

A NOTE ON THE MAGNITUDE OF WALSH FOURIER COEFFICIENTS

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Abstract: In this note, the order of magnitude of Walsh Fourier coefficients for functions of the classes $BV^{(p)}$ ($p \geq 1$), ϕBV , $\Lambda BV^{(p)}$ ($p \geq 1$) and $\phi \Lambda BV$ is studied. For the classes $BV^{(p)}$ and ϕBV , Taibleson-like technique for Walsh Fourier coefficients is developed.

However, for the classes $\Lambda BV^{(p)}$ and $\phi \Lambda BV$ this technique seems to be not working and hence classical technique is applied. In the case of ΛBV , it is also shown that the result is best possible in a certain sense.



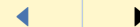
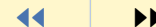
Walsh Fourier Coefficients

B.L. Ghodadra and
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1. Introduction

It appears that while the study of the order of magnitude of the trigonometric Fourier coefficients for the functions of various classes of generalized variations such as $BV^{(p)}$ ($p \geq 1$) [9], ϕBV [2], ΛBV [8], $\Lambda BV^{(p)}$ ($p \geq 1$) [5], $\phi \Lambda BV$ [4], etc. has been carried out, such a study for the Walsh Fourier coefficients has not yet been done. The only result available is due to N.J. Fine [1], who proves, using the second mean value theorem that, if $f \in BV[0, 1]$ then its Walsh Fourier coefficients $\hat{f}(n) = O(\frac{1}{n})$. In this note we carry out this study. Interestingly, here, no use of the second mean value theorem is made. We also prove that for the class ΛBV , our result is best possible in a certain sense.

Definition 1.1. Let $I = [a, b]$, $p \geq 1$ be a real number, $\{\lambda_k\}$, $k \in \mathbb{N}$, be a sequence of non-decreasing positive real numbers such that $\sum_{k=1}^{\infty} \frac{1}{\lambda_k}$ diverges and $\phi : [0, \infty) \rightarrow \mathbb{R}$, be a strictly increasing function. We say that:

1. $f \in BV^{(p)}(I)$ (that is, f is of p -bounded variation over I) if

$$V(f, p, I) = \sup_{\{I_k\}} \left\{ \left(\sum_k |f(b_k) - f(a_k)|^p \right)^{\frac{1}{p}} \right\} < \infty,$$

2. $f \in \phi BV(I)$ (that is, f is of ϕ -bounded variation over I) if

$$V(f, \phi, I) = \sup_{\{I_k\}} \left\{ \sum_k \phi(|f(b_k) - f(a_k)|) \right\} < \infty,$$



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3. $f \in \Lambda BV^{(p)}(I)$ (that is, f is of $p - \Lambda$ - bounded variation over I) if

$$V_{\Lambda}(f, p, I) = \sup_{\{I_k\}} \left\{ \left(\sum_k \frac{|f(b_k) - f(a_k)|^p}{\lambda_k} \right)^{\frac{1}{p}} \right\} < \infty,$$

4. $f \in \phi \Lambda BV(I)$ (that is, f is of $\phi - \Lambda$ - bounded variation over I) if

$$V_{\Lambda}(f, \phi, I) = \sup_{\{I_k\}} \left\{ \sum_k \frac{\phi(|f(b_k) - f(a_k)|)}{\lambda_k} \right\} < \infty,$$

in which $\{I_k = [a_k, b_k]\}$ is a sequence of non-overlapping subintervals of I .

In (2) and (4), it is customary to consider ϕ a convex function such that

$$\phi(0) = 0, \quad \frac{\phi(x)}{x} \rightarrow 0 \quad (x \rightarrow 0_+), \quad \frac{\phi(x)}{x} \rightarrow \infty \quad (x \rightarrow \infty);$$

such a function is necessarily continuous and strictly increasing on $[0, \infty)$.

