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MONODROMY AND THE BOHR–SOMMERFELD GEOMETRIC QUANTIZATION

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Abstract. We study the linear part of the monodromy of completely integrable Hamiltonian systems via Bohr–Sommerfeld Geometric Quantization. We relate monodromy to the ambiguity in the choice of the pre–quantum connection and to the action of the (connected component of the) gauge group.

1. Introduction

In the framework of Bohr–Sommerfeld geometric quantization, we study (quantum) monodromy from different viewpoints. Monodromy, together with the so–called Chern–Duistermaat class and the Lagrangian class, provides an obstruction to the global definition of action–angle variables for completely integrable Hamiltonian systems [7, 9]. Our specific contributions relate monodromy to the freedom of choice of a pre–quantum connection and to $\mathcal{G}_0$–equivalence of connections ($\mathcal{G}_0$ is the connected component of the identity of the gauge group $\mathcal{G}$ of a pre–quantum line bundle).

The present work is organized as follows. In Section 2 we first review Liouville–Arnold theorem and the obstructions to existence of global action–angles coordinates and then we quickly review the geometric quantization method. In Section 3 we state and prove the main results of the paper. A short section with conclusions and perspectives follows.

2. Liouville–Arnold Theorem and Geometric Quantization

In this section we review some basic facts about completely integrable Hamiltonian systems and geometric quantization. We will also introduce the notation that will be used throughout the paper.