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AN ALTERNATIVE MODEL OF THE GENERAL RELATIVITY THEORY

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Abstract. We establish in the offered paper that a 'displacement postulate' for the light sphere allows to obtain a metric in four–dimensional space–time from which the well–known Schwarzschild's metric may be simply deduced (by passing to the locational time). Probably, displacement postulate is a description of a profound mechanism of an interaction between the particles and the gravitational field.

1. The main metric. Let a mass point m_0 move under action of Newtonian gravitational potential of a rest–mass m. We assume m_0 moves from infinity, where it had zero velocity. It is easily shown at a point a the mass m_0 has the velocity

$$\vec{v} = -\sqrt{\frac{2Gm}{r}} \cdot \frac{\vec{x}}{r}\,, \tag{1}$$

where $\vec{x}$ is the radius–vector of a, $r = |\vec{x}|$, and $G \approx 6.67 \cdot 10^{-11} m^3 kg^{-1} s^{-2}$ is the gravitational constant.

We now distract our attention from movement of mass points and consider spreading of the light. If the gravitational field is infinitely small, a light

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signal emited at a time t from a is spreaded in all directions with the same velocity c. Hence the points which receive this light signal in a time dt are situated on the sphere of the radius $c\,dt$ centered at a. It is described by the equation $|d\vec{x}|^2 = c^2 dt^2$, where $d\vec{x}$ is the vector going from a to a point of the sphere. A mass point starting at the time t from a has a velocity less than c; hence in the time dt it is situated *inside* the sphere.

Let now an isolated mass m rest at the origin of a Cartesian coordinate system K. We state its action on light spreading is described by the following *displacement postulate*: the light velocity with respect to K is equal to $\vec{u} = \vec{c} + \vec{v}$, where *proper light velocity* $\vec{c}$ has the constant value $|\vec{c}| = c$ and $\vec{v}$ is the *displacement vector* defined by (1).

By displacement, the points which receive the light signal in the time dt are situated on the sphere of the same radius $c\,dt$ centered at $a' = a + \vec{v}\,dt$. This sphere is defined by the equation $|d\vec{x} - \vec{v}\,dt|^2 = c^2\,dt^2$, i.e.,

$$(c^2 - |\vec{v}|^2)\,dt^2 + 2\langle\vec{v},\ d\vec{x}\rangle\,dt - |d\vec{x}|^2 = 0. \tag{2}$$

Unlike the light, a mass point starting at the time t from a is situated in the time dt *inside* the sphere (2).

Denote the quadratic form in the left–hand side of (2) by ds^2. By virtue of (1),

$$ds^2 = \left(c^2 - \frac{2Gm}{r}\right) dt^2 - 2\sqrt{\frac{2Gm}{r^3}}\langle\vec{x},\ d\vec{x}\rangle\,dt - |d\vec{x}|^2. \tag{3}$$

This *main quadratic form* defines a pseudoeuclidean metric ds in the four–dimensional space W of the variable $(t, \vec{x}) = (t, x^1, x^2, x^3)$. We call it the *main metric* [1, 2, 3]. Thus along light trajectories $ds^2 = 0$, i.e., these trajectories have *isotropic* directions. For the mass points $ds^2 > 0$.

Let us write $|d\vec{x}|^2 = -h_{pq}\,dx^p\,dx^q$, where $h_{pq} = -1$ for $p = q$ and $h_{pq} = 0$ for $p \neq q$ (here and in the sequel the indices denoted by Latin letters take values 1, 2, 3 and summation over recurring indices from 1 to 3 is made).

The coordinates x^1, x^2, x^3 are contravariant. The *covariant* coordinates $x_p = h_{pq}x^q$ differ from them by the sign 'minus'. In this notation, the scalar product of vectors $\vec{x}, \vec{y}$ is equal to $\langle\vec{x}, \vec{y}\rangle = -h_{pq}x^p y^q = -x_q y^q$. Hence the main metric takes the form

$$ds^2 = \left(c^2 - \frac{2Gm}{r}\right) dx^0\,dx^0 + 2\sqrt{\frac{2Gm}{r^3}}x_p\,dx^p\,dx^0 + h_{pq}\,dx^p\,dx^q, \tag{4}$$

where $x^0 = t$ is the time.

Let now dt be the time for spreading of a light signal from a point $\vec{x}$ to $\vec{x}+d\vec{x}$ and dt' be the time for its spreading from $\vec{x}+d\vec{x}$ to $\vec{x}$. Theorem 1 below describes what occurs as we pass to the 'locational' time, i.e., as we postulate the time of light spreading between $\vec{x}$ and $\vec{x}+d\vec{x}$ (in both the directions) is equal to $dt_* = \frac{1}{2}(dt+dt')$.

