

THE SPIRIC SECTIONS OF PERSEUS AND THE UNIFORM PARAMETERIZATIONS OF THE CASSINIAN OVALS

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Abstract. Here we derive explicit formulas that parameterize the Cassinian ovals based on their recognition as the so called spiric sections of the standard tori in the three-dimensional Euclidean space which was suggested in the ancient time by Perseus. These formulas derived originally in terms of the toric parameters are expressed through the usual geometrical parameters that enter in the present day definition of the Cassinian curves. All three types of morphologically different curves are illustrated graphically using the corresponding sets of parameters and respective formulas. The geometry of the ovals can be studied in full details and this is done here to some extent. As examples explicit formulas for the embraced volume and the surface area of the dumbbell like surface generated by the oval are presented. Last, but not least, new alternative explicit parameterizations of the Cassinian ovals are derived in polar, and even in non-canonical Monge forms.

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