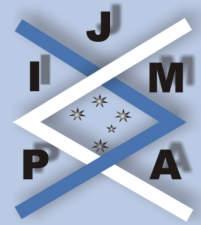
Let $\{\varphi_n\}$ ($n = 0, 1, 2, 3, \dots$) denote the complete orthonormal Walsh system [7], where the subscript denotes the number of zeros (that is, sign-changes) in the interior of the interval $[0, 1]$. For a 1-periodic f in $L[0, 1]$ its Walsh Fourier series is given by

$$f(x) \sim \sum_{n=0}^{\infty} \hat{f}(n) \varphi_n(x),$$

where the n^{th} Walsh Fourier coefficient $\hat{f}(n)$ is given by

$$\hat{f}(n) = \int_0^1 f(x) \varphi_n(x) dx \quad (n = 0, 1, 2, 3, \dots).$$

The Walsh system can be realized [1] as the full set of characters of the dyadic group $G = Z_2^\infty$, in which $Z_2 = \{0, 1\}$ is the group under addition modulo 2. We denote the operation of G by $\dot{+}$. $(G, \dot{+})$ is identified with $([0, 1], +)$ under the usual convention for the binary expansion of elements of $[0, 1]$ [1].



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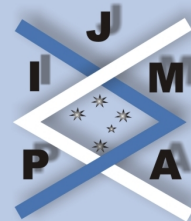
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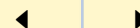
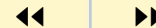
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2. Results

We prove the following theorems. In Theorem 2.5 it is shown that Theorem 2.3 with $p = 1$ is best possible in a certain sense.

Theorem 2.1. *If $f \in BV^{(p)}[0, 1]$ then $\hat{f}(n) = O\left(1 / \left(n^{\frac{1}{p}}\right)\right)$.*

Note. Theorem 2.1 with $p = 1$ gives the result of Fine [1, Theorem VI].

Theorem 2.2. *If $f \in \phi BV[0, 1]$ then $\hat{f}(n) = O(\phi^{-1}(1/n))$.*

Theorem 2.3. *If 1-periodic $f \in \Lambda BV^{(p)}[0, 1]$ ($p \geq 1$) then*

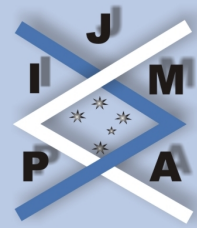
$$\hat{f}(n) = O\left(1 / \left(\sum_{j=1}^n \frac{1}{\lambda_j}\right)^{\frac{1}{p}}\right).$$

Theorem 2.4. *If 1-periodic $f \in \phi \Lambda BV[0, 1]$ then*

$$\hat{f}(n) = O\left(\phi^{-1}\left(1 / \left(\sum_{j=1}^n \frac{1}{\lambda_j}\right)\right)\right).$$

Theorem 2.5. *If $\Gamma BV[0, 1] \supseteq \Lambda BV[0, 1]$ properly then*

$$\exists f \in \Gamma BV[0, 1] \ni \hat{f}(n) \neq O\left(1 / \left(\sum_{j=1}^n \frac{1}{\lambda_j}\right)\right).$$



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Proof of Theorem 2.1. Let $n \in \mathbb{N}$. Let $k \in \mathbb{N} \cup \{0\}$ be such that $2^k \leq n < 2^{k+1}$ and put $a_i = (i/2^k)$ for $i = 0, 1, 2, 3, \dots, 2^k$. Since φ_n takes the value 1 on one half of each of the intervals (a_{i-1}, a_i) and the value -1 on the other half, we have

$$\int_{a_{i-1}}^{a_i} \varphi_n(x) dx = 0, \quad \text{for all } i = 1, 2, 3, \dots, 2^k.$$

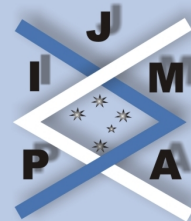
Define a step function g by $g(x) = f(a_{i-1})$ on $[a_{i-1}, a_i)$, $i = 1, 2, 3, \dots, 2^k$. Then

$$\int_0^1 g(x) \varphi_n(x) dx = \sum_{i=1}^{2^k} f(a_{i-1}) \int_{a_{i-1}}^{a_i} \varphi_n(x) dx = 0.$$

Therefore,

$$\begin{aligned} |\hat{f}(n)| &= \left| \int_0^1 [f(x) - g(x)] \varphi_n(x) dx \right| \\ (2.1) \quad &\leq \int_0^1 |f(x) - g(x)| dx \\ &\leq \|f - g\|_p \|1\|_q \\ &= \left(\sum_{i=1}^{2^k} \int_{a_{i-1}}^{a_i} |f(x) - f(a_{i-1})|^p dx \right)^{\frac{1}{p}} \end{aligned}$$

by Hölder's inequality as $f, g \in BV^{(p)}[0, 1]$ and $BV^{(p)}[0, 1] \subset L^p[0, 1]$.



