



DEFORMATION QUANTIZATION IN THE TEACHING OF LIE GROUP REPRESENTATIONS

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Abstract. We present straightforward and concrete computations of the unitary irreducible representations of the Euclidean motion group $M(2)$ employing the methods of deformation quantization. Deformation quantization is a quantization method of classical mechanics and is an autonomous approach to quantum mechanics, arising from the Wigner quasiprobability distributions and Weyl correspondence. We advertise the utility and power of deformation theory in Lie group representations. In implementing this idea, many aspects of the method of orbits are also learned, thus further adding to the mathematical toolkit of the beginning graduate student of physics. Furthermore, the essential unity of many topics in mathematics and physics (such as Lie theory, quantization, functional analysis and symplectic geometry) is witnessed, an aspect seldom encountered in textbooks, in an elementary way.

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