THEOREM 1. *Under the passage to locational time, the metric* (3) (*or* (4)) *turns into Schwarzschild's metric well–known in general relativity theory* (*see* (21) *below*).

A proof (less than one page!) is given in Section 4. Thus the displacement postulate accompanied by the passage to locational time leads to the formulas of classical general relativity theory! Undoubtedly, this is not an accidental coincidence. Maybe, the postulate describes a profound mechanism of interaction between particles and gravitational field ?

We recall experimental confirmation for conclusions of general relativity theory (movement of Mercury's perihelion, etc) was obtained with Schwarzschild's metric. The initial Einstein's idea (to explaine gravitation by a curvature of the four–dimensional time–space) was developed under the influence of Hilbert, who had discussions with Einstein (in many letters). As a result, the *Einsteinian gravitation law* was appeared with a very refined geometrical apparata (tensor calculus, Riemannian geometry, curvature tensor, Ricci tensor). It affirms in empty space (in particular, around an isolated rest–mass) *the Ricci tensor is identically equal to zero.*

Besides, Einstein's idea contains only a *geometrical* description of the gravitation. It does not explain what the rest–mass acts on the enclosing space with. The displacement postulate provides for a possibility to explain the gravitational field by a *flux* of superlight particles.

The reader can object the constancy of the light velocity is an *experimental* fact and movement with superlight velocity is impossible. But any physical experiment gives only the *average* velocity $\frac{ds}{dt_*} = \frac{2\,ds}{dt+dt'}$ of the light, dt, dt' being the spreading times 'there' and 'back'. Indeed, for finding dt it is necessary to synchronize the watches and for synchronization of the watches it is necessary to know dt. Einstein goes from this vicious circle out by a will act, postulating *coincidence* of the light velocity with its *average* value.

The reader can also object the assumtion on superlight movement leads to a contradiction with the causality low: if there are superlight particles, we can look into tomorrow and know what *will happend in future*! The author heard that objection on his reports and attempts to publish the article (the last review was obtained recently from "Journal of Geometry and Physics").

That objection is nonessential too, since causality is relized *stastically*, as many other phisical lows. Moreover, we do not have "superlight sight" and therefore tackiones (superlight particles) do not carry signals from future.

Nevertheless, our spirit live depends on *aura*, including hypnotic events, telepathy, foresight. The thinking is a common activity of the brain and aura. Maybe aura consists of tackions and we are able to focus them? Then it is possible to influence on the gravitation to some exit and the phenomenon of levitation is possible as indian yogis affirmed? And telekinesis as well, i.e., we can (at least in principle) move things by stress of thought? Finally, maybe by loss of some comfort and possibilities to apply our knowledge, we can see something from the future? Certainly, these are only fantasies, but on the other hand, maybe the objections of physicists on causality low are superfluously orthodox?

Let us now say something more serious. The alternative version for relativity theory sketched below shows employment of the main metric (4) leads to *the same* results which are regarded as experimental justification of classical general relativity theory. This version is a mathematical model only. It is kinematic, without dynamical interpretation for interaction of tackions and particles. But let the reader forgives me an analogy. In the beginning of May, 1905 great Poincaré had made a report on *space–time interval* $dx^2 + dy^2 + dz^2 - c^2dt^2$ and his kinematical understanding of simultaneity. In a month Einstein publishes his version of special relativity theory (without a reference to Poincaré's paper), where he supplements Poincaré's ideas by dynamical concepts. Maybe in a month after my paper, a great physicist takes its ideas and proposes a deeply developed 'tackionetics'? It is not essential we consider here an isolated rest–mass only: integration (accompanied by ideas of special relativity theory) allows to pass to arbitrary mass distribution.

By the way, a metric similar in a sense to (4) was considered by Lamaitre [7, 8]. But in contrast to (4), his metric uses a dependence of the radius r on time and is *invariantly connected* with Schwarzschild's one.

2. Geodesics of the main quadratic form. Write the right–hand side of (4) in the form $g_{\alpha\beta}dx^\alpha dx^\beta$:

$$g_{\alpha\beta}dx^\alpha dx^\beta = \left(c^2 - \frac{2Gm}{r}\right) dx^0\, dx^0 + \\ + 2\sqrt{\frac{2Gm}{r^3}} x_p\, dx^p\, dx^0 + h_{pq}\, dx^p\, dx^q \quad (5)$$

(here and in the sequel the summation from 0 to 3 over recurring Greek indices is made). Thus the components of the symmetric matrix $(g_{\alpha\beta})$ are:

$$g_{00} = c^2 - \frac{2Gm}{r}; \qquad g_{0p} = g_{p0} = \sqrt{\frac{2Gm}{r^3}}x_p; \qquad g_{pq} = h_{pq}. \tag{6}$$

We now refuse to consider the metric $ds = \sqrt{g_{\alpha\beta}dx^\alpha dx^\beta}$. Instead of that, we directly work with the quadratic form (5).

