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SOME GEOMETRICAL ASPECTS OF EINSTEIN, RICCI AND YAMABE SOLITONS

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Abstract. Under certain assumptions, we characterize the almost η-Einstein, η-Ricci and η-Yamabe solitons on a pseudo-Riemannian manifold when the potential vector field of the soliton is infinitezimal harmonic or torse-forming. Moreover, in the second case, if the manifold is Ricci symmetric of constant scalar curvature, then the soliton is completely determined.

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1. Preliminaries

Einstein, Ricci and Yamabe solitons may be regarded as generalized fixed points of Einstein, Ricci and Yamabe flow respectively, being important tools in particular areas of theoretical physics and mathematical physics. Recently, some generalizations have been considered: almost η-Einstein, almost η-Ricci and almost η-Yamabe solitons, having the previous ones as particular cases. We shall briefly recall these notions.

Let g be a pseudo-Riemannian metric on the smooth manifold M, ξ a vector field, η is a one-form, and λ and μ two smooth functions on M. If we denote by $\pounds_\xi$ the Lie derivative operator along the vector field ξ, Ric the Ricci curvature tensor field, r the scalar curvature, then the data $(g, \xi, \eta, \lambda, \mu)$ define

i) *an almost η-Einstein soliton* on M if they satisfy the equation

$$\frac{1}{2}\pounds_\xi g + \mathrm{Ric} + \left(\lambda - \frac{r}{2}\right) g + \mu\eta \otimes \eta = 0 \tag{1}$$

ii) *an almost η-Ricci soliton* on M if they satisfy the equation

$$\frac{1}{2}\pounds_\xi g + \mathrm{Ric} + \lambda g + \mu\eta \otimes \eta = 0 \tag{2}$$

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