

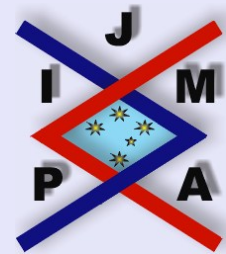
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APPROXIMATION OF FIXED POINTS OF ASYMPTOTICALLY DEMICONTRACTIVE MAPPINGS IN ARBITRARY BANACH SPACES

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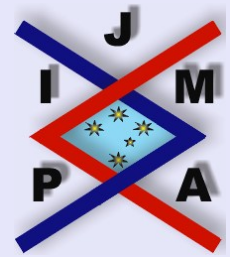


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Abstract

Let E be a real Banach Space and K a nonempty closed convex (not necessarily bounded) subset of E . Iterative methods for the approximation of fixed points of asymptotically demicontractive mappings $T : K \rightarrow K$ are constructed using the more general modified Mann and Ishikawa iteration methods with errors.

Our results show that a recent result of Osilike [3] (which is itself a generalization of a theorem of Qihou [4]) can be extended from real q -uniformly smooth Banach spaces, $1 < q < \infty$, to arbitrary real Banach spaces, and to the more general Modified Mann and Ishikawa iteration methods with errors. Furthermore, the boundedness assumption imposed on the subset K in ([3, 4]) are removed in our present more general result. Moreover, our iteration parameters are independent of any geometric properties of the underlying Banach space.

2000 Mathematics Subject Classification: 47H06, 47H10, 47H15, 47H17.

Key words: Asymptotically Demicontractive Maps, Fixed Points, Modified Mann and Ishikawa Iteration Methods with Errors.

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1. Introduction

Let E be an arbitrary real Banach space and let J denote the normalized duality mapping from E into 2^{E^*} given by $J(x) = \{f \in E^* : \langle x, f \rangle = \|x\|^2 = \|f\|^2\}$, where E^* denotes the dual space of E and $\langle \cdot, \cdot \rangle$ denotes the generalized duality pairing. If E^* is strictly convex, then J is single-valued. In the sequel, we shall denote the single-valued duality mapping by j .

Let K be a nonempty subset of E . A mapping $T : K \rightarrow K$ is called k -strictly asymptotically pseudocontractive mapping, with sequence $\{k_n\} \subseteq [1, \infty)$, $\lim_n k_n = 1$ (see for example [3, 4]), if for all $x, y \in K$ there exists $j(x - y) \in J(x - y)$ and a constant $k \in [0, 1)$ such that

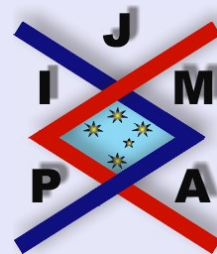
$$(1.1) \quad \langle (I - T^n)x - (I - T^n)y, j(x - y) \rangle \geq \frac{1}{2}(1 - k) \|(I - T^n)x - (I - T^n)y\|^2 - \frac{1}{2}(k_n^2 - 1)\|x - y\|^2,$$

for all $n \in \mathbb{N}$. T is called an asymptotically demicontractive mapping with sequence $k_n \subseteq [0, \infty)$, $\lim_n k_n = 1$ (see for example [3, 4]) if $F(T) = \{x \in K : Tx = x\} \neq \emptyset$ and for all $x \in K$ and $x^* \in F(T)$, there exists $k \in [0, 1)$ and $j(x - x^*) \in J(x - x^*)$ such that

$$(1.2) \quad \langle x - T^n x, j(x - x^*) \rangle \geq \frac{1}{2}(1 - k)\|x - T^n x\|^2 - \frac{1}{2}(k_n^2 - 1)\|x - x^*\|^2$$

for all $n \in \mathbb{N}$. Furthermore, T is uniformly L -Lipschitzian, if there exists a constant $L > 0$, such that

$$(1.3) \quad \|T^n x - T^n y\| \leq L\|x - y\|,$$



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for all $x, y \in K$ and $n \in \mathbb{N}$.

It is clear that a k -strictly asymptotically pseudocontractive mapping with a nonempty fixed point set $F(T)$ is asymptotically demicontractive. The classes of k -strictly asymptotically pseudocontractive and asymptotically demicontractive maps were first introduced in Hilbert spaces by Qihou [4]. In a Hilbert space, J is the identity and it is shown in [3] that (1.1) and (1.2) are respectively equivalent to the inequalities:

$$(1.4) \quad \|T^n x - T^n y\| \leq k_n^2 \|x - y\|^2 + k \|(I - T^n)x - (I - T^n)y\|^2$$

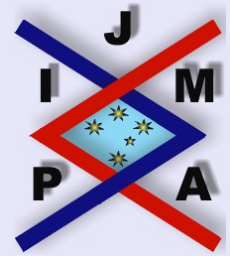
and

$$(1.5) \quad \|T^n x - T^n y\|^2 \leq k_n^2 \|x - y\|^2 + \|x - T^n x\|^2$$

which are the inequalities considered by Qihou [4].

In [4], Qihou using the *modified Mann* iteration method introduced by Schu [5], proved convergence theorem for the iterative approximation of fixed points of k -strictly asymptotically pseudocontractive mappings and asymptotically demicontractive mappings in Hilbert spaces. Recently, Osilike [3], extended the theorems of Qihou [4] concerning the iterative approximation of fixed points of k -strictly asymptotically demicontractive mappings from Hilbert spaces to much more general real q -uniformly smooth Banach spaces, $1 < q < \infty$, and to the much more general *modified Ishikawa iteration method*. More precisely, he proved the following:

Theorem 1.1. (Osilike [3, p. 1296]): *Let $q > 1$ and let E be a real q -uniformly smooth Banach space. Let K be a closed convex and bounded subset of E and*



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$T : K \rightarrow K$ a completely continuous uniformly L -Lipschitzian asymptotically demicontractive mapping with a sequence $k_n \subseteq [1, \infty)$ satisfying $\sum_{n=1}^{\infty} (k_n^2 - 1) < \infty$. Let $\{\alpha_n\}$ and $\{\beta_n\}$ be real sequences satisfying the conditions.

- (i) $0 \leq \alpha_n, \beta_n \leq 1, n \geq 1$;
- (ii) $0 < \epsilon \leq c_q \alpha_n^{q-1} (1 + L \beta_n)^q \leq \frac{1}{2} \{q(1 - k)(1 + L)^{-(q-2)}\} - \epsilon$, for all $n \geq 1$ and for some $\epsilon > 0$; and
- (iii) $\sum_{n=0}^{\infty} \beta_n < \infty$.