Let $X(\sigma) = \left(x^0(\sigma), x^1(\sigma), x^2(\sigma), x^3(\sigma)\right)$, $\sigma_0 \leq \sigma \leq \sigma_1$ be a twice differentiable arc. We say this arc is *geodesic* for the quadratic form (5) if the integral

$$Q = \int_{\sigma_0}^{\sigma_1} g_{\alpha\beta} \frac{dx^\alpha(\sigma)}{d\sigma} \frac{dx^\beta(\sigma)}{d\sigma}\, d\sigma \tag{7}$$

is stationary with respect to all arcs connecting the same endpoints $X(\sigma_0)$ and $X(\sigma_1)$. In other words, if $X(\sigma) + \delta X(\sigma)$, $\sigma_0 \leq \sigma \leq \sigma_1$ is a varied arc with $\delta x^\alpha(\sigma_0) = 0$, $\delta x^\alpha(\sigma_1) = 0$ for $\alpha = 0, 1, 2, 3$, then $\delta Q = 0$.

For conveniency, denote the derivative of a scalar function f with respect to the coordinate x^α by $f_{,\alpha}$. The *first Christoffel symbol* $\Gamma_{\gamma\alpha\beta}$ for the quadratic form $g_{\alpha\beta}dx^\alpha dx^\beta$ is defined by the formula

$$\Gamma_{\gamma\alpha\beta} = \frac{1}{2}\left(-g_{\alpha\beta,\gamma} + g_{\gamma\alpha,\beta} + g_{\beta\gamma,\alpha}\right). \tag{8}$$

Furthermore, let $(g^{\mu\nu})$ be the inverse matrix for $(g_{\alpha\beta})$, i.e., $g_{\alpha\beta}g^{\beta\nu} = \delta^\nu_\alpha$ (the Kronecker delta). The *second Christoffel symbol* $\Gamma^\nu_{\alpha\beta}$ is defined by

$$\Gamma^\nu_{\alpha\beta} = g^{\gamma\nu}\Gamma_{\gamma\alpha\beta}. \tag{9}$$

Theorem 2. *An arc* $X(\sigma) = \left(x^0(\sigma), x^1(\sigma), x^2(\sigma), x^3(\sigma)\right)$, $\sigma_0 \leq \sigma \leq \sigma_1$ *is geodesic for the quadratic form* $g_{\alpha\beta}dx^\alpha dx^\beta$ *if and only if it satisfies the system of differential equations*

$$\frac{d^2x^\nu}{d\sigma^2} + \Gamma^\nu_{\alpha\beta}\frac{dx^\alpha}{d\sigma}\frac{dx^\beta}{d\sigma} = 0, \qquad \nu = 0, 1, 2, 3. \tag{10}$$

Every geodesic arc $X(\sigma)$ *satisfies the relation*

$$g_{\alpha\beta}\frac{dx^\alpha}{d\sigma}\frac{dx^\beta}{d\sigma} = \text{const},$$

i.e., the left–hand side does not depend on σ.

The proof is contained in Section 4. We say a geodesic is *positropic, isotropic* or *negatropic* if the aforesaid constant is positive, equal to zero or negative correspondingly.

REMARK 1. Let $X(\sigma)$, $\sigma_0 \le \sigma \le \sigma_1$ be a geodesic. Introduce a parameter $\bar\sigma = k\sigma$ along it, $k \ne 0$ being a positive constant. The obtained arc $\bar X(\bar\sigma) = X(\frac{\bar\sigma}{k})$, $k\sigma_0 \le \bar\sigma \le k\sigma_1$ is again a geodesic, since it satisfies the system (10). Consequently choosing a suitable k, we may suppose

$$g_{\alpha\beta}\frac{dx^\alpha}{d\sigma}\frac{dx^\beta}{d\sigma} = V \tag{11}$$

for every geodesic, where V is equal to 1, 0, and -1 for positropic, isotropis, and negatropic geodesics respectively. For our aims we consider only the paramertizations with $\frac{dx^0}{d\sigma} > 0$. In the case of positropic or negatropic geodesics, the relation (11) defines the parameter σ uniquely up to a translation: $\sigma \to \sigma+$ const.

We remark positropic geodesics can be described by an another definition: the *length*

$$s = \int_{\sigma_0}^{\sigma_1} \sqrt{g_{\alpha\beta}\frac{dx^\alpha}{d\sigma}\frac{dx^\beta}{d\sigma}}\, d\sigma$$

is stationary. This way leads to *the same* equations (10) for geodesics. But the definition with stationarity of the integral (7) is more convenient, since it is applicable for any geodesics (positropic, isotropic or negatropic).

