\right)\right).$$

Figure 3: Time evolution of outputs—adaptive control (2) by comparison to exactly linearizing control (1).
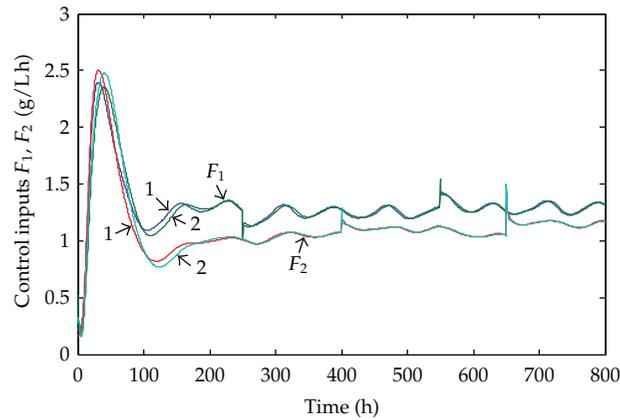

Figure 4: Profiles of control inputs—adaptive control (2) by comparison to exactly linearizing control (1).

Also, one of the kinetic parameters—the maximum specific growth rate—is considered time varying:

$$\mu_{\max}(t) = \mu_{\max 0} \cdot \left(1 - 0.1 \sin\left(\frac{\pi t}{40}\right)\right). \tag{4.2}$$

The values of the estimator design parameters, chosen so that to satisfy the tuning conditions of assumption (A9), are: $\lambda_1 = \lambda_2 = 0.45$, $\omega_1 = 1.75$, $\omega_2 = 0.75$, $\gamma_1 = 0.25$, $\gamma_2 = 0.1$.

The behavior of the indirect adaptive controlled system by comparison to the behavior of closed loop system with exactly linearizing controller is shown in Figure 3, the time evolution of the two controlled variables S_1 and S_2, respectively, and in Figure 4, the control inputs F_1 and F_2, respectively.

In order to test the behavior of the indirect adaptive controlled system in more realistic circumstances, we considered that the measurements of both controlled variables (S_1 and S_2) are corrupted with an additive white noise with zero average (5% from their nominal values). The simulation results in this case, conducted in the same conditions as in previous
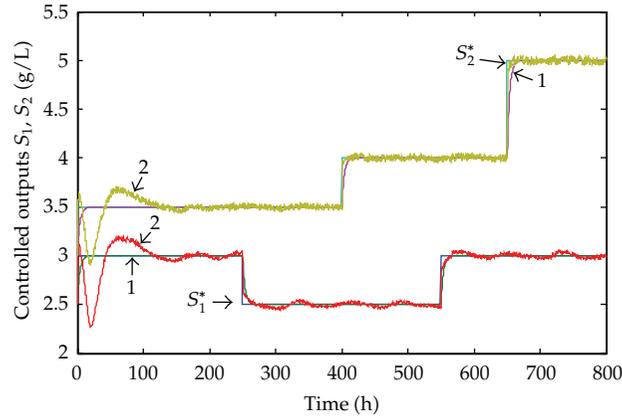
Figure 5: Evolution of outputs—adaptive control (noisy data) (2) by comparison to linearizing control (1).
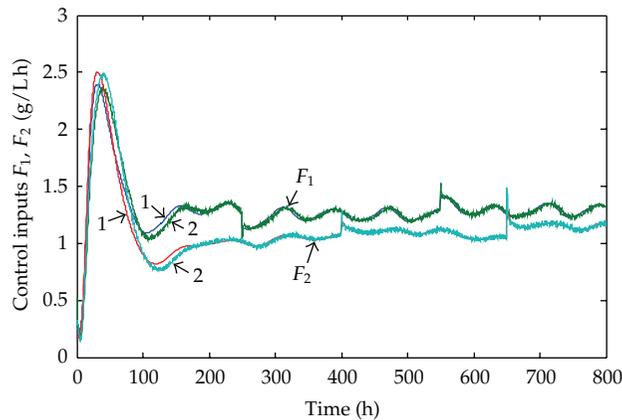

Figure 6: Control inputs—adaptive control (noisy data) (2) by comparison to linearizing control (1).

simulation, are presented in Figures 5 and 6, respectively. The simulation shows that the behavior of controlled variables and of control inputs is comparable with the results obtained in the free noise simulation.

