



# INCLUSION AND NEIGHBORHOOD PROPERTIES FOR CERTAIN CLASSES OF MULTIVALENTLY ANALYTIC FUNCTIONS ASSOCIATED WITH THE CONVOLUTION STRUCTURE

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**Multivalently Analytic Functions**

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vol. 8, iss. 1, art. 7, 2007

[Title Page](#)

[Contents](#)

[⏪](#) [⏩](#)

[◀](#) [▶](#)

Page 1 of 19

[Go Back](#)

[Full Screen](#)

[Close](#)

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vol. 8, iss. 1, art. 7, 2007

---

Title Page

Contents



Page 2 of 19

Go Back

Full Screen

Close

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*Abstract:*

Making use of the familiar convolution structure of analytic functions, in this paper we introduce and investigate two new subclasses of multivalently analytic functions of complex order. Among the various results obtained here for each of these function classes, we derive the coefficient bounds and coefficient inequalities, and inclusion and neighborhood properties, involving multivalently analytic functions belonging to the function classes introduced here.

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# Contents

1	Introduction, Definitions and Preliminaries	4
2	Coefficient Bounds and Coefficient Inequalities	9
3	Inclusion Properties	12
4	Neighborhood Properties	15



Multivalently Analytic Functions

J.K. Prajapat, R.K. Raina and

H.M. Srivastava

vol. 8, iss. 1, art. 7, 2007

Title Page

Contents



Page 3 of 19

Go Back

Full Screen

Close

journal of **inequalities**  
in pure and applied  
mathematics

issn: 1443-5756



Title Page

Contents

◀◀ ▶▶

◀ ▶

Page 4 of 19

Go Back

Full Screen

Close

# 1. Introduction, Definitions and Preliminaries

Let  $\mathcal{A}_p(n)$  denote the class of functions of the form:

$$(1.1) \quad f(z) = z^p + \sum_{k=n}^{\infty} a_k z^k \quad (p < n; n, p \in \mathbb{N} := \{1, 2, 3, \dots\}),$$

which are analytic and  $p$ -valent in the open unit disk

$$\mathbb{U} = \{z : z \in \mathbb{C} \text{ and } |z| < 1\}.$$

If  $f \in \mathcal{A}_p(n)$  is given by (1.1) and  $g \in \mathcal{A}_p(n)$  is given by

$$g(z) = z^p + \sum_{k=n}^{\infty} b_k z^k,$$

then the Hadamard product (or convolution)  $f * g$  of  $f$  and  $g$  is defined (as usual) by

$$(1.2) \quad (f * g)(z) := z^p + \sum_{k=n}^{\infty} a_k b_k z^k =: (g * f)(z).$$

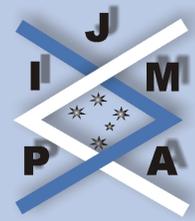
We denote by  $\mathcal{T}_p(n)$  the subclass of  $\mathcal{A}_p(n)$  consisting of functions of the form:

$$(1.3) \quad f(z) = z^p - \sum_{k=n}^{\infty} a_k z^k \quad (p < n; a_k \geq 0 \ (k \geq n); n, p \in \mathbb{N}),$$

which are  $p$ -valent in  $\mathbb{U}$ .

For a given function  $g(z) \in \mathcal{A}_p(n)$  defined by

$$(1.4) \quad g(z) = z^p + \sum_{k=n}^{\infty} b_k z^k \quad (p < n; b_k \geq 0 \ (k \geq n); n, p \in \mathbb{N}),$$



Title Page

Contents



Page 5 of 19

Go Back

Full Screen

Close

we introduce here a new class  $\mathcal{S}_g(p, n, b, m)$  of functions belonging to the subclass of  $\mathcal{T}_p(n)$ , which consists of functions  $f(z)$  of the form (1.3) satisfying the following inequality:

$$(1.5) \quad \left| \frac{1}{b} \left( \frac{z(f * g)^{(m+1)}(z)}{(f * g)^{(m)}(z)} - (p - m) \right) \right| < 1$$

$$(z \in \mathbb{U}; p \in \mathbb{N}; m \in \mathbb{N}_0; p > m; b \in \mathbb{C} \setminus \{0\}).$$