Then the sequence $\{x_n\}$ generated from an arbitrary $x_1 \in K$ by

$$y_n = (1 - \beta_n)x_n + \beta_n T^n x_n, n \geq 1,$$

$$x_{n+1} = (1 - \alpha_n)x_n + \alpha_n T^n y_n, n \geq 1$$

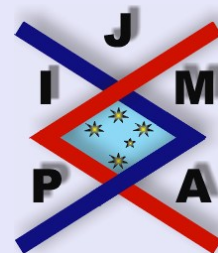
converges strongly to a fixed point of T .

In Theorem 1.1, c_q is a constant appearing in an inequality which characterizes q -uniformly smooth Banach spaces. In Hilbert spaces, $q = 2$, $c_q = 1$ and with $\beta_n = 0 \forall n$, Theorems 1 and 2 of Qihou [4] follow from Theorem 1.1 (see Remark 2 of [3]).

It is our purpose in this paper to extend Theorem 1.1 from real q -uniformly smooth Banach spaces to arbitrary real Banach spaces using the more general modified Ishikawa iteration method with errors in the sense of Xu [7] given by

$$(1.6) \quad y_n = a_n x_n + b_n T^n x_n + c_n u_n, n \geq 1,$$

$$x_{n+1} = a'_n x_n + b'_n T^n y_n + c'_n v_n, n \geq 1,$$



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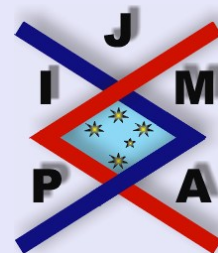
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where $\{a_n\}$, $\{b_n\}$, $\{c_n\}$, $\{a'_n\}$, $\{b'_n\}$, $\{c'_n\}$ are real sequences in $[0, 1]$. $a_n + b_n + c_n = 1 = a'_n + b'_n + c'_n$, $\{u_n\}$ and $\{v_n\}$ are bounded sequences in K . If we set $b_n = c_n = 0$ in (1.6) we obtain the *modified Mann iteration method with errors* in the sense of Xu [7] given by

$$(1.7) \quad x_{n+1} = a'_n x_n + b'_n T^n x_n + c'_n v_n, \quad n \geq 1.$$



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2. Main Results

In the sequel, we shall need the following:

Lemma 2.1. *Let E be a normed space, and K a nonempty convex subset of E . Let $T : K \rightarrow K$ be uniformly L -Lipschitzian mapping and let $\{a_n\}$, $\{b_n\}$, $\{c_n\}$, $\{a'_n\}$, $\{b'_n\}$ and $\{c'_n\}$ be sequences in $[0, 1]$ with $a_n + b_n + c_n = a'_n + b'_n + c'_n = 1$. Let $\{u_n\}$, $\{v_n\}$ be bounded sequences in K . For arbitrary $x_1 \in K$, generate the sequence $\{x_n\}$ by*

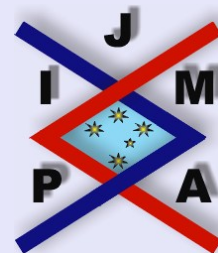
$$\begin{aligned} y_n &= a_n x_n + b_n T^n x_n + c_n u_n, \quad n \geq 1 \\ x_{n+1} &= a'_n x_n + b'_n T^n y_n + c'_n v_n, \quad n \geq 0. \end{aligned}$$

Then

$$\begin{aligned} \|x_n - T x_n\| &\leq \|x_n - T^n x_n\| + L(1 + L)^2 \|x_{n+1} - T^{n-1} x_{n-1}\| \\ &\quad + L(1 + L)c'_{n-1} \|v_{n-1} - x_{n-1}\| + L^2(1 + L)c_{n-1} \|u_{n-1} - x_n\| \\ (2.1) \quad &\quad + Lc'_{n-1} \|x_{n-1} - T^{n-1} x_{n-1}\|. \end{aligned}$$

Proof. Set $\lambda_n = \|x_n - T^n x_n\|$. Then

$$\begin{aligned} \|x_n - T x_n\| &\leq \|x_n - T^n x_n\| + L \|T^{n-1} x_n - x_n\| \\ &\leq \lambda_n + L^2 \|x_n - x_{n-1}\| + L \|T^{n-1} x_{n-1} - x_n\| \\ &= \lambda_n + L^2 \|a'_{n-1} x_n + b'_{n-1} T^{n-1} y_{n-1} + c'_{n-1} v_{n-1} - x_{n-1}\| \\ &\quad + L \|a'_{n-1} x_{n-1} + b'_{n-1} T^{n-1} y_{n-1} + c'_{n-1} v_{n-1} - T^{n-1} x_{n-1}\| \\ &= \lambda_n + L^2 \|b'_{n-1} (T^{n-1} y_{n-1} - x_{n-1}) + c'_{n-1} (v_{n-1} - x_{n-1})\| \\ &\quad + L \|a'_{n-1} (x_{n-1} - T^{n-1} x_{n-1}) + b'_{n-1} (T^{n-1} y_{n-1} - T^{n-1} x_{n-1}) \\ &\quad + c'_{n-1} (v_{n-1} - T^{n-1} x_{n-1})\| \end{aligned}$$