The time evolution of the estimates of the unknown functions (3.6) provided by the observer-based estimator is presented in Figures 7 and 8, in both simulation cases.

From Figures 7 and 8, it can be noticed that the time evolution of estimates for noisy measurements of S_1 and S_2 is similar with the time profiles in free noise case.

The production of lactic acid can be evaluated by using the data provided by the asymptotic observer (3.15) and (3.16). The time evolution of the lactic acid concentration estimates in the two reactors is depicted in Figure 9.

From graphics in Figures 3–6 it can be seen that the behavior of overall system with indirect adaptive controller, even if this controller uses much less a priori information, is good, being very close to the behavior of closed loop system in the ideal case when the process model is completely known. Note also the regulation properties and ability of the controller to maintain the controlled outputs S_1 and S_2 very close to their desired values, despite the very high variations for D_1, D_2, α_1^{in}, and μ_{max}, time variation of process parameters and the influence of noisy measurements.
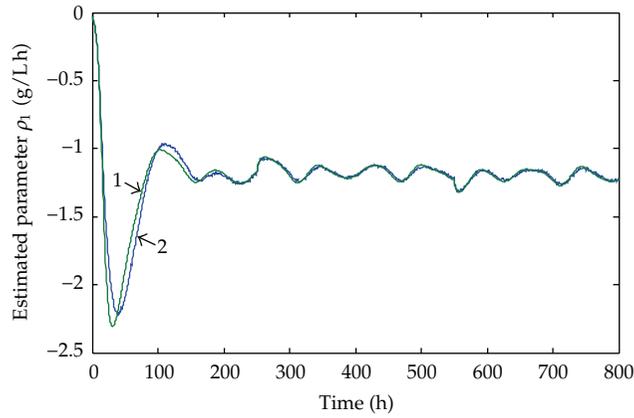
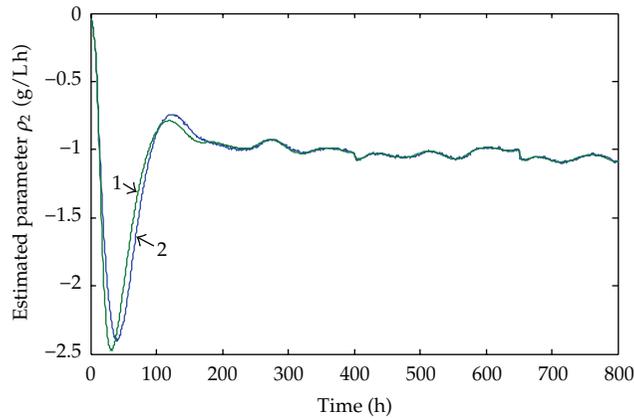**Figure 7:** Profile of estimates of unknown function ρ_1: (1) without and (2) with noisy measurements.

Figure 8: Profile of estimates of unknown function ρ_2: (1) without and (2) with noisy measurements.
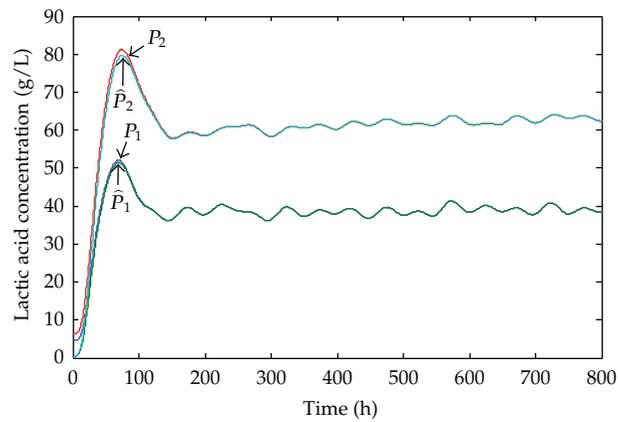