We note that there are several interesting new or known subclasses of our function class  $\mathcal{S}_g(p, n, b, m)$ . For example, if we set

$$m = 0 \quad \text{and} \quad b = p(1 - \alpha) \quad (p \in \mathbb{N}; 0 \leq \alpha < 1)$$

in (1.5), then  $\mathcal{S}_g(p, n, b, m)$  reduces to the class studied very recently by Ali *et al.* [1]. On the other hand, if the coefficients  $b_k$  in (1.4) are chosen as follows:

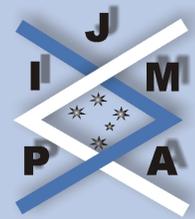
$$b_k = \binom{\lambda + k - 1}{k - p} \quad (\lambda > -p),$$

and  $n$  is replaced by  $n + p$  in (1.2) and (1.3), then we obtain the class  $\mathcal{H}_{n,m}^p(\lambda, b)$  of  $p$ -valently analytic functions (involving the familiar Ruscheweyh derivative operator), which was investigated by Raina and Srivastava [9]. Further, upon setting  $p = 1$  and  $n = 2$  in (1.2) and (1.3), if we choose the coefficients  $b_k$  in (1.4) as follows:

$$b_k = k^l \quad (l \in \mathbb{N}_0),$$

then the class  $\mathcal{S}_g(1, 2, 1 - \alpha, 0)$  would reduce to the function class  $\mathcal{TS}_l^*(\alpha)$  (involving the familiar Sălăgean derivative operator [11]), which was studied in [1]. Moreover, when

$$(1.6) \quad g(z) = z^p + \sum_{k=n}^{\infty} \frac{(\alpha_1)_{k-p} \cdots (\alpha_q)_{k-p}}{(\beta_1)_{k-p} \cdots (\beta_s)_{k-p} (k-p)!} z^k$$



Title Page

Contents

◀◀ ▶▶

◀ ▶

Page 6 of 19

Go Back

Full Screen

Close

$$(\alpha_j \in \mathbb{C} \ (j = 1, \dots, q); \beta_j \in \mathbb{C} \setminus \{0, -1, -2, \dots\} \ (j = 1, \dots, s)),$$

with the parameters

$$\alpha_1, \dots, \alpha_q \quad \text{and} \quad \beta_1, \dots, \beta_s$$

being so chosen that the coefficients  $b_k$  in (1.4) satisfy the following condition:

$$b_k = \frac{(\alpha_1)_{k-p} \cdots (\alpha_q)_{k-p}}{(\beta_1)_{k-p} \cdots (\beta_s)_{k-p} (k-p)!} \geq 0,$$

then the class  $\mathcal{S}_g(p, n, b, m)$  transforms into a (presumably) new class  $\mathcal{S}^*(p, n, b, m)$  defined by

$$(1.7) \quad \mathcal{S}^*(p, n, b, m)$$

$$:= \left\{ f : f \in \mathcal{T}_p(n) \quad \text{and} \quad \left| \frac{1}{b} \left( \frac{z(H_s^q[\alpha_1]f)^{(m+1)}(z)}{(H_s^q[\alpha_1]f)^{(m)}(z)} - (p-m) \right) \right| < 1 \right\}$$

$$(z \in \mathbb{U}; q \leq s+1; m, q, s \in \mathbb{N}_0; p \in \mathbb{N}; b \in \mathbb{C} \setminus \{0\}).$$

The operator

$$(H_s^q[\alpha_1]f)(z) := H_s^q(\alpha_1, \dots, \alpha_q; \beta_1, \dots, \beta_s)f(z),$$

involved in the definition (1.7), is the Dziok-Srivastava linear operator (see, for details, [4]; see also [5] and [6]), which contains such well-known operators as the Hohlov linear operator, Saitoh's generalized linear operator, the Carlson-Shaffer linear operator, the Ruscheweyh derivative operator as well as its generalized version, the Bernardi-Libera-Livingston operator, and the Srivastava-Owa fractional derivative operator. One may refer to the papers [4] to [6] for further details and references for these operators. The Dziok-Srivastava linear operator defined in [4] was further extended by Dziok and Raina [2] (see also [3] and [8]).