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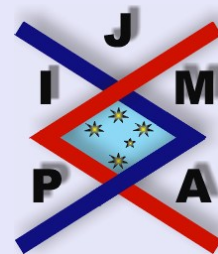
$$\begin{aligned}
&\leq \lambda_n + L^2 \|T^{n-1}y_{n-1} - x_{n-1}\| + L^2 c'_{n-1} \|v_{n-1} - x_{n-1}\| \\
&\quad + L \|x_{n-1} - T^{n-1}x_{n-1}\| + L^2 \|y_{n-1} - x_{n-1}\| + Lc'_{n-1} \|v_{n-1} - x_{n-1}\| \\
&\quad + Lc'_{n-1} \|x_{n-1} - T^{n-1}x_{n-1}\| \\
&= \lambda_n + L\lambda_{n-1} + L(1+L)c'_{n-1} \|v_{n-1} - x_{n-1}\| + L^2 \|T^{n-1}y_{n-1} - x_{n-1}\| \\
&\quad + L^2 \|y_{n-1} - x_{n-1}\| + Lc'_{n-1} \|x_{n-1} - T^{n-1}x_{n-1}\| \\
&\leq \lambda_n + 2L\lambda_{n-1} + L(1+L)c'_{n-1} \|v_{n-1} - x_{n-1}\| + L^2(1+L) \|y_{n-1} - x_{n-1}\| \\
&\quad + L^2 \|T^{n-1}x_{n-1} - x_{n-1}\| + Lc'_{n-1} \|x_{n-1} - T^{n-1}x_{n-1}\| \\
&= \lambda_n + L(1+L)\lambda_{n-1} + L(1+L)c'_{n-1} \|v_{n-1} - x_{n-1}\| \\
&\quad + L^2(1+L) \|b_{n-1}(T^{n-1}x_{n-1} - x_{n-1}) + c_{n-1}(u_{n-1} - x_{n-1})\| \\
&\quad + Lc'_{n-1} \|x_{n-1} - T^{n-1}x_{n-1}\| \\
&\leq \lambda_n + L(L^2 + 2L + 1)\lambda_{n-1} + L(1+L)c'_{n-1} \|v_{n-1} - x_{n-1}\| \\
&\quad + L^2(1+L)c_{n-1} \|u_{n-1} - x_{n-1}\| + Lc'_{n-1} \|x_{n-1} - T^{n-1}x_{n-1}\|,
\end{aligned}$$

completing the proof of Lemma 1. \square

Lemma 2.2. *Let $\{a_n\}$, $\{b_n\}$ and $\{\delta_n\}$ be sequences of nonnegative real numbers satisfying*

$$(2.2) \quad a_{n+1} \leq (1 + \delta_n)a_n + b_n, \quad n \geq 1.$$

If $\sum_{n=1}^{\infty} \delta_n < \infty$ and $\sum_{n=1}^{\infty} b_n < \infty$ then $\lim_{n \rightarrow \infty} a_n$ exists. If in addition $\{a_n\}$ has a subsequence which converges strongly to zero then $\lim_{n \rightarrow \infty} a_n = 0$.



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Proof. Observe that

$$\begin{aligned}
 a_{n+1} &\leq (1 + \delta_n)a_n + b_n \\
 &\leq (1 + \delta_n)[(1 + \delta_{n-1})a_{n-1} + b_{n-1}] + b_n \\
 &\leq \dots \leq \prod_{j=1}^n (1 + \delta_j)a_1 + \prod_{j=1}^n (1 + \delta_j) \sum_{j=1}^n b_j \\
 &\leq \dots \leq \prod_{j=1}^{\infty} (1 + \delta_j)a_1 + \prod_{j=1}^{\infty} (1 + \delta_j) \sum_{j=1}^{\infty} b_j < \infty.
 \end{aligned}$$

Hence $\{a_n\}$ is bounded. Let $M > 0$ be such that $a_n \leq M$, $n \geq 1$. Then

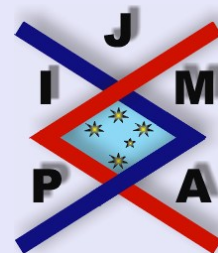
$$a_{n+1} \leq (1 + \delta_n)a_n + b_n \leq a_n + M\delta_n + b_n = a_n + \sigma_n$$

where $\sigma_n = M\delta_n + b_n$. It now follows from Lemma 2.1 of ([6, p. 303]) that $\lim_n a_n$ exists. Consequently, if $\{a_n\}$ has a subsequence which converges strongly to zero then $\lim_n a_n = 0$ completing the proof of Lemma 2.2. \square

Lemma 2.3. *Let E be a real Banach space and K a nonempty convex subset of E . Let $T : K \rightarrow K$ be uniformly L -Lipschitzian asymptotically demicontractive mapping with a sequence $\{k_n\} \subseteq [1, \infty)$, such that $\lim_n k_n = 1$, and $\sum_{n=1}^{\infty} (k_n^2 - 1) < \infty$. Let $\{a_n\}$, $\{b_n\}$, $\{c_n\}$, $\{a'_n\}$, $\{b'_n\}$, $\{c'_n\}$ be real sequences in $[0, 1]$ satisfying:*

$$(i) \quad a_n + b_n + c_n = 1 = a'_n + b'_n + c'_n,$$

$$(ii) \quad \sum_{n=1}^{\infty} b'_n = \infty,$$



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(iii) $\sum_{n=1}^{\infty} (b'_n)^2 < \infty$, $\sum_{n=1}^{\infty} c'_n < \infty$, $\sum_{n=1}^{\infty} b_n < \infty$, and $\sum_{n=1}^{\infty} c_n < \infty$.

Let $\{u_n\}$ and $\{v_n\}$ be bounded sequences in K and let $\{x_n\}$ be the sequence generated from an arbitrary $x_1 \in K$ by

$$\begin{aligned} y_n &= a_n x_n + b_n T^n x_n + c_n u_n, \quad n \geq 1, \\ x_{n+1} &= a'_n + b'_n T^n y_n + c'_n v_n, \quad n \geq 1, \end{aligned}$$

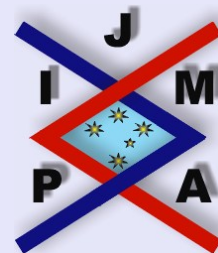
then $\liminf_n \|x_n - T x_n\| = 0$.

Proof. It is now well-known (see e.g. [1]) that for all $x, y \in E$, there exists $j(x + y) \in J(x + y)$ such that

$$(2.3) \quad \|x + y\|^2 \leq \|x\|^2 + 2\langle y, j(x + y) \rangle.$$

Let $x^* \in F(T)$ and let $M > 0$ be such that $\|u_n - x^*\| \leq M$, $\|v_n - x^*\| \leq M$, $n \geq 1$. Using (1.6) and (2.3) we obtain

$$\begin{aligned} & \|x_{n+1} - x^*\|^2 \\ &= \|(1 - b'_n - c'_n)x_n + b'_n T^n y_n + c'_n v_n - x^*\|^2 \\ &= \|(x_n - x^*) + b'_n(T^n y_n - x_n) + c'_n(v_n - x_n)\|^2 \\ &\leq \|x_n - x^*\|^2 + 2\langle b'_n(T^n y_n - x_n) + c'_n(v_n - x_n), j(x_{n+1} - x^*) \rangle \\ &= \|x_n - x^*\|^2 - 2b'_n \langle x_{n+1} - T^n x_{n+1}, j(x_{n+1} - x^*) \rangle \\ &\quad + 2b'_n \langle x_{n+1} - T^n x_{n+1}, j(x_{n+1} - x^*) \rangle \\ &\quad + 2b'_n \langle T^n y_n - x_n, j(x_{n+1} - x^*) \rangle + 2c'_n \langle v_n - x_n, j(x_{n+1} - x^*) \rangle \end{aligned}$$



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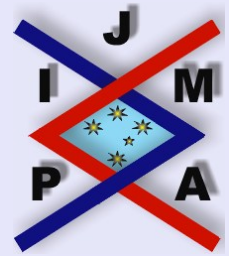


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$$\begin{aligned}
 &= \|(x_n - x^*)\|^2 - 2b'_n \langle x_{n+1} - T^n x_{n+1}, j(x_{n+1} - x^*) \rangle \\
 &\quad + 2b'_n \langle x_{n+1} - x_n, j(x_{n+1} - x^*) \rangle \\
 &\quad + 2b'_n \langle T^n y_n - T^n x_{n+1}, j(x_{n+1} - x^*) \rangle \\
 (2.4) \quad &\quad + 2c'_n \langle v_n - x_n, j(x_{n+1} - x^*) \rangle.
 \end{aligned}$$

Observe that

$$x_{n+1} - x_n = b'_n (T^n y_n - x_n) + c'_n (v_n - x_n).$$

Using this and (1.2) in (2.4) we have

$$\begin{aligned}
 &\|x_{n+1} - x^*\|^2 \\
 &\leq \|(x_n - x^*)\|^2 - b'_n (1 - k) \|x_{n+1} - T^n x_{n+1}\|^2 \\
 &\quad + b'_n (k_n^2 - 1) \|x_{n+1} - x^*\|^2 \\
 &\quad + 2(b'_n)^2 \langle T^n y_n - x_n, j(x_{n+1} - x^*) \rangle \\
 &\quad + 2b'_n \langle T^n y_n - T^n x_{n+1}, j(x_{n+1} - x^*) \rangle \\
 &\quad + 3c'_n \langle v_n - x_n, j(x_{n+1} - x^*) \rangle \\
 &\leq \|(x_n - x^*)\|^2 - b'_n (1 - k) \|x_{n+1} - T^n x_{n+1}\|^2 \\
 &\quad + (k_n^2 - 1) \|x_{n+1} - x^*\|^2 \\
 &\quad + 2(b'_n)^2 \|T^n y_n - x_n\| \|x_{n+1} - x^*\| \\
 &\quad + 2b'_n L \|x_{n+1} - y_n\| \|x_{n+1} - x^*\| \\
 &\quad + 3c'_n \|v_n - x_n\| \|x_{n+1} - x^*\| \\
 &= \|(x_n - x^*)\|^2 - b'_n (1 - k) \|x_{n+1} - T^n x_{n+1}\|^2 \\
 &\quad + (k_n^2 - 1) \|x_{n+1} - x^*\|^2 + [2(b'_n)^2 \|T^n y_n - x_n\| \\
 (2.5) \quad &\quad + 2b'_n L \|x_{n+1} - y_n\| + 3c'_n \|v_n - x_n\|] \|x_{n+1} - x^*\|.
 \end{aligned}$$

Observe that

$$\begin{aligned}
 \|y_n - x^*\| &= \|a_n(x_n - x^*) + b_n(T^n x_n - x^*) + c_n(u_n - x^*)\| \\
 &\leq \|x_n - x^*\| + L\|x_n - x^*\| + M \\
 (2.6) \qquad &= (1 + L)\|x_n - x^*\| + M,
 \end{aligned}$$

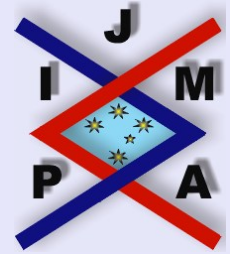
so that

$$\begin{aligned}
 \|T^n y_n - x_n\| &\leq L\|y_n - x^*\| + \|x_n - x^*\| \\
 &\leq L[(1 + L)\|x_n - x^*\| + M] + \|x_n - x^*\| \\
 (2.7) \qquad &\leq [1 + L(1 + L)]\|x_n - x^*\| + ML,
 \end{aligned}$$

$$\begin{aligned}
 \|x_{n+1} - x^*\| &= \|a'_n(x_n - x^*) + b'_n(T^n y_n - x^*) + c'_n(v_n - x^*)\| \\
 &\leq \|x_n - x^*\| + L\|y_n - x^*\| + M \\
 &\leq \|x_n - x^*\| + L[(1 + L)\|x_n - x^*\| + M] + M \\
 (2.8) \qquad &= [1 + L(1 + L)]\|x_n - x^*\| + (1 + L)M,
 \end{aligned}$$

and

$$\begin{aligned}
 \|x_{n+1} - y_n\| &= \|a'_n(x_n - y_n) + b'_n(T^n y_n - y_n) + c'_n(v_n - y_n)\| \\
 &\leq \|x_n - y_n\| + b'_n[\|T^n y_n - x^*\| + \|y_n - x^*\|] \\
 &\quad + c'_n[\|v_n - x^*\| + \|y_n - x^*\|] \\
 &= \|b_n(T^n x_n - x_n) + c_n(u_n - x_n)\| + b'_n[L\|y_n - x_n\| + \|y_n - x^*\|] \\
 &\quad + c'_n M + c'_n \|y_n - x^*\| \\
 &\leq b_n(1 + L)\|x_n - x^*\| + c_n M + c_n \|x_n - x^*\| \\
 &\quad + [b'_n(1 + L) + c'_n]\|y_n - x^*\| + c'_n M
 \end{aligned}$$



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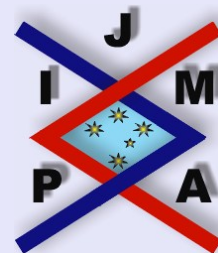
$$\begin{aligned}
&\leq [b_n(1+L) + c_n]\|x_n - x^*\| + c_n M \\
&\quad + [b'_n(1+L) + c'_n][(1+L)\|x_n - x^*\| + M] + c'_n M \\
(2.9) \quad &\leq \{[b_n(1+L) + c_n] + [b'_n(1+L) + c'_n](1+L)\}\|x_n - x^*\| \\
&\quad + M[b'_n(1+L) + 2c'_n + c_n].
\end{aligned}$$

