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FEKETE-SZEGÖ FUNCTIONAL FOR SOME SUBCLASS OF NON-BAZILEVIČ FUNCTIONS

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ABSTRACT. In this present investigation, the authors obtain a sharp Fekete-Szegö's inequality for certain normalized analytic functions $f(z)$ defined on the open unit disk for which $(1+\beta)\left(\frac{z}{f(z)}\right)^{\alpha} - \beta f'(z)\left(\frac{z}{f(z)}\right)^{1+\alpha}$, $(\beta \in \mathbb{C},\ 0 < \alpha < 1)$ lies in a region starlike with respect to 1 and is symmetric with respect to the real axis. Also, certain applications of our results for a class of functions defined by convolution are given. As a special case of this result, Fekete-Szegö's inequality for a class of functions defined through fractional derivatives is also obtained.

Key words and phrases: Analytic functions, Starlike functions, Subordination, Coefficient problem, Fekete-Szegö inequality.

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1. INTRODUCTION

Let $\mathcal{A}$ denote the class of all *analytic* functions $f(z)$ of the form

$$f(z) = z + \sum_{k=2}^{\infty} a_k z^k \quad (z \in \Delta := \{z \in \mathbb{C} / \, |z| < 1\}) \tag{1.1}$$

and $\mathcal{S}$ be the subclass of $\mathcal{A}$ consisting of univalent functions. Let $\phi(z)$ be an analytic function with positive real part on Δ with $\phi(0) = 1$, $\phi'(0) > 0$ which maps the unit disk Δ onto a region

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starlike with respect to 1 and is symmetric with respect to the real axis. Let $S^*(\phi)$ be the class of functions $f \in \mathcal{S}$ for which

$$\frac{zf'(z)}{f(z)} \prec \phi(z), \quad (z \in \Delta)$$

and $C(\phi)$ be the class of functions $f \in \mathcal{S}$ for which

$$1 + \frac{zf''(z)}{f'(z)} \prec \phi(z), \quad (z \in \Delta),$$

where $\prec$ denotes the subordination between analytic functions. These classes were introduced and studied by Ma and Minda [3]. They have obtained the Fekete-Szegö inequality for the functions in the class $C(\phi)$. Since $f \in C(\phi)$ if and only if $zf' \in S^*(\phi)$, we get the Fekete-Szegö inequality for functions in the class $S^*(\phi)$. Recently, Shanmugam and Sivasubramanian [9] obtained Fekete- Szegö inequalities for the class of functions $f \in \mathcal{A}$ such that

$$\frac{zf'(z) + \alpha z^2 f''(z)}{(1-\alpha)f(z) + \alpha z f'(z)} \prec \phi(z) \quad (0 \le \alpha < 1).$$

Also, Ravichandran et al. [7] obtained the Fekete-Szegö inequality for the class of Bazilevič functions. For a brief history of the Fekete-Szegö problem for the class of starlike, convex and close-to-convex functions, see the recent paper by Srivastava *et al.* [11]. Obradovic [4] introduced a class of functions $f \in \mathcal{A}$, such that, for $0 < \alpha < 1$,

$$\Re\left\{ f'(z) \left(\frac{z}{f(z)} \right)^{\alpha} \right\} > 0, \quad z \in \Delta.$$

He called this class of function as "Non-Bazilevič" type. Tuneski and Darus [14] obtained the Fekete-Szegö inequality for the non-Bazilevič class of functions. Using this non-Bazilevič class, Wang et al.[15] studied many subordination results for the class $N(\alpha, \beta, A, B)$ defined as

$$N(\alpha,\beta,A,B) := \left\{ f \in \mathcal{A} : (1+\beta)\left(\frac{z}{f(z)}\right)^{\alpha} - \beta f'(z)\left(\frac{z}{f(z)}\right)^{1+\alpha} \prec \frac{1+Az}{1+Bz} \right\},$$

where $\beta \in \mathbb{C}, -1 \le B \le 1, A \ne B, \quad 0 < \alpha < 1$.

