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A REMARK ON COMPACT MINIMAL SURFACES IN S^5 WITH NON-NEGATIVE GAUSSIAN CURVATURE

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Abstract. The purpose of this paper is to show that a generalized Clifford immersion with non-negative Gaussian curvature has constant contact angle, thus extending previous results.

1. Introduction

In [4] we introduced the notion of contact angle, which can be considered as a new geometric invariant useful for investigating the geometry of immersed surfaces in S^3. Geometrically, the contact angle β is the complementary angle between the contact distribution and the tangent space of the surface. Also in [4], we derived formulae for the Gaussian curvature and the Laplacian of an immersed minimal surface in S^3, and we gave a characterization of the Clifford Torus as the only minimal surface in S^3 with constant contact angle.

Recently, in [5], we constructed a family of minimal tori in S^5 with constant contact and holomorphic angles. These tori are parametrized by the following circle equation

$$a^2 + \left(b - \frac{\cos\beta}{1+\sin^2\beta}\right)^2 = 2\frac{\sin^4\beta}{(1+\sin^2\beta)^2} \tag{1}$$

where a and b are given in Section 3 (equation (9)). In particular, when $a = 0$, we recover the examples found by Kenmotsu [3]. These examples are defined for $0 < \beta < \frac{\pi}{2}$. Also, when $b = 0$, we find a new family of minimal tori in S^5, and these tori are defined for $\frac{\pi}{4} < \beta < \frac{\pi}{2}$. Also, in [5], when $\beta = \frac{\pi}{2}$, we give an alternative proof of this classification of a Theorem proved by Blair in [1], and Yamaguchi, Kon and Miyahara in [6] for Legendrian minimal surfaces in S^5 with constant Gaussian curvature.

The immersions that we investigate in this paper are those that satisfy the following conditions: