ON SPECTRAL THEORY OF LAX OPERATORS ON SYMMETRIC SPACES: VANISHING VERSUS CONSTANT BOUNDARY CONDITIONS

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Abstract. We outline several specific issues concerning the theory of multi-component nonlinear Schrödinger equations with vanishing and constant boundary conditions. We start with the spectral properties of the Lax operator L for vanishing boundary conditions. We introduce the fundamental analytic solutions (FAS) and demonstrate their importance for relating the scattering problem to a Riemann-Hilbert problem, and for the construction of the resolvent of L. Then we generalize this procedure to constant boundary conditions case. We start with the structure of the class of allowed potentials $\mathcal{M}$ and give a recipe of how FAS can be constructed on each of the leafs of the relevant Riemannian surface. This allows us to relate the scattering problem to a Riemann-Hilbert problem posed on a Riemannian surface. Next we use these FAS to construct the resolvent of L and study its spectral properties. We also introduce the minimal set of scattering data on the continuous spectrum of L which generically has varying multiplicity. The general construction is illustrated by three representative examples related to A.III, C.II and D.III symmetric spaces. Finally we consider regularized Wronskian relations which allow us to analyze the mapping between the potential of L and the scattering data.

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