Substituting (2.7)-(2.9) in (2.5) we obtain,

$$\begin{aligned}
&\|x_{n+1} - x^*\|^2 \\
&\leq \|(x_n - x^*)\|^2 - b'_n(1-k)\|x_{n+1} - T^n x_{n+1}\|^2 \\
&\quad + (k_n^2 - 1)\{[1 + L(1+L)]\|x_{n+1} - x^*\| + M(1+L)\}^2 \\
&\quad + \{(b'_n[[1 + L(1+L)]\|x_n - x^*\| + ML] + 3c'_n[M + \|x_n - x^*\|] \\
&\quad + 2b'_n L[[b_n(1+L) + c_n] + [b'_n(1+L + c'_n)](1+L)]\|x_n - x^*\| \\
&\quad + M[b'_n(1+L) + 2c'_n + c_n]\}\{[1 + L(1+L)]\|x_n - x^*\| + M(1+L)\} \\
&\leq \|(x_n - x^*)\|^2 - b'_n(1-k)\|x_{n+1} - T^n x_{n+1}\|^2 \\
&\quad + (k_n^2 - 1)\{[1 + L(1+L)]^2\|x_{n+1} - x^*\|^2 \\
&\quad + 2M(1+L)[1 + L(1+L)]\|x_n - x^*\| + M^2(1+L)^2\} \\
&\quad + 2(b'_n)^2[[1 + L(1+L)]\|x_n - x^*\| + ML][[1 + L(1+L)]\|x_n - x^*\| \\
&\quad + M(1+L)] + 3c'_n[M + \|x_n - x^*\|][[1 + L(1+L)]\|x_n - x^*\| \\
&\quad + M(1+L)] + 2b'_n L\{[[b_n(1+L) + c_n] \\
&\quad + [b'_n(1+L) + c'_n](1+L)]\|x_n - x^*\| \\
&\quad + M[b'_n(1+L) + 2c'_n + c_n]\}\{[1 + L(1+L)]\|x_n - x^*\| + M(1+L)\}.
\end{aligned}$$

Since $\|x_n - x^*\| \leq 1 + \|x_n - x^*\|^2$, we have

$$(2.10) \quad \|x_{n+1} - x^*\|^2 \leq [1 + \delta_n]\|x_n - x^*\|^2 + \sigma_n - b'_n(1-k)\|x_{n+1} - T^n x_{n+1}\|^2,$$



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where

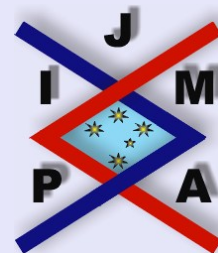
$$\begin{aligned} \delta_n = & (k_n^2 - 1)\{[1 + L(1 + L)]^2 + 2M(1 + L)[1 + L(1 + L)]\} \\ & + 2(b'_n)^2\{[1 + L(1 + L)]^2 + M(1 + L)[1 + L(1 + L)] \\ & + ML[1 + L(1 + L)]\} \\ & + 3c'_n\{[1 + L(1 + L)] + M[1 + L(1 + L)] + M(1 + L)\} \\ & + 2b'_nL\{[b_n(1 + L) + c_n] + [b'_n(1 + L) + c'_n](1 + L)\} \\ & \times \{[1 + L(1 + L)] + M(1 + L)\} \\ & + M[b'_n(1 + L) + 2c'_n + c_n][1 + L(1 + L)] \} \end{aligned}$$

and

$$\begin{aligned} \sigma_n = & (k_n^2 - 1)\{2M(1 + L)[1 + L(1 + L)] + M^2(1 + L)^2\} \\ & + 2(b'_n)^2\{[1 + L(1 + L)]M(1 + L) + ML[1 + L(1 + L)] + M^2L(1 + L)\} \\ & + 3c'_n\{M[1 + L(1 + L)] + M^2(1 + L) + M(1 + L)\} \\ & + 2b'_nL\{[b_n(1 + L) + c_n] + [b'_n(1 + L) + c'_n](1 + L)\}[M(1 + L)] \\ & + M[b'_n(1 + L) + 2c'_n + c_n][[1 + L(1 + L)] + M(1 + L)] \}. \end{aligned}$$

Since $\sum_{n=1}^{\infty} (k_n^2 - 1) < \infty$, condition (iii) implies that $\sum_{n=1}^{\infty} \delta_n < \infty$ and $\sum_{n=1}^{\infty} \sigma_n < \infty$. From (2.10) we obtain

$$\begin{aligned} \|x_{n+1} - x^*\|^2 & \leq [1 + \delta_n]\|x_n - x^*\|^2 + \sigma_n \\ & \leq \dots \leq \prod_{j=1}^n [1 + \delta_j]\|x_1 - x^*\|^2 + \prod_{j=1}^n [1 + \delta_j] \sum_{j=1}^n \sigma_j \\ & \leq \prod_{j=1}^{\infty} [1 + \delta_j]\|x_1 - x^*\|^2 + \prod_{j=1}^{\infty} [1 + \delta_j] \sum_{j=1}^{\infty} \sigma_j < \infty, \end{aligned}$$



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since $\sum_{n=1}^{\infty} \delta_n < \infty$ and $\sum_{n=1}^{\infty} \sigma_n < \infty$. Hence $\{\|x_n - x^*\|\}_{n=1}^{\infty}$ is bounded. Let $\|x_n - x^*\| \leq M$, $n \geq 1$. Then it follows from (2.10) that

$$(2.11) \quad \|x_{n+1} - x^*\|^2 \leq \|x_n - x^*\|^2 + M^2\delta_n + \sigma_n - b'_n(1-k)\|x_{n+1} - T^n x_{n+1}\|^2, \quad n \geq 1$$

Hence,

$$b'_n(1-k)\|x_{n+1} - T^n x_{n+1}\|^2 \leq \|x_n - x^*\|^2 - \|x_{n+1} - x^*\|^2 + \mu_n,$$

where $\mu_n = M^2\delta_n + \sigma_n$ so that,

$$(1-k) \sum_{j=1}^n b'_j \|x_{j+1} - T^j x_{j+1}\|^2 \leq \|x_1 - x^*\|^2 + \sum_{j=1}^n \mu_j < \infty,$$