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Hence,

$$\begin{aligned}
 |\hat{f}(n)|^p &\leq \sum_{i=1}^{2^k} \int_{a_{i-1}}^{a_i} |f(x) - f(a_{i-1})|^p dx. \\
 &\leq \sum_{i=1}^{2^k} \int_{a_{i-1}}^{a_i} (V(f, p, [a_{i-1}, a_i]))^p dx \\
 &= \sum_{i=1}^{2^k} (V(f, p, [a_{i-1}, a_i]))^p \left(\frac{1}{2^k}\right) \\
 &\leq \left(\frac{1}{2^k}\right) (V(f, p, [0, 1]))^p \\
 &\leq \left(\frac{2}{n}\right) (V(f, p, [0, 1]))^p,
 \end{aligned}$$

which completes the proof of Theorem 2.1. □

Proof of Theorem 2.2. Let $c > 0$. Using Jensen's inequality and proceeding as in Theorem 2.1, we get

$$\begin{aligned}
 \phi\left(c \int_0^1 |f(x) - g(x)| dx\right) &\leq \int_0^1 \phi(c|f(x) - g(x)|) dx \\
 &= \sum_{i=1}^{2^k} \int_{a_{i-1}}^{a_i} \phi(c|f(x) - f(a_{i-1})|) dx \\
 &\leq \sum_{i=1}^{2^k} \int_{a_{i-1}}^{a_i} V(cf, \phi, [a_{i-1}, a_i]) dx
 \end{aligned}$$

$$\begin{aligned}
&= \sum_{i=1}^{2^k} V(cf, \phi, [a_{i-1}, a_i]) \left(\frac{1}{2^k}\right) \\
&\leq \left(\frac{2}{n}\right) V(cf, \phi, [0, 1]).
\end{aligned}$$

Since ϕ is convex and $\phi(0) = 0$, for sufficiently small $c \in (0, 1)$, $V(cf, \phi, [0, 1]) < 1/2$.

This completes the proof of Theorem 2.2 in view of (2.1). □

Remark 1. If $\phi(x) = x^p$, $p \geq 1$, then the class ϕBV coincides with the class $BV^{(p)}$ and Theorem 2.2 with Theorem 2.1.

Remark 2. Note that in the proof of Theorems 2.1 and 2.2, we have used the fact that if $a = a_0 < a_1 < \dots < a_n = b$, then

$$\sum_{i=1}^n (V(f, p, [a_{i-1}, a_i]))^p \leq (V(f, p, [a, b]))^p$$

and

$$\sum_{i=1}^n V(f, \phi, [a_{i-1}, a_i]) \leq V(f, \phi, [a, b]),$$

for any $n \geq 2$ (see [2, 1.17, p. 15]). Such inequalities for functions of the class $\Lambda BV^{(p)}$ ($p \geq 1$) (resp., $\phi \Lambda BV$), which contain $BV^{(p)}$ (resp., ϕBV) properly, do not hold true.

In fact, the following proposition shows that the validity of such inequalities for the class $\Lambda BV^{(p)}$ (resp., $\phi \Lambda BV$) virtually reduces the class to $BV^{(p)}$ (resp., ϕBV). Hence we prove Theorem 2.3 and Theorem 2.4 applying a technique different from the Taibleson-like technique [6] which we have applied in proving Theorem 2.1 and Theorem 2.2.



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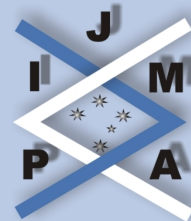
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Proposition 2.6. Let $f \in \phi\Lambda BV[a, b]$. If there is a constant C such that

$$\sum_{i=1}^n V_{\Lambda}(f, \phi, [a_{i-1}, a_i]) \leq CV_{\Lambda}(f, \phi, [a, b]),$$

for any sequence of points $\{a_i\}_{i=0}^n$ with $a = a_0 < a_1 < \dots < a_n = b$, then $f \in \phi BV[a, b]$.

Proof. For any partition $a = x_0 < x_1 < \dots < x_n = b$ of $[a, b]$, we have

$$\begin{aligned} \sum_{i=1}^n \phi(|f(x_i) - f(x_{i-1})|) &= \lambda_1 \sum_{i=1}^n \frac{\phi(|f(x_i) - f(x_{i-1})|)}{\lambda_1} \\ &\leq \lambda_1 \sum_{i=1}^n V_{\Lambda}(f, \phi, [x_{i-1}, x_i]) \\ &\leq \lambda_1 CV_{\Lambda}(f, \phi, [a, b]), \end{aligned}$$

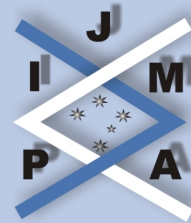
which shows that $f \in \phi BV[a, b]$. □

Remark 3. $\phi(x) = x^p$ ($p \geq 1$) in this proposition will give an analogous result for $\Lambda BV^{(p)}$.

