THE PHASE SPACE FOR CONFORMAL PARTICLE
WITH ZERO MASS AND SPIRALITY AND ITS
RELATION WITH THE KEPLER'S PROBLEM

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Abstract. We show that the two orbits in the coadjoint representation of the group SO(4,2), the one, related with the Kepler's problem, and the other - modelling the phase space of a conformal particle with zero mass and spirality zero, coincide.

1. Introduction

The the idea for the present work arose naturally in the development of two important trends in the Classical Mechanics and Differential Geometry. The first one is related with the so-called Kirillov-Kostant-Souriau theorem, that allows to identify connected symplectic spaces having sufficiently large dynamical group with orbits in the coadjoint representation of these groups, [12, 21]. The second is the circle of ideas related to the geometric quantization [13, 21]. If the symplectic manifold is given, we expect to quantize some subalgebra of functions (usually finite dimensional) and naturally such a subalgebra is associated with some Lie group. Then the idea of the Kirillov-Kostant-Souriau theorem can be developed in somewhat opposite direction, that is, to start with certain Lie algebra $\mathcal{G}$, to which corresponds a connected Lie group G, and then using it, to find a phase space having G as dynamical group. Natural candidate for it will be some orbit in the coadjoint representation of G. One usually imposes also the condition that such G must contain the Poincaré or the Galilean group in order to be able to give physical interpretations. It is clear that the above procedure can be performed even for quantum systems that have no classical counterparts in the usual sense. The phase space (symplectic manifold) will be simply an orbit in the coadjoint representation of the group, picked up by fixing the values of some important physical observables (such as mass for example), [21]. It is natural to expect, that some of the spaces, obtained in this way may coincide with some classical phase spaces. The Hydrogen atom problem (Kepler's problem) turned