Following a recent investigation by Frasin and Darus [7], if  $f(z) \in \mathcal{T}_p(n)$  and  $\delta \geq 0$ , then we define the  $(q, \delta)$ -neighborhood of the function  $f(z)$  by

$$(1.8) \quad \mathcal{N}_{n,\delta}^q(f) := \left\{ h : h \in \mathcal{T}_p(n), h(z) = z^p - \sum_{k=n}^{\infty} c_k z^k \text{ and } \sum_{k=n}^{\infty} k^{q+1} |a_k - c_k| \leq \delta \right\}.$$

It follows from the definition (1.8) that, if

$$(1.9) \quad e(z) = z^p \quad (p \in \mathbb{N}),$$

then

$$(1.10) \quad \mathcal{N}_{n,\delta}^q(e) = \left\{ h : h \in \mathcal{T}_p(n), h(z) = z^p - \sum_{k=n}^{\infty} c_k z^k \text{ and } \sum_{k=n}^{\infty} k^{q+1} |c_k| \leq \delta \right\}.$$

We observe that

$$\mathcal{N}_{2,\delta}^0(f) = \mathcal{N}_\delta(f)$$

and

$$\mathcal{N}_{2,\delta}^1(f) = \mathcal{M}_\delta(f),$$

where  $\mathcal{N}_\delta(f)$  and  $\mathcal{M}_\delta(f)$  denote, respectively, the  $\delta$ -neighborhoods of the function

$$(1.11) \quad f(z) = z - \sum_{k=2}^{\infty} a_k z^k \quad (a_k \geq 0; z \in \mathbb{U}),$$

as defined by Ruscheweyh [10] and Silverman [12].

Title Page

Contents

◀◀ ▶▶

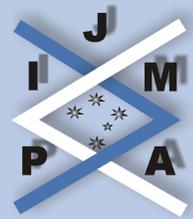
◀ ▶

Page 7 of 19

Go Back

Full Screen

Close



[Title Page](#)

[Contents](#)

[◀◀](#) [▶▶](#)

[◀](#) [▶](#)

Page 8 of 19

[Go Back](#)

[Full Screen](#)

[Close](#)

Finally, for a given function

$$g(z) = z^p + \sum_{k=n}^{\infty} b_k z^k \in \mathcal{A}_p(n) \quad (b_k > 0 \quad (k \geq n)),$$

let  $\mathcal{P}_g(p, n, b, m; \mu)$  denote the subclass of  $\mathcal{T}_p(n)$  consisting of functions  $f(z)$  of the form (1.3) which satisfy the following inequality:

$$(1.12) \quad \left| \frac{1}{b} \left[ p(1 - \mu) \left( \frac{(f * g)(z)}{z} \right)^{(m)} + \mu(f * g)^{(m+1)}(z) - (p - m) \right] \right| < p - m$$

$$(z \in \mathbb{U}; m \in \mathbb{N}_0; p \in \mathbb{N}; p > m; b \in \mathbb{C} \setminus \{0\}; \mu \geq 0).$$

Our object in the present paper is to investigate the various properties and characteristics of functions belonging to the above-defined subclasses

$$\mathcal{S}_g(p, n, b, m) \quad \text{and} \quad \mathcal{P}_g(p, n, b, m; \mu)$$

of  $p$ -valently analytic functions in  $\mathbb{U}$ . Apart from deriving coefficient bounds and coefficient inequalities for each of these function classes, we establish several inclusion relationships involving the  $(n, \delta)$ -neighborhoods of functions belonging to these subclasses.



Title Page

Contents



Page 9 of 19

Go Back

Full Screen

Close

## 2. Coefficient Bounds and Coefficient Inequalities

We begin by proving a necessary and sufficient condition for the function  $f(z) \in \mathcal{T}_p(n)$  to be in each of the classes

$$\mathcal{S}_g(p, n, b, m) \quad \text{and} \quad \mathcal{P}_g(p, n, b, m; \mu).$$

**Theorem 1.** *Let  $f(z) \in \mathcal{T}_p(n)$  be given by (1.3). Then  $f(z)$  is in the class  $\mathcal{S}_g(p, n, b, m)$  if and only if*

$$(2.1) \quad \sum_{k=n}^{\infty} a_k b_k (k - p + |b|) \binom{k}{m} \leq |b| \binom{p}{m}.$$

*Proof.* Assume that  $f(z) \in \mathcal{S}_g(p, n, b, m)$ . Then, in view of (1.3) to (1.5), we obtain

$$\Re \left( \frac{z(f * g)^{(m+1)}(z) - (p - m)(f * g)^{(m)}(z)}{(f * g)^{(m)}(z)} \right) > -|b| \quad (z \in \mathbb{U}),$$

which yields

$$(2.2) \quad \Re \left( \frac{\sum_{k=n}^{\infty} a_k b_k (p - k) \binom{k}{m} z^{k-p}}{\binom{p}{m} - \sum_{k=n}^{\infty} a_k b_k \binom{k}{m} z^{k-p}} \right) > -|b| \quad (z \in \mathbb{U}).$$