Figure 9: Profile of estimates of lactic acid concentration: (1) without and (2) with noisy measurements.
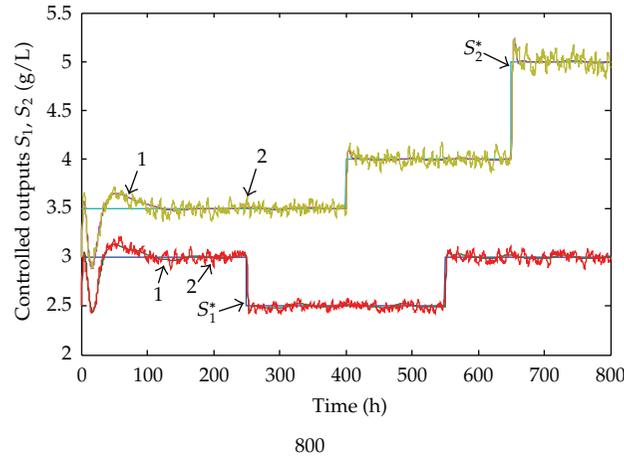
800

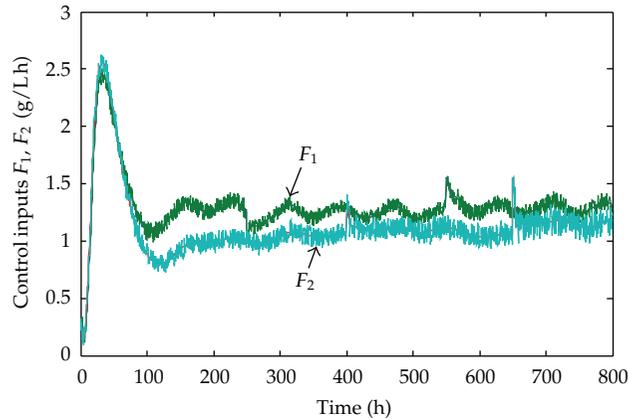**Figure 10:** Evolution of outputs—PI control (noisy data) (2) by comparison to PI (free noise) (1).

Figure 11: Control inputs—PI control (noisy data) by comparison to PI (free noise).

In order to compare our proposed strategy with some classical control technique, several simulations were performed. As exemplification, Figures 10 and 11 present the outputs and control inputs for the case when a classical PI (proportional-integrative) control is used, with the best tuning of control parameters. As can be noticed, the behavior of the classical control is strongly influenced by the noisy measurements and by the variation of kinetic and process parameters.

5. Conclusion

This paper is concerned with the design of an indirect multivariable adaptive control for a lactic acid fermentation process that is carried out in a cascade of two stirred tank fermentation reactors.

The controller was achieved by assumption that the consumption rates of two limiting substrates are completely unknown and are summarized in two fully unknown time varying

parameters which are estimated by means of appropriately observer-based estimators. A simple tuning method of kinetic estimator design parameters was achieved by stability and convergence analysis of the control scheme.

The simulation results showed that the performances of the proposed multivariable adaptive controller are very good. The derived control method can be applied also for a large class of square nonlinear bioprocesses.
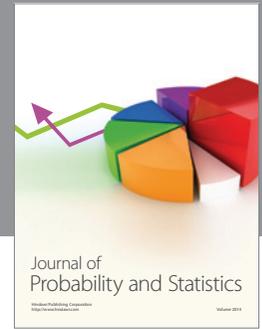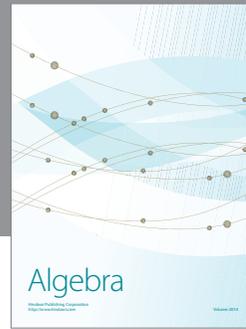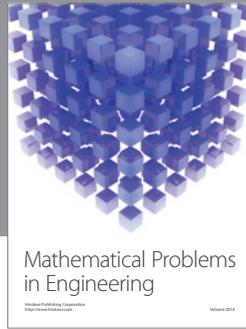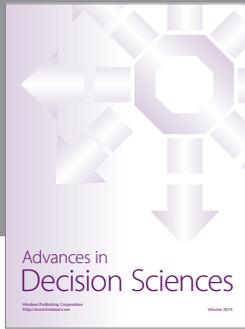
Acknowledgment

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