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RECURSION OPERATORS AND EXPANSIONS OVER ADJOINT SOLUTIONS FOR THE CAUDREY-BEALS-COIFMAN SYSTEM WITH $\mathbb{Z}_P$ REDUCTIONS OF MIKHAILOV TYPE

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Abstract. We consider the Caudrey-Beals-Coifman linear problem and the theory of the Recursion Operators (Generating Operators) related to it in the presence of $\mathbb{Z}_p$ reduction of Mikhailov type.

1. Introduction

1.1. The Generalized Zakharov-Shabat and Caudrey-Beals-Coifman Systems

As it is well known nonlinear evolution equations (NLEEs) of soliton type are equations (systems) that can be written into the form $[L, A] = 0$ (Lax represensta-tion) where L, A are linear operators on ∂_x, ∂_t depending also on some functions $q_\alpha(x,t)$, $1 \leq \alpha \leq s$ (called 'potentials') and the spectral parameter λ. The corresponding system is of a course system of partial differential equations on $q_\alpha(x,t)$. Usually the equation is a part of a hierarchy of NLEEs related to $L\psi = 0$ (auxiliary linear problem) which consists of the equations that can be obtained by changing A and fixing L, [7, 15]. The soliton equations possess many interesting properties but for our purposes we shall mention only that they can be solved explicitly through various schemes, most of which share the property that the Lax representation permits to pass from the original evolution to the evolution of some spectral data related to the problem $L\psi = 0$. The Caudrey-Beals-Coifman (CBC) system, called the Generalized Zakharov-Shabat (GZS) system in the case when the element J is real, is one of the best known auxiliary linear problems

$$L\psi = (\mathrm{i}\partial_x + q(x) - \lambda J)\,\psi = 0. \tag{1}$$

The system has a long history of study and generalizations see [2–6, 29, 30], finally it has been realized that one can assume that $q(x)$ and J belong to a fixed simple Lie algebra $\mathfrak{g}$ in some finite dimensional irreducible representation, [17]. Then the element J should be regular, that is $\ker(\mathrm{ad}_J)$ $(\mathrm{ad}_J(X) \equiv [J, X]$, $X \in \mathfrak{g})$ is a

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