THEOREM 3. *For the main quadratic form* (5), *the differential equations of geodesics have the form*

$$\frac{d^2x^n}{d\sigma^2} + \frac{GmV}{c^2r^3}x^n - \frac{3Gm}{c^2r^3}x^n\left(h_{pq}\frac{dx^p}{d\sigma}\frac{dx^q}{d\sigma} + \frac{1}{r^2}\left(x_p\frac{dx^p}{d\sigma}\right)^2\right) = 0, \qquad n = 1,2,3, \tag{12}$$

$$\left(c^2 - \frac{2Gm}{r}\right)\frac{dx^0}{d\sigma}\frac{dx^0}{d\sigma} + 2\sqrt{\frac{2Gm}{r^3}}x_p\frac{dx^p}{d\sigma}\frac{dx^0}{d\sigma} + h_{pq}\frac{dx^p}{d\sigma}\frac{dx^q}{d\sigma} = V, \tag{13}$$

where the constant V is equal to 1, 0, *and* -1 *for positropic, isotropic, and negatropic geodesics correspondingly.*

The proof is contained in Section 4.

We now remark by spherical symmetry of the main quadratic form (5) (i.e., by spherical symmetry of the gravitational field for the rest–mass m) every geodesic is a *planar* curve. Supposing it is situated in the plane $x^3 = 0$, we obtain

THEOREM 4. *The equations of the geodesics in the plane* $x^3 = 0$ *have the form*

$$\frac{d^2r}{d\sigma^2} = -\frac{GmV}{c^2r^2} + r\left(1 - \frac{3Gm}{c^2r}\right)\left(\frac{d\varphi}{d\sigma}\right)^2, \tag{14}$$

$$\frac{d^2\varphi}{d\sigma^2} = -\frac{2}{r}\frac{dr}{d\sigma}\frac{d\varphi}{d\sigma}, \tag{15}$$

$$\left(c^2 - \frac{2Gm}{r}\right)\frac{dx^0}{d\sigma}\frac{dx^0}{d\sigma} - 2\sqrt{\frac{2Gm}{r}}\frac{dr}{d\sigma}\frac{dx^0}{d\sigma} + \\ - \left(\left(\frac{dr}{d\sigma}\right)^2 + r^2\left(\frac{d\varphi}{d\sigma}\right)^2\right) = V, \tag{16}$$

where r, φ *are polar coordinates in the plane* $x^3 = 0$.

Proof. This is obtained from Theorem 3 by passing to the polar coordinates: $x^1 = r\,\cos\,\varphi,\ \ x^2 = r\,\sin\,\varphi,\ \ x^3 = 0.$ □

REMARK 2. In Theorems 3 and 4, the parameter σ has the dimensionality of *length.* But it is possible to pass to another parameter such that the constant V takes the value c^2 for positropic geodesics and the value $-c^2$ for negatropic ones. In this case, σ has the dimensionality of the *time.* Sometimes (in particular, in Examples 4 and 7 below) it may be more preferable.

3. Particles trajectories.

EXAMPLE 1. Assume a mass point moves in a region of weak gravitation and its velocity is not large, i.e., $\sqrt{\frac{2Gm}{r}}$ and $|\frac{d\vec{x}}{dt}|$ are small with respect to c. Then by (4),

$$\left(\frac{ds}{c\,dt}\right)^2 = \left(1 - \frac{1}{c^2}\frac{2Gm}{r}\right) - \frac{2}{c^2}\sqrt{\frac{2Gm}{r}}\left(\frac{\vec{x}}{r}\cdot\frac{d\vec{x}}{dt}\right) - \left(\frac{1}{c}\frac{d\vec{x}}{dt}\right)^2 \approx 1,$$

i.e., $ds \approx c\,dt = c\,dx^0$. Moreover since $V = 1$, it follows from (4), (13) $d\sigma = ds \approx c\,dt$. Consequently the equations (14), (15) of geodesics take the form

$$\frac{d^2r}{dt^2} \approx -\frac{Gm}{r^2} + r\left(\frac{d\varphi}{dt}\right)^2, \qquad \frac{d^2\varphi}{dt^2} = -\frac{2}{r}\frac{dr}{dt}\frac{d\varphi}{dt},$$

i.e., coincide with the *Newtonian equations* of planet movings. In this connection, it is naturally to postulate in general (not only for weak gravitational field and small velocities), *the world trajectories of mass points in the* *gravitational field of the rest–mass coincide with the positropic geodesics of the main quadratic form* (5). This postulate is connected with the classical stationary action principle, since from mechanical point of view, $\int ds$ is the *action*, i.e., stationary action principle $\delta \int ds = 0$ means geometrically the trajectory is a geodesic with respect to the main metric (or, what is the same, with respect to the main quadratic form, cf. the Remark 1).