Putting  $z = r$  ( $0 \leq r < 1$ ) in (2.2), the expression in the denominator on the left-hand side of (2.2) remains positive for  $r = 0$  and also for all  $r \in (0, 1)$ . Hence, by letting  $r \rightarrow 1-$ , the inequality (2.2) leads us to the desired assertion (2.1) of Theorem 1.

Conversely, by applying the hypothesis (2.1) of Theorem 1, and setting  $|z| = 1$ , we find that

$$\begin{aligned} \left| \frac{z(f * g)^{(m+1)}(z)}{(f * g)^{(m)}(z)} - (p - m) \right| &= \left| \frac{\sum_{k=n}^{\infty} a_k b_k (k - p) \binom{k}{m} z^{k-m}}{\binom{p}{m} z^{p-m} - \sum_{k=n}^{\infty} a_k b_k \binom{k}{m} z^{k-m}} \right| \\ &\leq \frac{|b| \left[ \binom{p}{m} - \sum_{k=n}^{\infty} a_k b_k \binom{k}{m} \right]}{\binom{p}{m} - \sum_{k=n}^{\infty} a_k b_k \binom{k}{m}} \\ &= |b|. \end{aligned}$$

Hence, by the *maximum modulus principle*, we infer that  $f(z) \in \mathcal{S}_g(p, n, b, m)$ , which completes the proof of Theorem 1. ■

**Remark 1.** *In the special case when*

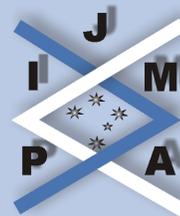
$$(2.3) \quad b_k = \binom{\lambda + k - 1}{k - p} \quad (\lambda > -p; k \geq n; n, p \in \mathbb{N}; n \mapsto n + p),$$

*Theorem 1 corresponds to the result given recently by Raina and Srivastava [9, p. 3, Theorem 1]. Furthermore, if we set*

$$(2.4) \quad m = 0 \quad \text{and} \quad b = p(1 - \alpha) \quad (p \in \mathbb{N}; 0 \leq \alpha < 1),$$

*Theorem 1 yields a recently established result due to Ali et al. [1, p. 181, Theorem 1].*

The following result involving the function class  $\mathcal{P}_g(p, n, b, m; \mu)$  can be proved on similar lines as detailed above for Theorem 1.



**Multivalently Analytic Functions**

J.K. Prajapat, R.K. Raina and

H.M. Srivastava

vol. 8, iss. 1, art. 7, 2007

[Title Page](#)

[Contents](#)



Page 10 of 19

[Go Back](#)

[Full Screen](#)

[Close](#)

journal of **inequalities**  
in pure and applied  
mathematics

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**Theorem 2.** Let  $f(z) \in \mathcal{T}_p(n)$  be given by (1.3). Then  $f(z)$  is in the class  $\mathcal{P}_g(p, n, b, m; \mu)$  if and only if

$$(2.5) \quad \sum_{k=n}^{\infty} a_k b_k [\mu(k-p) + p] \binom{k-1}{m} \leq (p-m) \left[ \frac{|b|-1}{m!} + \binom{p}{m} \right].$$

**Remark 2.** Making use of the same substitutions as mentioned above in (2.3), Theorem 2 yields the corrected version of another known result due to Raina and Srivastava [9, p. 4, Theorem 2].

Multivalently Analytic Functions

J.K. Prajapat, R.K. Raina and

H.M. Srivastava

vol. 8, iss. 1, art. 7, 2007

Title Page

Contents



Page 11 of 19

Go Back

Full Screen

Close

journal of **inequalities**  
in pure and applied  
mathematics

issn: 1443-5756



Title Page

Contents



Page 12 of 19

Go Back

Full Screen

Close

### 3. Inclusion Properties

We now establish some inclusion relationships for each of the function classes

$$\mathcal{S}_g(p, n, b, m) \quad \text{and} \quad \mathcal{P}_g(p, n, b, m; \mu)$$

involving the  $(n, \delta)$ -neighborhood defined by (1.8).