In the present paper, we obtain the Fekete-Szegö inequality for functions in a more general class $N_{\alpha,\beta}(\phi)$ of functions which we define below. Also we give applications of our results to certain functions defined through convolution (or Hadamard product) and in particular we consider a class $N^{\lambda}_{\alpha,\beta}(\phi)$ of functions defined by fractional derivatives. The aim of this paper is to give a generalization the Fekete-Szegö inequalities for some subclass of Non-Bazilevič functions .

Definition 1.1. Let $\phi(z)$ be an univalent starlike function with respect to 1 which maps the unit disk Δ onto a region in the right half plane which is symmetric with respect to the real axis, $\phi(0) = 1$ and $\phi'(0) > 0$. A function $f \in \mathcal{A}$ is in the class $N_{\alpha,\beta}(\phi)$ if

$$(1+\beta)\left(\frac{z}{f(z)}\right)^{\alpha} - \beta f'(z)\left(\frac{z}{f(z)}\right)^{1+\alpha} \prec \phi(z), \quad (\beta \in \mathbb{C}, 0 < \alpha < 1).$$

For fixed $g \in \mathcal{A}$, we define the class $N^{g}_{\alpha,\beta}(\phi)$ to be the class of functions $f \in \mathcal{A}$ for which $(f * g) \in N_{\alpha,\beta}(\phi)$.

Remark 1.1. $N_{\alpha,-1}\left(\frac{1+z}{1-z}\right)$ is the class of Non-Bazilevič functions introduced by Obradovic [4].

Remark 1.2. $N_{\alpha,-1}\left(\frac{1+(1-2\gamma)z}{1-z}\right), \quad 0 \le \gamma < 1$ is the class of Non-Bazilevič functions of order γ introduced and studied by Tuneski and Darus [14].

Remark 1.3. We call $N_{\alpha,\beta}\left\{1+\frac{2}{\pi^2}\left(\log\frac{1+\sqrt{z}}{1-\sqrt{z}}\right)^2\right\}$ the class of "Non-Bazilevič parabolic starlike functions".

To prove our main result, we need the following:

Lemma 1.4 ([3]). *If $p_1(z)=1+c_1z+c_2z^2+\cdots$ is an analytic function with a positive real part in Δ, then*

$$|c_2-vc_1^2|\le\begin{cases}-4v+2 & \text{if } v\le 0,\\ 2 & \text{if } 0\le v\le 1,\\ 4v-2 & \text{if } v\ge 1.\end{cases}$$

When $v<0$ or $v>1$, the equality holds if and only if $p_1(z)$ is $(1+z)/(1-z)$ or one of its rotations. If $0<v<1$, then the equality holds if and only if $p_1(z)$ is $(1+z^2)/(1-z^2)$ or one of its rotations. If $v=0$, the equality holds if and only if

$$p_1(z)=\left(\frac{1}{2}+\frac{1}{2}\lambda\right)\frac{1+z}{1-z}+\left(\frac{1}{2}-\frac{1}{2}\lambda\right)\frac{1-z}{1+z}\quad(0\le\lambda\le 1)$$

or one of its rotations. If $v=1$, the equality holds if and only if p_1 is the reciprocal of one of the functions such that equality holds in the case of $v=0$.

Also the above upper bound is sharp, and it can be improved as follows when $0<v<1$:

$$|c_2-vc_1^2|+v|c_1|^2\le 2\quad(0<v\le 1/2)$$

and

$$|c_2-vc_1^2|+(1-v)|c_1|^2\le 2\quad(1/2<v\le 1).$$

2. FEKETE-SZEGÖ PROBLEM

Our main result is the following:

Theorem 2.1. *Let $\phi(z)=1+B_1z+B_2z^2+B_3z^3+\cdots$. If f given by (1.1) belongs to $N_{\alpha,\beta}(\phi)$, then*

$$|a_3-\mu a_2^2|\le\begin{cases}-\frac{B_2}{(\alpha+2\beta)}-\frac{\mu B_1^2}{2(\alpha+\beta)^2}+\frac{(1+\alpha)}{2(\alpha+\beta)^2}B_1^2 & \text{if } \mu\le\sigma_1;\\ -\frac{B_1}{(\alpha+2\beta)} & \text{if } \sigma_1\le\mu\le\sigma_2;\\ \frac{B_2}{(\alpha+2\beta)}+\frac{\mu B_1^2}{2(\alpha+\beta)^2}-\frac{(1+\alpha)}{2(\alpha+\beta)^2}B_1^2 & \text{if } \mu\ge\sigma_2,\end{cases}$$