Example 2. Let us consider circular movement of a planet with a constant radius r of its orbit and constant angular velocity $\omega = \frac{d\varphi}{dt} = \frac{d\varphi}{dx^0}$ under the action of the central rest–mass (the Sun). From (14) we find

$$r\left(1 - \frac{3Gm}{c^2 r}\right)\left(\frac{d\varphi}{d\sigma}\right)^2 = \frac{Gm}{c^2 r^2}. \tag{17}$$

Furthermore, according to (16) and taking into account r is constant, we have

$$\left(c^2 - \frac{2Gm}{r}\right)\left(\frac{dx^0}{d\sigma}\right)^2 - r^2\left(\frac{d\varphi}{d\sigma}\right)^2 = c^2.$$

Two last relations imply

$$\left(\frac{d\varphi}{dx^0}\right)^2 = \left(\frac{d\varphi}{d\sigma}\right)^2\left(\frac{dx^0}{d\sigma}\right)^{-2} = \frac{Gm}{r^3}. \tag{18}$$

In other words, $\omega^2 = \frac{Gm}{r^3}$, in complete conformity with the Newtonian gravitational law, i.e., in this case the geodesics exactly coincide with the Newtonian trajectories.

Example 3. Denote $\sqrt{\frac{Gm}{c^2 r}}$ by ξ. For Mercury (with the average radius of the trajectory $r \approx 5.7 \cdot 10^{10}$m), we have $\xi \approx 1.6 \cdot 10^{-4}$ and for other planets ξ is essentially lesser. If a Newtonian trajectory is distinct from a circle, it does not coincidence exactly with the geodesic. Nevertheless, for small ξ and almost circular trajectory the deviation is not large. Writing (14), (15) as a system of equations with a small parameter ξ and solving it approximately, we can find the rotation angle of the perihelion during each period. Such a calculation explains (cf. for details [6], [7]) the presence of additional displacement of the Mercury's perihelion (after taking into account the disturbing influence of other planets) at the angle of $42''$ during a century, in complete conformity with experimental data.

We remark the above–mentioned formulas for circular movement hold as $1 - \frac{3Gm}{c^2 r} > 0$ only (cf. (17)), i.e., $r > \frac{3Gm}{c^2}$. Moreover according to (18), the angular velocity $\frac{d\varphi}{dx^0}$ remains bounded above as $r \to \frac{3Gm}{c^2}$.

EXAMPLE 4. Let us consider the relations

$$\frac{dr}{d\sigma} = -\sqrt{\frac{2Gm}{c^2 r}}, \qquad \frac{d\varphi}{d\sigma} = 0, \qquad \frac{dx^0}{d\sigma} = \frac{dt}{d\sigma} = \frac{1}{c}. \tag{19}$$

It is easily shown the equations (19) satisfy the relations (14)–(16), i.e., they describe a movement of a mass point along a ray $\varphi =$const. From (19), we find $\frac{d^2r}{dt^2} = -\frac{Gm}{r^2}$, i.e., (19) is a *Newtonian movement* of the mass point along the considered ray in the gravitational field of the rest–mass m.

For Schwarzschild's metric (i.e., after the passage to the average time dx^0_*, cf. (21) below), we find

$$\frac{dx^0_*}{d\sigma} = c\left(c^2 - \frac{2Gm}{r}\right)^{-1}.$$

The derivative $\frac{dx^0_*}{d\sigma}$ tends to infinity and $x^0_* \to \infty$ as r approaches to the critical radius $r_{\text{cr}} = \frac{2Gm}{c^2}$. This means the region $r \le \frac{2Gm}{c^2}$ (named a *black hole*) may be reached only in infinite time. The description of this movement in Schwarzschild's metric has a 'singularity' at the critical radius (cf. [5]).

For the geodesics of the main quadratic form (5) (or for positropic geodesics of the main metric (4)) there is no singularity and this shows the main metric is more convenient than Schwarzschild's one. Indeed, as $r \to r_{\text{cr}}$ we have $c - \sqrt{\frac{2Gm}{r}} \to 0$, i.e., the velocity of spreading for the light signal going *away from* the black hole tends to zero. Therefore the average (locational) time x^0_* increases indefinitely as $r \to r_{\text{cr}}$. Nevertheless, the time $x^0 = \frac{\sigma}{c}$ remains *bounded* in the main metric as a mass point reaches the black hole and continues the fall. Moreover, $c - \sqrt{\frac{2Gm}{r}}$ is *negative* in the interior of the black hole, i.e., *both* the light velocities directed along the considered ray (and defined in the main metric by the relation $ds^2 = 0$) are directed inward the black hole. This means the light signals do not go from the black hole to the exterior region.