To prove Theorem 2.3 and Theorem 2.4, we need the following lemma.

Lemma 2.7. For any $n \in \mathbb{N}$, $|\hat{f}(n)| \leq \omega_p(1/n; f)$, where $\omega_p(\delta; f)$ ($\delta > 0$, $p \geq 1$) denotes the integral modulus of continuity of order p of f given by

$$\omega_p(\delta; f) = \sup_{|h| \leq \delta} \left(\int_0^1 |f(x+h) - f(x)|^p dx \right)^{\frac{1}{p}}.$$

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Proof. The inequality [1, Theorem IV, p. 382] $|\hat{f}(n)| \leq \omega_1(1/n; f)$ and the fact that $\omega_1(1/n; f) \leq \omega_p(1/n; f)$ for $p \geq 1$ immediately proves the lemma. \square

Proof of Theorem 2.3. For any $n \in \mathbb{N}$, put $\theta_n = \sum_{j=1}^n 1/\lambda_j$. Let $f \in \Lambda BV^{(p)}[0, 1]$. For $0 < h \leq 1/n$, put $k = [1/h]$. Then for a given $x \in \mathbb{R}$, all the points $x + jh$, $j = 0, 1, \dots, k$ lie in the interval $[x, x + 1]$ of length 1 and

$$\int_0^1 |f(x) - f(x + h)|^p dx = \int_0^1 |f_j(x)|^p dx, \quad j = 1, 2, \dots, k,$$

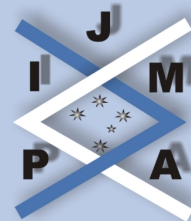
where $f_j(x) = f(x + (j - 1)h) - f(x + jh)$, for all $j = 1, 2, \dots, k$. Since the left hand side of this equation is independent of j , multiplying both sides by $1/(\lambda_j \theta_k)$ and summing over $j = 1, 2, \dots, k$, we get

$$\begin{aligned} \int_0^1 |f(x) - f(x + h)|^p dx &\leq \left(\frac{1}{\theta_k}\right) \int_0^1 \sum_{j=1}^k \left(\frac{|f_j(x)|^p}{\lambda_j}\right) dx \\ &\leq \frac{(V_\Lambda(f, p, [0, 1]))^p}{\theta_k} \\ &\leq \frac{(V_\Lambda(f, p, [0, 1]))^p}{\theta_n}, \end{aligned}$$

because $\{\lambda_j\}$ is non-decreasing and $0 < h \leq 1/n$. The case $-1/n \leq h < 0$ is similar and we get using Lemma 2.7,

$$|\hat{f}(n)|^p \leq (\omega_p(1/n; f))^p \leq \frac{(V_\Lambda(f, p, [0, 1]))^p}{\theta_n}.$$

This proves Theorem 2.3. \square



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Proof of Theorem 2.4. Let $f \in \phi\Lambda BV[0, 1]$. Then for h, k and $f_j(x)$ as in the proof of Theorem 2.3 and for $c > 0$ by Jensen's inequality,

$$\begin{aligned} \phi\left(c \int_0^1 |f(x) - f(x+h)| dx\right) &\leq \int_0^1 \phi(c|f(x) - f(x+h)|) dx \\ &= \int_0^1 \phi(c|f_j(x)|) dx, \quad j = 1, 2, \dots, k. \end{aligned}$$

Multiplying both sides by $1/(\lambda_j\theta_k)$ and summing over $j = 1, 2, \dots, k$, we get

$$\begin{aligned} \phi\left(c \int_0^1 |f(x) - f(x+h)| dx\right) &\leq \left(\frac{1}{\theta_k}\right) \int_0^1 \sum_{j=1}^k \left(\frac{\phi(c|f_j(x)|)}{\lambda_j}\right) dx \\ &\leq \frac{V_\Lambda(cf, \phi, [0, 1])}{\theta_k} \\ &\leq \frac{V_\Lambda(cf, \phi, [0, 1])}{\theta_n}. \end{aligned}$$

