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LINEAR CONNECTIONS AND EXTENDED ELECTRODYNAMICS

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Abstract. In this paper we give a presentation of the basic vacuum relations of Extended Electrodynamics in terms of linear connections.

1. Linear Connections

Linear connections are first-order differential operators in vector bundles. If such a connection ∇ is given and σ is a section of the bundle, then $\nabla\sigma$ is one-form on the base space valued in the space of sections of the vector bundle, so if X is a vector field on the base space then $i(X)\nabla\sigma = \nabla_X\sigma$ is a new section of the same bundle [2]. If f is a smooth function on the base space then $\nabla(f\sigma) = \mathrm{d}f \otimes \sigma + f\nabla\sigma$, which justifies the differential operator nature of ∇: the components of σ are differentiated and the basis vectors in the bundle space are linearly transformed.

Let e_a and $\varepsilon^b, a, b = 1, 2, \ldots, r$ be two dual local bases of the corresponding spaces of sections $<\varepsilon^b, e_a>= \delta^b_a$, then we can write

$$\sigma = \sigma^a e_a, \qquad \nabla = \mathbf{d} \otimes \mathrm{id} + \Gamma^b_{\mu a}\mathrm{d}x^\mu \otimes (\varepsilon^a \otimes e_b), \qquad \nabla(e_a) = \Gamma^b_{\mu a}\mathrm{d}x^\mu \otimes e_b$$

and therefore

$$\nabla(\sigma^m e_m) = \mathbf{d}\sigma^m \otimes e_m + \sigma^m\Gamma^b_{\mu a}\mathrm{d}x^\mu <\varepsilon^a, e_m> \otimes\, e_b = \left[\mathbf{d}\sigma^b + \sigma^a\Gamma^b_{\mu a}\mathrm{d}x^\mu\right]\otimes e_b$$

where $\Gamma^b_{\mu a}$ are the components of ∇ with respect to the coordinates $\{x^\mu\}$ on the base space and with respect to the bases $\{e_a\}$ and $\{\varepsilon^b\}$.

Since the elements $(\varepsilon^a \otimes e_b)$ define a basis of the space of (local) linear maps of the local sections, it becomes clear that in order to define locally a linear connection it is sufficient to specify some one-form θ on the base space and a