We nou call our attention to spreading of the light and postulate the light trajectories coincide with the isotropic geodesics of the main quadratic form (5). This postulate is connected with the classical Fermat principle. Indeed, acording to this principle, the light spreads from a point p to an another point q along a trajectory (in the space of variables x^1, x^2, x^3) such that the spreading time $\int dx^0$ is stationary with respect to trajectories connecting the same points p and q. In other words, for a varied trajectory, if the relations $\delta x^1 = 0$, $\delta x^2 = 0$, $\delta x^3 = 0$ hold at the endpoints p, q, then by Fermat principle, $\delta x^0 = 0$ at the endpoints too. Moreover, if the varied trajectory is isotropic, the relation $\delta Q = 0$ occurs as well. Thus if an isotropic world trajectory is stationary (with respect to the main quadratic form), then it satisfies the Fermat principle. We remark the stationarity with respect to the main quadratic form is a more strict requrement than the Fermat principle, since we compare the trajectory with *all* varied trajectories (not only isotropic).

EXAMPLE 5. The equations (14)–(15) allow to show (cf. [6], [7]) the light trajectories going from stars and passing close the surface of the Sun are rotated in the angle $\psi \approx \frac{4Gm_\odot}{c^2 r_\odot}$ where $m_\odot \approx 1.99 \cdot 10^{30}\, kg$ is the mass of the Sun and $r_\odot \approx 6.95 \cdot 10^8\, m$ is its radius. Thus, $\psi \approx 8.49 \cdot 10^{-6}$ (radian), i.e., $\psi \approx 1,75''$, in good conformity with experiments.

The gravitational bend of the light trajectories passing close to the rest–mass allows to explane the nature of quasars [4]. Indeed, assume m is a black hole resting at a very far point. For every star S (situated behind m with respect to us) there exists a light trajectory that starts at S, passes close to m, and comes up to us. In other words, the black hole m *focuses* the light of all stars. This focusing is greater as the mass m is larger. Thus immense black holes have vast brightnesses. And we have to see a dark spot in the center of this quasar since the light trajectories bend m. This corresponds to the observations.

Let us now consider superlight particles. They are usually named *tachiones*, although the physicists do not take them seriously, since $\sqrt{ds^2}$ is imaginary in this case. Our passing to geodesics of the main quadratic form (instead of geodesics of a metric) allows to avoid this 'imaginary'. The trajectories of tachions are *negatropic.*

EXAMPLE 6. Consider the movement of a tachion along a circumference with a constant angular velocity $\omega = \frac{d\varphi}{dt}$ under the action of a central rest–mass m (black hole). In this case by $V = -1$, we have from (14) $1 - \frac{3Gm}{c^2 r} < 0$, i.e., $r < \frac{3Gm}{c^2}$. As in Example 2 for the angular velocity $\omega = \frac{d\varphi}{dt} = \frac{d\varphi}{dx^0}$, we find the 'Newtonian' value $\omega^2 = \frac{Gm}{r^3}$. Thus the region $r < \frac{3Gm}{c^2}$ is completed by possible circular trajectories for tachions.

EXAMPLE 7. Let us now consider the relations

$$\frac{dr}{d\sigma} = \frac{1}{c}\sqrt{c^2 - \frac{2Gm}{r}}, \qquad \frac{d\varphi}{d\sigma} = 0, \qquad \frac{dx^0}{d\sigma} = \frac{\sqrt{\frac{2Gm}{r}}}{c\sqrt{c^2 - \frac{2Gm}{r}}}. \tag{20}$$

It is easily shown the equations (20) satisfy (14)–(16) with $V = -1$, i.e., they describe a movement of a tackion along a ray $\varphi = $const. This movement starts at the distance of $\frac{2Gm}{c^2}$ from the center of the black hole and is directed outside, i.e., it describes the *birth* of a tackion. A similar reasoning shows the birth of tackions is possible at *lesser* (not greater!) distances from the center of the black hole as well.

The aforesaid gives an explanation for nature of the gravitation. The world space is pierced by a flux of tackions. The central rest–mass creates a 'shadow' in the flux. And a mass point receives *weaker* flux of tackions from the side of the rest–mass than from the opposite direction. As a result, the mass point is pushed by the tackions *to* the rest–mass a little more strong than in the opposite direction. And statistically, the shadow effect generates the picture described by the main quadratic form.

There is an another explanation. In the depth of infrastructure for the atomic nucleus (by quite another geometric properties of the space for very small distances), a flux of tackions arises. The take–off running tackions puch each other but in a distance, the tackions feel more freely and their superlight velocities become smaller (as in Example 7). This is an 'aura' of the atom. Each mass point, each star has its aura consisting of tackions. Assume the masses (and impulses) of tackions are *negative.* Then we understand the tackions do not *push* a mass point in the direction of their moving but *pull* it in the opposite direction. This generates the picture described by the main quadratic form.