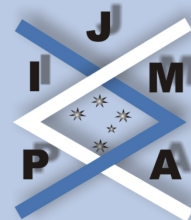
Since ϕ is convex and $\phi(0) = 0$, $\phi(\alpha x) \leq \alpha\phi(x)$ for $0 < \alpha < 1$. So we may choose c sufficiently small so that $V_\Lambda(cf, \phi, [0, 1]) \leq 1$. But then we have

$$\int_0^1 |f(x) - f(x+h)| dx \leq \frac{1}{c} \phi^{-1}\left(\frac{1}{\theta_n}\right).$$

Thus it follows in view of Lemma 2.7 that

$$|\hat{f}(n)| \leq \omega_1(1/n; f) \leq \frac{1}{c} \phi^{-1}\left(\frac{1}{\theta_n}\right),$$

which proves Theorem 2.4. □



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Proof of Theorem 2.5. It is known [3] that if ΓBV contains ΛBV properly with $\Gamma = \{\gamma_n\}$ then $\theta_n \neq O(\rho_n)$, where $\rho_n = \sum_{j=1}^n \frac{1}{\gamma_j}$ for each n . Also, if $c_0 = 0$, $c_{n+1} = 1$ and $c_1 < c_2 < \dots < c_n$ denote all the n points of $(0, 1)$ where the function φ_n changes its sign in $(0, 1)$, $n_0 \in \mathbb{N}$ is such that $\rho_n \geq \frac{1}{2}$ for all $n \geq n_0$ and $E = \{n \in \mathbb{N} : n \geq n_0 \text{ is even}\}$, then for each $n \in E$, for the function

$$f_n = \sum_{k=1}^{n+1} \frac{(-1)^{k-1}}{4\rho_n} \chi_{[c_{k-1}, c_k)}$$

extended 1-periodically on \mathbb{R} ,

$$V_\Gamma(f_n, [0, 1]) = \sum_{k=1}^{n+1} \frac{|f_n(c_k) - f_n(c_{k-1})|}{\gamma_k} = \sum_{k=1}^n \frac{1}{\gamma_k} \cdot \frac{1}{2\rho_n} = \frac{1}{2}$$

because

$$f_n(c_{n+1}) = f_n(1) = f_n(0) = \frac{1}{4\rho_n} = f_n(c_n)$$

as $\varphi_n \equiv 1$ on $[c_0, c_1)$. Hence $\|f_n\| = \frac{1}{4\rho_n} + \frac{1}{2} \leq 1$ for each $n \in E$ in the Banach space $\Gamma BV[0, 1]$ with $\|f\| = |f(0)| + V_\Gamma(f, [0, 1])$. Observe that for $f \in \Gamma BV[0, 1]$

$$\|f\|_1 \leq \int_0^1 \left(\frac{|f(x) - f(0)|}{\gamma_1} \gamma_1 + |f(0)| \right) dx \leq C\|f\|, \quad C = \max\{1, \gamma_1\},$$

and hence, for each $n \in \mathbb{N}$ the linear map $T_n : \Gamma BV[0, 1] \rightarrow \mathbb{R}$ defined by $T_n(f) = \theta_n \hat{f}(n)$ is bounded as

$$|T_n(f)| = \theta_n |\hat{f}(n)| \leq \theta_n \|f\|_1 \leq \theta_n C \|f\|, \quad \forall f \in \Gamma BV[0, 1].$$

Next, for each $n \in E$ since $f_n \cdot \varphi_n = \frac{1}{4\rho_n}$ on $[0, 1)$, we see that

$$T_n(f_n) = \theta_n \hat{f}_n(n) = \theta_n \int_0^1 f_n(x) \varphi_n(x) dx = \frac{1}{4} \left(\frac{\theta_n}{\rho_n} \right) \neq O(1)$$

and hence

$$\sup\{|T_n| : n \in \mathbb{N}\} \geq \sup\{|T_n| : n \in E\} \geq \sup\{|T_n(f_n)| : n \in E\} = \infty.$$

Therefore, an application of the Banach-Steinhaus theorem gives an $f \in \Gamma BV[0, 1]$ such that $\sup\{|T_n(f)| : n \in \mathbb{N}\} = \infty$. It follows that $\theta_n \hat{f}(n) = T_n(f) \neq O(1)$ and hence the theorem is proved. \square



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