



ON VENTCEL'S TYPE BOUNDARY CONDITION FOR LAPLACE OPERATOR IN A SECTOR

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Abstract. This paper deals with classical solutions of the Dirichlet-Ventcel boundary value problem (BVP) for the Laplace operator in bounded sector in the plane having opening of the corresponding angle $\varphi_0 > 0$. Ventcel BVP is given by second order differential operator on the boundary satisfying Lopatinskii condition there. As the boundary is non smooth, two different cases appear: $\frac{\pi}{\varphi_0}$ is irrational and $\frac{\pi}{\varphi_0}$ is an integer. At first we prove uniqueness result via the maximum principle and then existence of the classical solution. To do this we apply two different approaches: the machinery of the small denominators and the concept of Green function.

1. Introduction

This paper deals with existence and uniqueness of the classical solution for the Laplace operator equipped with Ventcel's type boundary condition in a bounded sector in the plane. Ventcel boundary conditions are second order differential conditions appearing in asymptotic models proposed by Feller and Ventcel [4], [7, 8] (interpreted as a surface diffusion). The opening of the angle $\varphi_0 > 0$ with vertex at the origin is such that $\pi/\varphi_0 \notin \mathbb{Q}$ or $\frac{\pi}{\varphi_0} \in \mathbb{N}$, \mathbb{Q} being the set of rational numbers and \mathbb{N} standing for the set of positive integers. At first we state the problem and prove uniqueness result (comparison principle) via the maximum principle for elliptic equations. Our second step is to prove existence result for classical solutions. To do this we apply the machinery of small denominators. Another approach is from the theory of ordinary differential equations (see Section 3 of the paper).

Under different conditions imposed on φ_0 we prove existence of a C^2 solution in the bounded domain as well regularity results, including C^∞ solutions. The solution is found in the form of convergent series in $r^m \sin(\frac{n\pi\varphi}{\varphi_0})$, $m, n \in \mathbb{N}$. As Laplace operator is C^∞ and even analytic hypoelliptic, the main difficulties are in proving regularity up to the boundary. Laplace-Dirichlet-Ventcel problem in a disc, in a ring and in a bounded smooth domain were studied via pseudo-differential operators approach in [2].