4. Proofs.

Proof of Theorem 1. Since the light trajectories are isotropic, dt is the positive root of the equation $ds^2 = 0$ (cf. (3)). Furthermore, dt' is the positive root of the equation obtained as we replace $d\vec{x}$ by $-d\vec{x}$. In other words, $-dt'$ is the negative root of the equation $ds^2 = 0$. According to the Vieta formula, we have

$$dt - dt_* = dt - \frac{dt + dt'}{2} = \frac{dt + (-dt')}{2} = \frac{\sqrt{\frac{2Gm}{r^3}}\,\langle \vec{x},\, d\vec{x}\rangle}{c^2 - \frac{2Gm}{r}},$$

and consequently,

$$dt = dt_* + \frac{\sqrt{\frac{2Gm}{r^3}}}{c^2 - \frac{2Gm}{r}}\,\langle \vec{x},\, d\vec{x}\rangle.$$

Substituing this value of dt into (3), we obtain the metric

$$ds^2 = \left(c^2 - \frac{2Gm}{r}\right) dt_*^2 - \left(|dx|^2 + \frac{\frac{2Gm}{r^3}}{c^2 - \frac{2Gm}{r}}\langle \vec{x},\, d\vec{x}\rangle^2\right),$$

i.e., in the coordinate form

$$ds^2 = \left(c^2 - \frac{2Gm}{r}\right) dx_*^0 dx_*^0 + \left(h_{pq} - \frac{\frac{2Gm}{r^3} x_p x_q}{c^2 - \frac{2Gm}{r}}\right) dx^p dx^q. \qquad (21)$$

But this is just Schwarzschild's metric (cf., for example, [5, 6, 7]). □

We remark that a similar reasoning proves the following assertion: *Let us consider the metric*

$$ds^2 = (c^2 - |\vec{v}|^2)\, dt^2 + 2\langle \vec{v},\ d\vec{x}\rangle\, dt - |d\vec{x}|^2, \qquad (22)$$

where $\vec{v}(x)$ *is a vector field defined outside of the origin of the coordinate system* K. *The metric obtained from* (22) *by passing to the locational time coincides with Schwarzschild's metric, if and only if* $\vec{v}(x)$ *is defined* (*up to a sign*) *by the formula* (1).

Proof of Theorem 2. Take the variation of the quadratic form $g_{\alpha\beta} dx^\alpha dx^\beta$:

$$\delta(g_{\alpha\beta} dx^\alpha dx^\beta) = (\delta g_{\alpha\beta}) dx^\alpha dx^\beta + g_{\alpha\beta}(\delta\, dx^\alpha) dx^\beta + g_{\alpha\beta} dx^\alpha (\delta\, dx^\beta).$$

Two last summands coincide (since $g_{\alpha\beta} = g_{\beta\alpha}$). Besides, $\delta(dx^\beta) = d(\delta x^\beta)$. Hence, we find

$$\delta(g_{\alpha\beta} dx^\alpha dx^\beta) = g_{\alpha\beta,\gamma} dx^\alpha dx^\beta \delta x^\gamma + 2 g_{\alpha\gamma} dx^\alpha d(\delta x^\gamma).$$

Consequently, by stationarity, we obtain the following integral equation of geodesics:

$$\delta Q = \int_{\sigma_0}^{\sigma_1} \delta \left(g_{\alpha\beta} \frac{dx^\alpha}{d\sigma} \frac{dx^\beta}{d\sigma} \right) d\sigma =$$
$$\int_{\sigma_0}^{\sigma_1} \left(g_{\alpha\beta,\gamma} \frac{dx^\alpha}{d\sigma} \frac{dx^\beta}{d\sigma} \delta x^\gamma + 2 g_{\alpha\gamma} \frac{dx^\alpha}{d\sigma} \frac{d(\delta x^\gamma)}{d\sigma} \right) d\tau = 0.$$

Integrating the second summand in the right–hand side by parts and taking into account that $\delta x^\gamma = 0$ at the endpoints, we rewrite the equation in the form

$$\delta Q = \int_{\sigma_0}^{\sigma_1} \left(g_{\alpha\beta,\gamma} \frac{dx^\alpha}{d\sigma} \frac{dx^\beta}{d\sigma} - 2 \frac{d}{d\sigma} \left(g_{\alpha\gamma} \frac{dx^\alpha}{d\sigma} \right) \right) \delta x^\gamma \, d\sigma = 0.$$