Hence,

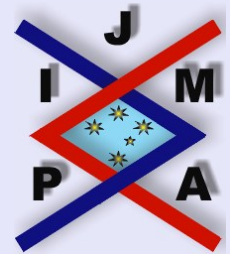
$$\sum_{n=1}^{\infty} b'_n \|x_{n+1} - T^n x_{n+1}\|^2 < \infty,$$

and condition (ii) implies that $\liminf_{n \rightarrow \infty} \|x_{n+1} - T^n x_{n+1}\| = 0$. Observe that

$$(2.12) \quad \begin{aligned} \|x_{n+1} - T^n x_{n+1}\|^2 &= \|(1 - b'_n - c'_n)x_n + b'_n T^n y_n + c'_n v_n - T^n x_{n+1}\|^2 \\ &= \|x_n - T^n x_n + b'_n(T^n y_n - x_n) + T^n x_n - T^n x_{n+1} \\ &\quad + c'_n(v_n - x_n)\|^2. \end{aligned}$$

For arbitrary $u, v \in E$, set $x = u + v$ and $y = -v$ in (2.3) to obtain

$$(2.13) \quad \|v + u\|^2 \geq \|u\|^2 + 2\langle v, j(u) \rangle.$$



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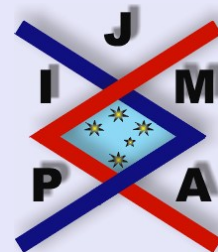
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From (2.12) and (2.13), we have

$$\begin{aligned} & \|x_{n+1} - T^n x_{n+1}\|^2 \\ &= \|x_n - T^n x_n + b'_n(T^n y_n - x_n) + T^n x_n - T^n x_{n+1} + c'_n(v_n - x_n)\|^2 \\ &\geq \|x_n - T^n x_n\|^2 + 2\langle b'_n(T^n y_n - x_n) + T^n x_n - T^n x_{n+1} \\ &\quad + c'_n(v_n - x_n), j(x_n - T^n x_n) \rangle. \end{aligned}$$

Hence

$$\begin{aligned} & \|x_n - T^n x_n\|^2 \\ &\leq \|x_{n+1} - T^n x_{n+1}\|^2 + 2\|b'_n(T^n y_n - x_n) \\ &\quad + T^n x_n - T^n x_{n+1} + c'_n(v_n - x_n)\| \|x_n - T^n x_n\| \\ &\leq \|x_{n+1} - T^n x_{n+1}\|^2 + 2\{b'_n\|T^n y_n - x_n\| + L\|x_{n+1} - x_n\| \\ &\quad + c'_n\|v_n - x_n\|\} \|x_n - T^n x_n\| \\ &\leq \|x_{n+1} - T^n x_{n+1}\|^2 + 2\{b'_n\|T^n y_n - x_n\| + Lb'_n\|T^n y_n - x_n\| \\ &\quad + Lc'_n\|v_n - x_n\| + c'_n\|v_n - x_n\|\} \|x_n - T^n x_n\| \\ &\leq \|x_{n+1} - T^n x_{n+1}\|^2 + 2(1+L)\|x_n - x^*\| \\ &\quad \times \{(1+L)b'_n\|T^n y_n - x_n\| + (1+L)c'_n\|v_n - x_n\|\} \\ &\leq \|x_{n+1} - T^n x_{n+1}\|^2 + 2(1+L)\|x_n - x^*\| \\ &\quad \times \{(1+L)b'_n[[1+L(1+L)]\|x_n - x^*\| + ML] \\ &\quad + (1+L)c'_n[M + \|x_n - x^*\|]\}, \quad (\text{using (2.6)}) \\ &\leq \|x_{n+1} - T^n x_{n+1}\|^2 \\ &\quad + 2(1+L)M\{(1+L)b'_n[[1+L(1+L)]M + ML] \\ &\quad + (1+L)c'_n[M + M]\}, \quad (\text{since } \|x_n - x^*\| \leq M) \end{aligned}$$



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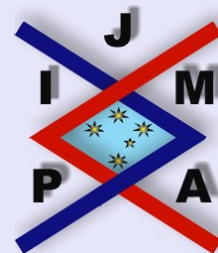


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$$(2.14) \quad = \|x_{n+1} - T^n x_{n+1}\|^2 + 2b'_n(1+L)^4 M^2 + 4c'_n(1+L)^2 M.$$

Since $\lim_{n \rightarrow \infty} b'_n = 0$, $\lim_{n \rightarrow \infty} c'_n = 0$ and $\liminf_{n \rightarrow \infty} \|x_{n+1} - T^n x_{n+1}\| = 0$, it follows from (2.14) that $\liminf_{n \rightarrow \infty} \|x_n - T^n x_n\| = 0$. It then follows from Lemma 1 that $\liminf_{n \rightarrow \infty} \|x_n - T x_n\| = 0$, completing the proof of Lemma 2.3. \square

Corollary 2.4. *Let E be a real Banach space and K a nonempty convex subset of E . Let $T : K \rightarrow K$ be a k -strictly asymptotically pseudocontractive map with $F(T) \neq \emptyset$ and sequence $\{k_n\} \subset [1, \infty)$ such that $\lim_{n \rightarrow \infty} k_n = 1$, $\sum_{n=1}^{\infty} (k_n^2 - 1) < \infty$. Let $\{a_n\}$, $\{b_n\}$, $\{c_n\}$, $\{a'_n\}$, $\{b'_n\}$, $\{c'_n\}$, $\{u_n\}$, and $\{v_n\}$ be as in Lemma 2.3 and let $\{x_n\}$ be the sequence generated from an arbitrary $x_1 \in K$ by*

$$\begin{aligned} y_n &= a_n x_n + b_n T^n x_n + c_n u_n, \quad n \geq 1, \\ x_{n+1} &= a'_n + b'_n T^n y_n + c'_n v_n, \quad n \geq 1, \end{aligned}$$

Then $\liminf_{n \rightarrow \infty} \|x_n - T x_n\| = 0$.

