JGSP **19** (2010) 43–72

JOURNAL OF
Geometry and Symmetry
in Physics

GROUP THEORETICAL APPROACHES TO VECTOR PARAMETERIZATION OF ROTATIONS

ANDREAS MÜLLER

Communicated by Jan J. Slawianowski

Abstract. Known parametrizations of rotations are derived from the LIE group theoretical point of view considering the two groups $\mathrm{SO}(3)$ and $\mathrm{SU}(2)$. The concept of coordinates of the first and second kind for these groups is used to derive the axis and angle as well as the three-angle description of rotation matrices. With the homomorphism of the two groups the EULER parameter description arises from the axis and angle description of $\mathrm{SU}(2)$. Due to the topology of $\mathrm{SO}(3)$ any three-angle description gives only a local parametrization like EULER angles such that the mapping from their time derivatives to the algebra $\mathfrak{so}(3)$, i.e., to the angular velocity tensor, exhibits singularities. All these parametrizations are based on the generation of the respective group by the $\exp$ map from their algebras. Alternatively the CAYLEY transformation also maps algebra elements to group elements. This fact is well know on $\mathrm{SO}(3)$ and yields a representation of rotation matrices in terms for RODRIGUES parameter, which is, however, not continuous. Generalizing this transformation to $\mathrm{SU}(2)$ allows for a singularity-free description of all rotations, which does not contain transcendental functions. While in the considered range the exponential map is of class C^∞ the cay map on $\mathrm{SU}(2)$ is only of class C^1 and on $\mathrm{SO}(3)$ it is not even continuous. Simulation results exemplify the resultant numerical benefits for the simulation of rigid body dynamics. The problem caused by a lack of a continuous transformation from generalized accelerations to angular accelerations can be avoided for rigid body motions using the BOLZMANN-HAMEL equations.

Contents