**Theorem 3.** *If*

$$(3.1) \quad b_k \geq b_n \quad (k \geq n) \quad \text{and} \quad \delta := \frac{n|b|\binom{p}{m}}{(n-p+|b|)\binom{n}{m}b_n} \quad (p > |b|),$$

then

$$(3.2) \quad \mathcal{S}_g(p, n, b, m) \subset \mathcal{N}_{n,\delta}^0(e).$$

*Proof.* Let  $f(z) \in \mathcal{S}_g(p, n, b, m)$ . Then, in view of the assertion (2.1) of Theorem 1, and the given condition that

$$b_k \geq b_n \quad (k \geq n),$$

we get

$$(n-p+|b|)\binom{n}{m}b_n \sum_{k=n}^{\infty} a_k \leq \sum_{k=n}^{\infty} a_k b_k (k-p+|b|)\binom{k}{m} < |b|\binom{p}{m},$$

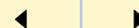
which implies that

$$(3.3) \quad \sum_{k=n}^{\infty} a_k \leq \frac{|b|\binom{p}{m}}{(n-p+|b|)\binom{n}{m}b_n}.$$



Title Page

Contents



Page 13 of 19

Go Back

Full Screen

Close

Applying the assertion (2.1) of Theorem 1 again, in conjunction with (3.3), we obtain

$$\begin{aligned} \binom{n}{m} b_n \sum_{k=n}^{\infty} k a_k &\leq |b| \binom{p}{m} + (p - |b|) \binom{n}{m} b_n \sum_{k=n}^{\infty} a_k \\ &\leq |b| \binom{p}{m} + (p - |b|) \binom{n}{m} b_n \frac{|b| \binom{p}{m}}{(n - p + |b|) \binom{n}{m} b_n} \\ &= \frac{n|b| \binom{p}{m}}{n - p + |b|}. \end{aligned}$$

Hence

$$(3.4) \quad \sum_{k=n}^{\infty} k a_k \leq \frac{n|b| \binom{p}{m}}{(n - p + |b|) \binom{n}{m} b_n} =: \delta \quad (p > |b|),$$

which, by virtue of (1.10), establishes the inclusion relation (3.2) of Theorem 3. ■

In an analogous manner, by applying the assertion (2.5) of Theorem 2, instead of the assertion (2.1) of Theorem 1, to the functions in the class  $\mathcal{P}_g(p, n, b, m; \mu)$ , we can prove the following inclusion relationship.

**Theorem 4.** *If*

$$(3.5) \quad b_k \geq b_n \quad (k \geq n) \quad \text{and} \quad \delta := \frac{n(p - m) \left[ \frac{|b|-1}{m!} + \binom{p}{m} \right]}{[\mu(n - p) + p] \binom{n-1}{m} b_n} \quad (\mu > 1),$$

then

$$(3.6) \quad \mathcal{P}_g(p, n, b, m; \mu) \subset \mathcal{N}_{n,\delta}^0(e).$$

**Remark 3.** Applying the parametric substitutions listed in (2.3), Theorem 3 yields a known result of Raina and Srivastava [9, p. 4, Theorem 3], while Theorem 4 would yield the corrected form of another known result [9, p. 5, Theorem 4].



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**Multivalently Analytic Functions**

J.K. Prajapat, R.K. Raina and

H.M. Srivastava

vol. 8, iss. 1, art. 7, 2007

---

Title Page

Contents



Page 14 of 19

Go Back

Full Screen

Close

journal of **inequalities**  
in pure and applied  
mathematics

issn: 1443-5756



Title Page

Contents



Page 15 of 19

Go Back

Full Screen

Close

## 4. Neighborhood Properties

In this concluding section, we determine the neighborhood properties for each of the function classes

$$\mathcal{S}_g^{(\alpha)}(p, n, b, m) \quad \text{and} \quad \mathcal{P}_g^{(\alpha)}(p, n, b, m; \mu),$$

which are defined as follows.

A function  $f(z) \in \mathcal{T}_p(n)$  is said to be in the class  $\mathcal{S}_g^{(\alpha)}(p, n, b, m)$  if there exists a function  $h(z) \in \mathcal{S}_g(p, n, b, m)$  such that

$$(4.1) \quad \left| \frac{f(z)}{h(z)} - 1 \right| < p - \alpha \quad (z \in \mathbb{U}; 0 \leq \alpha < p).$$

Analogously, a function  $f(z) \in \mathcal{T}_p(n)$  is said to be in the class  $\mathcal{P}_g^{(\alpha)}(p, n, b, m; \mu)$  if there exists a function  $h(z) \in \mathcal{P}_g(p, n, b, m; \mu)$  such that the inequality (4.1) holds true.