Proof. From (1.1) we obtain

$$\begin{aligned} & \|(I - T^n)x - (I - T^n)y\| \|x - y\| \\ & \geq \frac{1}{2} \{ (1 - k) \|(I - T^n)x - (I - T^n)y\|^2 - (k_n^2 - 1) \|x - y\|^2 \} \\ & = \frac{1}{2} [\sqrt{1 - k} \|(I - T^n)x - (I - T^n)y\| \\ & \quad + \sqrt{k_n^2 - 1} \|x - y\|] [\sqrt{1 - k} \|(I - T^n)x - (I - T^n)y\| - \sqrt{k^2 - 1} \|x - y\|] \end{aligned}$$

$$\geq \frac{1}{2}[\sqrt{1-k}\|(I-T^n)x - (I-T^n)y\|$$

$$[\sqrt{1-k}\|(I-T^n)x - (I-T^n)y\| - \sqrt{k^2-1}\|x-y\|]$$

so that

$$\frac{1}{2}\sqrt{1-k}[\sqrt{1-k}\|(I-T^n)x - (I-T^n)y\| - \sqrt{k^2-1}\|x-y\|] \leq \|x-y\|.$$

Hence

$$\|(I-T^n)x - (I-T^n)y\| \leq \left[\frac{2 + \sqrt{\{(1-k)(k_n^2-1)\}}}{1-k} \right] \|x-y\|.$$

Furthermore,

$$\|T^n x - T^n y\| - \|x-y\| \leq \|(I-T^n)x - (I-T^n)y\|$$

$$\leq \left[\frac{2 + \sqrt{\{(1-k)(k_n^2-1)\}}}{1-k} \right] \|x-y\|,$$

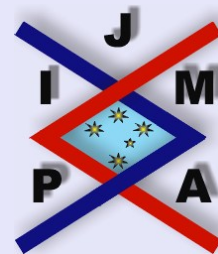
from which it follows that

$$\|T^n x - T^n y\| \leq \left[1 + \frac{2 + \sqrt{\{(1-k)(k_n^2-1)\}}}{1-k} \right] \|x-y\|.$$

Since $\{k_n\}$ is bounded, let $k_n \leq D, \forall n \geq 1$. Then

$$\|T^n x - T^n y\| \leq \left[1 + \frac{2 + \sqrt{\{(1-k)(D^2-1)\}}}{1-k} \right] \|x-y\|$$

$$\leq L\|x-y\|,$$



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where

$$L = 1 + \frac{2 + \sqrt{\{(1 - k)(D^2 - 1)\}}}{1 - k}.$$

Hence T is uniformly L -Lipschitzian. Since $F(T) \neq \emptyset$, then T is uniformly L -Lipschitzian and asymptotically demicontractive and hence the result follows from Lemma 2.3. \square

Remark 2.1. It is shown in [3] that if E is a Hilbert space and $T : K \rightarrow K$ is k -asymptotically pseudocontractive with sequence $\{k_n\}$ then

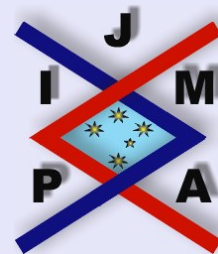
$$\|T^n x - T^n y\| \leq \frac{D + \sqrt{k}}{1 - \sqrt{k}} \|x - y\| \quad \forall x, y \in K, \text{ where } k_n \leq D, \forall n \geq 1.$$

Theorem 2.5. Let E be a real Banach space and K a nonempty closed convex subset of E . Let $T : K \rightarrow K$ be a completely continuous uniformly L -Lipschitzian asymptotically demicontractive mapping with sequence $\{k_n\} \subset [1, \infty)$ such that $\lim_n k_n = 1$ and $\sum_{n=1}^{\infty} (k_n^2 - 1) < \infty$. Let $\{a_n\}, \{b_n\}, \{c_n\}, \{a'_n\}, \{b'_n\}, \{c'_n\}, \{u_n\}$, and $\{v_n\}$ be as in Lemma 2.3. Then the sequence $\{x_n\}$ generated from an arbitrary $x_1 \in K$ by

$$\begin{aligned} y_n &= a_n x_n + b_n T^n x_n + c_n u_n, \quad n \geq 1, \\ x_{n+1} &= a'_n x_n + b'_n T^n y_n + c'_n v_n, \quad n \geq 1, \end{aligned}$$

converges strongly to a fixed point of T .

Proof. From Lemma 2.3, $\liminf_n \|x_n - T^n x_n\| = 0$, hence there exists a subsequence $\{x_{n_j}\}$ of $\{x_n\}$ such that $\lim_n \|x_{n_j} - T x_{n_j}\| = 0$.



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Since $\{x_{n_j}\}$ is bounded and T is completely continuous, then $\{Tx_{n_j}\}$ has a subsequence $\{Tx_{j_k}\}$ which converges strongly. Hence $\{x_{n_{j_k}}\}$ converges strongly. Suppose $\lim_{k \rightarrow \infty} x_{n_{j_k}} = p$. Then $\lim_{k \rightarrow \infty} Tx_{n_{j_k}} = Tp$. $\lim_{k \rightarrow \infty} \|x_{n_{j_k}} - Tx_{n_{j_k}}\| = \|p - Tp\| = 0$ so that $p \in F(T)$. It follows from (2.11) that

$$\|x_{n+1} - p\|^2 \leq \|x_n - p\|^2 + \mu_n$$

Lemma 2.2 now implies $\lim_{n \rightarrow \infty} \|x_n - p\| = 0$ completing the proof of Theorem 2.5. □

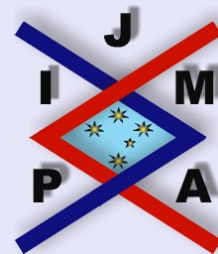
Corollary 2.6. *Let E be an arbitrary real Banach space and K a nonempty closed convex subset of E . let $T : K \rightarrow K$ be a k -strictly asymptotically pseudocontractive mapping with $F(T) \neq \emptyset$ and sequence $\{k_n\} \subset [1, \infty)$ such that $\lim_n k_n = 1$, and $\sum_{n=1}^{\infty} (k_n^2 - 1) < \infty$. Let $\{a_n\}, \{b_n\}, \{c_n\}, \{a'_n\}, \{b'_n\}, \{c'_n\}, \{u_n\}$, and $\{v_n\}$ be as in Lemma 2.3. Then the sequence $\{x_n\}$ generated from an arbitrary $x_1 \in K$ by*

$$\begin{aligned} y_n &= a_n x_n + b_n T^n x_n + c_n u_n, \quad n \geq 1, \\ x_{n+1} &= a'_n x_n + b'_n T^n y_n + c'_n v_n, \quad n \geq 1, \end{aligned}$$

converges strongly to a fixed point of T .