where,

$$\sigma_1:=\frac{(1+\alpha)(2\beta+\alpha)B_1^2-2(B_2-B_1)(\beta+\alpha)^2}{2(2\beta+\alpha)B_1^2},$$
$$\sigma_2:=\frac{(1+\alpha)(2\beta+\alpha)B_1^2-2(B_2+B_1)(\beta+\alpha)^2}{2(2\beta+\alpha)B_1^2}.$$

The result is sharp.

Proof. For $f\in N_{\alpha,\beta}(\phi)$, let

$$p(z):=(1+\beta)\left(\frac{z}{f(z)}\right)^{\alpha}-\beta f'(z)\left(\frac{z}{f(z)}\right)^{1+\alpha}=1+b_1z+b_2z^2+\cdots. \tag{2.1}$$

From (2.1), we obtain

$$-(\alpha+\beta)a_2=b_1$$

$$(2\beta+\alpha)\left(\frac{\alpha+1}{2}a_2^2-a_3\right)=b_2.$$

Since $\phi(z)$ is univalent and $p \prec \phi$, the function

$$p_1(z)=\frac{1+\phi^{-1}(p(z))}{1-\phi^{-1}(p(z))}=1+c_1z+c_2z^2+\cdots$$

is analytic and has a positive real part in Δ. Also we have

$$p(z)=\phi\left(\frac{p_1(z)-1}{p_1(z)+1}\right) \tag{2.2}$$

and from this equation (2.2), we obtain

$$b_1=\frac{1}{2}B_1c_1$$

and

$$b_2=\frac{1}{2}B_1\left(c_2-\frac{1}{2}c_1^2\right)+\frac{1}{4}B_2c_1^2.$$

Therefore we have

$$a_3-\mu a_2^2=-\frac{B_1}{2(2\beta+\alpha)}\left\{c_2-vc_1^2\right\}$$

where

$$v:=\frac{1}{2}\left[1-\frac{B_2}{B_1}+\frac{(2\beta+\alpha)(\alpha+1-2\mu)}{2(\beta+\alpha)^2}B_1\right].$$

Our result now follows by an application of Lemma 1.4. To show that the bounds are sharp, we define the functions $K_{\alpha,\beta}^{\phi_n}$ $(n=2,3,\dots)$ by

$$(1+\beta)\left(\frac{z}{K_{\alpha,\beta}^{\phi_n}(z)}\right)^{\alpha}-\beta\left(K_{\alpha,\beta}^{\phi_n}\right)'(z)\left(\frac{z}{K_{\alpha,\beta}^{\phi_n}(z)}\right)^{1+\alpha}=\phi(z^{n-1}),$$
$$K_{\alpha,\beta}^{\phi_n}(0)=0=[K_{\alpha,\beta}^{\phi_n}]'(0)-1$$

and the function $F_{\alpha,\beta}^{\lambda}$ and $G_{\alpha,\beta}^{\lambda}$ $(0<\alpha<1)$ by

$$(1+\beta)\left(\frac{z}{F_{\alpha,\beta}^{\lambda}(z)}\right)^{\alpha}-\beta[F_{\alpha,\beta}^{\lambda}]'(z)\left(\frac{z}{F_{\alpha,\beta}^{\lambda}(z)}\right)^{1+\alpha}=\phi(z^{n-1}),$$
$$[F_{\alpha,\beta}^{\lambda}](0)=0=[F_{\alpha,\beta}^{\lambda}]]'(0)-1$$

and

$$(1+\beta)\left(\frac{z}{G_{\alpha,\beta}^{\lambda}}\right)^{\alpha}-\beta[G_{\alpha,\beta}^{\lambda}]'(z)\left(\frac{z}{G_{\alpha,\beta}^{\lambda}(z)}\right)^{1+\alpha}=\phi(z^{n-1}),$$
$$[G_{\alpha,\beta}^{\lambda}](0)=0=[G_{\alpha,\beta}^{\lambda}]'(0)-1.$$

Clearly, the functions $K_{\alpha,\beta}^{\phi_n}$, $[F_{\alpha,\beta}^{\lambda}]$ and $[G_{\alpha,\beta}^{\lambda}]\in N_{\alpha,\beta}(\phi)$

Also we write $K_{\alpha,\beta}^{\phi}:=K_{\alpha,\beta}^{\phi_2}$.

