BOOK REVIEW

Differential Geometry and Lie Groups for Physicists, by Marian Fecko, Cambridge University Press, Cambridge, 2006, xv + 697 pp. ISBN 978-0-521-84507-6

A modern mathematical physicist could hardly imagine doing serious studies in the frame of contemporary theoretical and mathematical physics without making substantial use of concepts like vector and tensor fields, differential forms, Stokes theorem on manifolds, Lie derivative and symmetries, Lie (super)groups and Lie (super)algebras and their representations, bundles with connection, curvature and Chern classes, Hamiltonian and Poisson structures, Frobenius integrability. Differential topology has already also made serious steps inside theoretical physics through topological charges, helicity properties of magnetic fields in studying plasma properties, topological soliton theory and other homology/cohomology characteristics of physical systems wherever they are available and important. Any physical system has spatial structure and shows definite stability properties, so, it can support its existence and compensate in definite degree the external disturbances through appropriate shape changes and kinematical behavior without losing identity. Shortly speaking, its time existence is a dynamical process being strongly connected with various and continuous internal and external energy-momentum exchange processes. All these processes are real phenomena and any attempt for their description should be based on appropriate mathematical structures. The more than a century intensive interaction between differential geometry and Lie group theory on one hand, and the theoretical and mathematical physics, on the other hand, turned out to be exclusively useful, suggestive and creative process. The author of the book under review has very successfully and in a very appropriate way presented the idea that mathematics is not just abstract thinking and physics is not just working in labs. It could be said that physics has always stimulated mathematics to develop new branches, and mathematics has always been very responsive and warm-hearted to the needs of physics. Starting with Newton and Leibniz the greatest mathematicians of all times have contributed considerably to deeply understanding and solving difficult physical problems.

The book under review consists of 22 chapters and two Appendices. *Chapter 1* starts with a short review of some topological concepts introduces and illustrates the concepts of smooth manifold and smooth maps of manifolds, and ends with a description of smooth surfaces in $\mathbb{R}^n$. The following two chapters consider the basic tangent structures (the tensor algebra on the manifold) and mappings of tangent structures generated by mappings of manifolds. Among the illustrative examples the metric tensor and related considerations are given more attention. *Chapter 4* is devoted to a detailed consideration of Lie derivative, its properties and application in searching for isometries and conformal transformations. The following four chapters five to eight consider the differential and integral calculus on manifold: exterior algebra and orientability (the Hodge star also), exterior derivative, coderivative and corresponding Laplace – de-Rham operator, simplices and chains and Stokes theorem, classical vector analysis. A short chapter on de-Rham cohomology (the Poincare lemma) follows. The next four chapters consider the standard basic facts concerning Lie groups: left invariance, exponential map, representations, intertwining operators, actions of Lie groups and Lie algebras on manifolds, G-spaces. After a short chapter on Hamiltonian (symplectic) mechanics (including the moment map and reduction techniques) the considerations continue mainly along the connection-curvature structures in vector and principal bundles and their applications in physical theories – the so called gauge theories. *Chapter 17* is devoted to the differential geometry of tangent and cotangent bundles, *Chapter 18* deals with Euler-Lagrange formulation of field theory, and *Chapter 22* considers Clifford algebra approach to spinors. Finally, something that deserves to be specially noted: every chapter ends with a Summary text representing the most essential points and results of the chapter and all important formulas. The first *Appendix* gives essentials on some algebraic structures and the second *Appendix* (called "Starring") gives a list of 83 names of mathematicians and physicists with considerable contributions to the development of geometry and physics. The *Bibliography* is limited to 31 names (and corresponding titles).

All basic material that is necessary for a young scientist in the field of geometrical formulation of physical theories is included and, which is more important, is ordered and represented in a very appropriate manner, I would say, with a great respect to the reader. Much of the material is given in the form of problems with hints, so that the reader could test himself in the right understanding of what he has already read. A physically educated reader will find, for example, a detailed formal and actual explanation of the sense of Lie derivative (*Chapter 4* and *Section 8.2*) and why it is so important as a powerful tool for symmetry considerations of physical systems and how to find and study such symmetries. The role of local

isometries, i.e., Killing vector fields, on Riemannian manifolds, is also carefully considered from the point of view how these vector fields are used in field theory to generate integral conserved quantities like energy, momentum, angular momentum if the local stress-energy-momentum tensor is at hand.

For those who want to get into modern physical gauge theory the *Chapters 19-21* may serve as wonderful introduction. The basic geometrical concepts here are the connection and the curvature, which are presented in a very clear way in the both approaches: through splitting of the tangent bundles of some principal *G*-bundles to horizontal and vertical subbundles (*Chapters 19-20*), and through parallel transport and covariant derivative operator in vector bundles (the tangent bundle is mainly considered in *Chapter 15*). In the principal bundle approach the role of Frobenius integrability theorem is shown how it clarifies the most general sense of curvature as a measure of nonintegrability, which determines the basic usage of curvature as adequate quantity describing interaction, i.e. energy-momentum exchange, or force field, between physical systems (or subsystems of a system). In my view, most of the physicists still do not pay due respect to Frobenius integrability theorem as powerful adequate tool to be used in physical theories. For example, if a free physical system could be mathematically considered as integrable distribution (in a definite sense), any interaction of its two subsystems may be considered as nonintegrable (sub)distributions and the corresponding curvatures to be used as measures of the corresponding internal energy-momentum exchange. Although the soliton theory specialists feel glad when succeed in representing some nonlinear evolution equation as "zero curvature condition", the time-stability of the internal dynamical structure of the corresponding inter-related exchange processes (which are supposed to occur inside a small spatial region) clearly suggests that in order to get a better understanding of the corresponding spatially localized dynamical structure of the soliton, nonintegrability is so much important as integrability. This view clearly distinguishes the primary importance of the curvature concept in describing the internal physical energy-momentum exchange processes, since every physical process is necessarily accompanied by some energy-momentum space-time transport.

I truly believe that reading this book will bring a real pleasure to all physically inclined young mathematicians and mathematically inclined young physicists, moreover, I am fully convinced that getting appropriately acquainted with its contents really deserves. Therefore I would allow myself to characterize it as a very good high level textbook and to recommend it to all young scientists being interested in finding correspondence between harmony in the physical world and harmony in geometrical structures.

As a whole, (together with the exercises presented) the book is well written, very well ordered and the exposition is very clear. It could serve as a good stand-by introduction to modern differential geometry and to geometrical formulation of physical theories. I would be glad to see in the second edition of the book a chapter on Jet Bundles and their applications in mathematical physics.

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