Proof. As shown in Corollary 2.4, T is uniformly L -Lipschitzian and since $F(T) \neq \emptyset$ then T is asymptotically demicontractive and the result follows from Theorem 2.5. □

Remark 2.2. *If we set $b_n = c_n = 0, \forall n \geq 1$ in Lemma 2.3, Theorem 2.5 and Corollaries 2.4 and 2.6, we obtain the corresponding results for the modified Mann iteration method with errors in the sense of Xu [7].*



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Remark 2.3. *Theorem 2.5 extends the results of Osilike [3] (which is itself a generalization of a theorem of Qihou [4]) from real q -uniformly smooth Banach space to arbitrary real Banach space.*

Furthermore, our Theorem 2.5 is proved without the boundedness condition imposed on the subset K in ([3, 4]) and using the more general modified Ishikawa Iteration method with errors in the sense of Xu [7]. Also our iteration parameters $\{a_n\}$, $\{b_n\}$, $\{c_n\}$, $\{a'_n\}$, $\{b'_n\}$, $\{c'_n\}$, $\{u_n\}$, and $\{v_n\}$ are completely independent of any geometric properties of underlying Banach space.

Remark 2.4. *Prototypes for our iteration parameters are:*

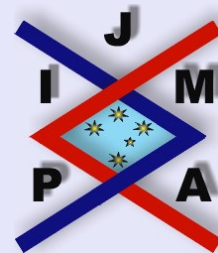
$$b'_n = \frac{1}{3(n+1)}, c'_n = \frac{1}{3(n+1)^2}, a'_n = 1 - (b'_n + c'_n),$$

$$b_n = c_n = \frac{1}{3(n+1)^2}, a_n = 1 - \frac{1}{3(n+1)^2}, n \geq 1.$$

The proofs of the following theorems and corollaries for the Ishikawa iteration method with errors in the sense of Liu [2] are omitted because the proofs follow by a straightforward modifications of the proofs of the corresponding results for the Ishikawa iteration method with errors in the sense of Xu [7].

Theorem 2.7. *Let E be a real Banach space and let $T : E \rightarrow E$ be a uniformly L -Lipschitzian asymptotically demicontractive mapping with sequence $\{k_n\} \subset [1, \infty)$ such that $\lim_n k_n = 1$, and $\sum_{n=1}^{\infty} (k_n^2 - 1) < \infty$. Let $\{u_n\}$ and $\{v_n\}$ be sequences in E such that $\sum_{n=1}^{\infty} \|u_n\| < \infty$ and $\sum_{n=1}^{\infty} \|v_n\| < \infty$, and let $\{\alpha_n\}$ and $\{\beta_n\}$ be sequences in $[0, 1]$ satisfying the conditions:*

(i) $0 \leq \alpha_n, \beta_n \leq 1, n \geq 1;$



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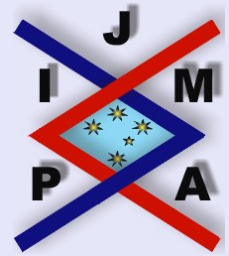


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$$(ii) \sum_{n=1}^{\infty} \alpha_n = \infty$$

$$(iii) \sum_{n=1}^{\infty} \alpha_n^2 < \infty \text{ and } \sum_{n=1}^{\infty} \beta_n < \infty.$$

Let $\{x_n\}$ be the sequence generated from an arbitrary $x_1 \in E$ by

$$y_n = (1 - \beta_n)x_n + \beta_n T^n x_n + u_n, \quad n \geq 1,$$

$$x_{n+1} = (1 - \alpha_n)x_n + \alpha_n T^n y_n + v_n, \quad n \geq 1,$$

Then $\liminf_{n \rightarrow \infty} \|x_n - T x_n\| = 0$.

Corollary 2.8. Let E be a real Banach space and let $T : E \rightarrow E$ be a k -strictly asymptotically pseudocontractive map with $F(T) \neq \emptyset$ and sequence $\{k_n\} \subset [1, \infty)$ such that $\lim_n k_n = 1$, and $\sum_{n=1}^{\infty} (k_n^2 - 1) < \infty$. Let $\{u_n\}$, $\{v_n\}$, $\{\alpha_n\}$ and $\{\beta_n\}$ be as in Theorem 2.7 and let $\{x_n\}$ be the sequence generated from an arbitrary $x_1 \in E$ by

$$y_n = (1 - \beta_n)x_n + \beta_n T^n x_n + u_n, \quad n \geq 1,$$

$$x_{n+1} = (1 - \alpha_n)x_n + \alpha_n T^n y_n + v_n, \quad n \geq 1,$$

Then $\liminf_{n \rightarrow \infty} \|x_n - T x_n\| = 0$.

Theorem 2.9. Let E , T , $\{u_n\}$, $\{v_n\}$, $\{\alpha_n\}$ and $\{\beta_n\}$ be as in Theorem 2.7. If in addition $T : E \rightarrow E$ is completely continuous then the sequence $\{x_n\}$ generated from an arbitrary $x_1 \in E$ by

$$y_n = (1 - \beta_n)x_n + \beta_n T^n x_n + u_n, \quad n \geq 1,$$

$$x_{n+1} = (1 - \alpha_n)x_n + \alpha_n T^n y_n + v_n, \quad n \geq 1,$$

converges strongly to a fixed point of T .

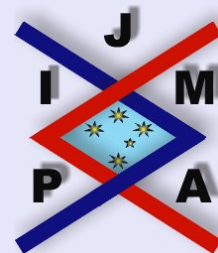
Corollary 2.10. *Let $E, T, \{u_n\}, \{v_n\}, \{\alpha_n\}$ and $\{\beta_n\}$ be as in Corollary 2.8. If in addition T is completely continuous, then the sequence $\{x_n\}$ generated from an arbitrary $x, y \in E$ by*

$$\begin{aligned} y_n &= (1 - \beta_n)x_n + \beta_n T^n x_n + u_n, \quad n \geq 1, \\ x_{n+1} &= (1 - \alpha_n)x_n + \alpha_n T^n y_n + v_n, \quad n \geq 1, \end{aligned}$$

converges strongly to a fixed point of T .

Remark 2.5. (a) *If K is a nonempty closed convex subset of E and $T : K \rightarrow K$, then Theorems 2.7 and 2.9 and Corollaries 2.8 and 2.10 also hold provided that in each case the sequence $\{x_n\}$ lives in K .*

(b) *If we set $\beta_n = 0, \forall n \geq 1$ in Theorems 2.7 and 2.9 and Corollaries 2.8 and 2.10, we obtain the corresponding results for modified Mann iteration method with errors in the sense of Liu [2].*



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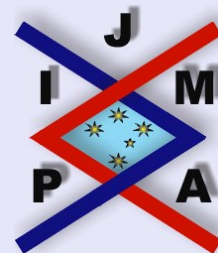
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