**Theorem 5.** *If  $h(z) \in \mathcal{S}_g(p, n, b, m)$  and*

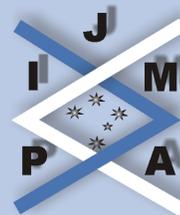
$$(4.2) \quad \alpha = p - \frac{\delta}{n^{q+1}} \cdot \frac{(n - p + |b|) \binom{n}{m} b_n}{(n - p + |b|) \binom{n}{m} b_n - |b| \binom{n}{m}},$$

then

$$(4.3) \quad \mathcal{N}_{n,\delta}^q(h) \subset \mathcal{S}_g^{(\alpha)}(p, n, b, m).$$

*Proof.* Suppose that  $f(z) \in \mathcal{N}_{n,\delta}^q(h)$ . We then find from (1.8) that

$$\sum_{k=n}^{\infty} k^{q+1} |a_k - c_k| \leq \delta,$$



[Title Page](#)

[Contents](#)



Page 16 of 19

[Go Back](#)

[Full Screen](#)

[Close](#)

which readily implies that

$$(4.4) \quad \sum_{k=n}^{\infty} |a_k - c_k| \leq \frac{\delta}{n^{q+1}} \quad (n \in \mathbb{N}).$$

Next, since  $h(z) \in \mathcal{S}_g(p, n, b, m)$ , we find from (3.3) that

$$(4.5) \quad \sum_{k=n}^{\infty} c_k \leq \frac{|b| \binom{p}{m}}{(n-p+|b|) \binom{n}{m} b_n},$$

so that

$$\begin{aligned} \left| \frac{f(z)}{h(z)} - 1 \right| &\leq \frac{\sum_{k=n}^{\infty} |a_k - c_k|}{1 - \sum_{k=n}^{\infty} c_k} \\ &\leq \frac{\delta}{n^{q+1}} \cdot \frac{1}{1 - \frac{|b| \binom{p}{m}}{(n-p+|b|) \binom{n}{m} b_n}} \\ &\leq \frac{\delta}{n^{q+1}} \cdot \frac{(n-p+|b|) \binom{n}{m} b_n}{(n-p+|b|) \binom{n}{m} b_n - |b| \binom{n}{m}} \\ &= p - \alpha, \end{aligned}$$

provided that  $\alpha$  is given by (4.2). Thus, by the above definition,  $f \in \mathcal{S}_g^{(\alpha)}(p, n, b, m)$ , where  $\alpha$  is given by (4.2). This evidently proves Theorem 5. ■

The proof of Theorem 6 below is similar to that of Theorem 5 above. We, therefore, omit the details involved.



Title Page

Contents



Page 17 of 19

Go Back

Full Screen

Close

**Theorem 6.** If  $h(z) \in \mathcal{P}_g(p, n, b, m; \mu)$  and

$$(4.6) \quad \alpha = p - \frac{\delta}{n^{q+1}} \cdot \frac{[\mu(n-p) + p] \binom{n-1}{m} b_n}{\left[ [\mu(n-p) + p] \binom{n-1}{m} b_n - (p-m) \left( \frac{|b|-1}{m!} + \binom{p}{m} \right) \right]},$$

then

$$(4.7) \quad \mathcal{N}_{n,\delta}^q(h) \subset \mathcal{P}_g^{(\alpha)}(p, n, b, m; \mu).$$

**Remark 4.** Applying the parametric substitutions listed in (2.3), Theorems 5 and 6 would yield the corresponding results of Raina and Srivastava [9, p. 6, Theorem 5 and (the corrected form of) Theorem 6].

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Multivalently Analytic Functions

J.K. Prajapat, R.K. Raina and

H.M. Srivastava

vol. 8, iss. 1, art. 7, 2007

---

Title Page

Contents



Page 18 of 19

Go Back

Full Screen

Close

journal of **inequalities**  
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mathematics

issn: 1443-5756



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vol. 8, iss. 1, art. 7, 2007

---

Title Page

Contents



Page 19 of 19

Go Back

Full Screen

Close

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in pure and applied  
mathematics

issn: 1443-5756