Since this relation holds for arbitrary δx^γ we obtain the following differential equations of geodesics

$$-g_{\alpha\beta,\gamma} \frac{dx^\alpha}{d\sigma} \frac{dx^\beta}{d\sigma} + 2 \frac{d}{d\sigma} \left(g_{\alpha\gamma} \frac{dx^\alpha}{d\sigma} \right) = 0,$$

i.e., after removing the parenthesis and multiplying by $\frac{1}{2}$,

$$g_{\alpha\gamma}\frac{d^2x^\alpha}{d\sigma^2}+\left(-\frac{1}{2}g_{\alpha\beta,\gamma}+\frac{1}{2}g_{\gamma\alpha,\beta}+\frac{1}{2}g_{\beta\gamma,\alpha}\right)\frac{dx^\alpha}{d\sigma}\frac{dx^\beta}{d\sigma}=0.$$

By virtue of (8), these equations take the form

$$g_{\alpha\gamma}\frac{d^2x^\alpha}{d\sigma^2}+\Gamma_{\gamma\alpha\beta}\frac{dx^\alpha}{d\sigma}\frac{dx^\beta}{d\sigma}=0.$$

Multiplying this relation by $g^{\gamma\nu}$ and summing over γ, we obtain, by (9), the equations (10). Finally, from (10) we obtain, by direct calculation, $\frac{d}{d\sigma}\left(g_{\alpha\beta}\frac{dx^\alpha}{d\sigma}\frac{dx^\beta}{d\sigma}\right)=0$. Hence, the relation (11) holds. □

Proof of Theorem 3. Using the obvious formulas

$$x^p{}_{,q}=\delta^p_q,\quad x_{p,\,q}=h_{pq},\quad h_{pq}x^px^q=x_px^p=-r^2,\quad r_{,p}=-\frac{1}{r}x_p,$$

we obtain from (6)

$$g_{\alpha\beta,0}=0,\qquad g_{pq,n}=0,\qquad g_{00,p}=-\frac{2Gm}{r^3}x_p,$$

$$g_{0p,q}=g_{p0,q}=\frac{3}{2}\sqrt{\frac{2Gm}{r^7}}x_px_q+\sqrt{\frac{2Gm}{r^3}}h_{pq}.$$

Now, according to (8), we find the formulas for the first Christoffel symbols:

$$\Gamma_{000}=0,\quad \Gamma_{pq0}=\Gamma_{p0q}=0,\quad \Gamma_{pqn}=0,\quad \Gamma_{p00}=-\Gamma_{0p0}=-\Gamma_{00p}=\frac{Gm}{r^3}x_p,$$

$$\Gamma_{0pq}=\frac{3}{2}\sqrt{\frac{2Gm}{r^7}}x_px_q+\sqrt{\frac{2Gm}{r^3}}h_{pq}.$$

Furthermore, the inverse matrix $(g^{\mu\nu})$ has the following elements:

$$g^{00}=\frac{1}{c^2},\quad g^{0p}=-\frac{1}{c^2}\sqrt{\frac{2Gm}{r^3}}\,x^p,\quad g^{pq}=\frac{2Gm}{c^2r^3}x^px^q+h^{pq},$$

where (h^{pq}) is the inverse matrix for (h_{mn}), i.e., $h^{pq}=-1$ for $p=q$ and $h^{pq}=0$ for $p\neq q$. So, by (9) we find the formulas for the second Christoffel symbols of the main quadratic form (5):

$$\Gamma^0_{00} = \frac{Gm}{c^2}\sqrt{\frac{2Gm}{r^5}}; \qquad \Gamma^n_{00} = \frac{Gm}{c^2r^3}\left(c^2 - \frac{2Gm}{r}\right)x^n; \tag{23}$$

$$\Gamma^0_{0p} = \Gamma^0_{p0} = -\frac{Gm}{c^2r^3}x_p; \qquad \Gamma^n_{0p} = \Gamma^n_{p0} = \frac{Gm}{c^2r^3}\sqrt{\frac{2Gm}{r^3}}x^n x_p; \tag{24}$$

$$\Gamma^0_{pq} = \frac{3}{2c^2}\sqrt{\frac{2Gm}{r^7}}x_p x_q + \frac{1}{c^2}\sqrt{\frac{2Gm}{r^3}}h_{pq}; \tag{25}$$

$$\Gamma^n_{pq} = -\frac{3Gm}{c^2r^5}x^n x_p x_q - \frac{2Gm}{c^2r^3}x^n h_{pq}.$$

We now can complete the proof. The relation (13) follows immediately from (5), (11) by our agreement for parametrization. Furthermore, substituting (23)–(25) into (10) and taking into account the relation (13), we obtain the equations (12). We do not wrire the equation for $\frac{d^2x^0}{d\sigma^2}$, i.e., the equation (10) for $\alpha = 0$, since the relation (13) gives $\frac{dx^0}{d\sigma}$. □

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