



SPACES REALIZED AND NON-REALIZED AS DOLD-LASHOF CLASSIFYING SPACES

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Abstract. Let X be a simply connected CW-complex of finite type. Denote by $\text{Baut}_1(X)$ the Dold-Lashof classifying space of fibrations with fiber X . This paper is a survey about the problem of realizing Dold-Lashof classifying spaces. We will also present some new results: we show that not all rank-two rational H -spaces can be realized as $\text{Baut}_1(X)$ for simply connected, rational elliptic space X . Moreover, we construct an infinite family of rational spaces X , such that $\text{Baut}_1(X)$ is rationally a finite H -space of rank-two (up to rational homotopy type).

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1. Introduction

In this paper, all spaces are simply connected CW-complex and are of finite type over \mathbb{Q} , i.e., have finite-dimensional rational homology in each degree.