If $\mu<\sigma_1$ or $\mu>\sigma_2$, then the equality holds if and only if f is $K_{\alpha,\beta}^{\phi}$ or one of its rotations. When $\sigma_1<\mu<\sigma_2$, the equality holds if and only if f is $K_{\alpha,\beta}^{\phi_3}$ or one of its rotations. If $\mu=\sigma_1$ then the equality holds if and only if f is $F_{\alpha,\beta}^{\lambda}$ or one of its rotations. If $\mu=\sigma_2$ then the equality holds if and only if f is $G_{\alpha,\beta}^{\lambda}$ or one of its rotations. □

Corollary 2.2. *Let* $\phi(z) = 1 + \frac{2}{\pi^2}\left(\log \frac{1+\sqrt{z}}{1-\sqrt{z}}\right)^2$. *If* f *given by (1.1) belongs to* $N_{\alpha,\beta}(\phi)$, *then*

$$|a_3 - \mu a_2^2| \leq \begin{cases} -\frac{8}{3\pi^2(\alpha+2\beta)} - \frac{8\mu}{\pi^4(\alpha+\beta)^2} + \frac{(1+\alpha)}{(\alpha+\beta)^2}\frac{8}{\pi^4} & \textit{if} \quad \mu \leq \sigma_1 \\ -\frac{4}{\pi^2(\alpha+2\beta)} & \textit{if} \quad \sigma_1 \leq \mu \leq \sigma_2 \\ \frac{8}{3\pi^2(\alpha+2\beta)} + \frac{8\mu}{\pi^4(\alpha+\beta)^2} - \frac{(1+\alpha)}{(\alpha+\beta)^2}\frac{8}{\pi^4} & \textit{if} \quad \mu \geq \sigma_2 \end{cases}$$

where,

$$\sigma_1 := \frac{(1+\alpha)(2\beta+\alpha)\frac{16}{\pi^4} - 2\left(\frac{8}{3\pi^2} - \frac{4}{\pi^2}\right)(\beta+\alpha)^2}{2(2\beta+\alpha)\frac{16}{\pi^4}}$$

$$\sigma_2 := \frac{(1+\alpha)(2\beta+\alpha)\frac{16}{\pi^4} - 2\left(\frac{8}{3\pi^2} + \frac{4}{\pi^2}\right)(\beta+\alpha)^2}{2(2\beta+\alpha)\frac{16}{\pi^4}}.$$

The result is sharp.

Corollary 2.3. *For* $\beta = -1$, $\phi(z) = \frac{1+(1-2\gamma)z}{1-z}$, $0 \leq \gamma < 1$ *in Theorem 2.1, we get the results obtained by Tuneski and Darus* [14].

Remark 2.4. If $\sigma_1 \leq \mu \leq \sigma_2$, then, in view of Lemma 1.4, Theorem 2.1 can be improved. Let σ_3 be given by

$$\sigma_3 := \frac{(1+\alpha)(2\beta+\alpha)B_1^2 - 2B_2(\beta+\alpha)^2}{2(2\beta+\alpha)B_1^2}.$$

If $\sigma_1 \leq \mu \leq \sigma_3$, then

$$|a_3 - \mu a_2^2| - \frac{(\beta+\alpha)^2}{(2\beta+\alpha)B_1^2}\left[B_1 - B_2 + B_1^2\frac{(\alpha+1-2\mu)(2\beta+\alpha)}{2(\beta+\alpha)^2}\right]|a_2|^2 \leq -\frac{B_1}{(2\beta+\alpha)}.$$

If $\sigma_3 \leq \mu \leq \sigma_2$, then

$$|a_3 - \mu a_2^2| - \frac{(\beta+\alpha)^2}{(2\beta+\alpha)B_1^2}\left[B_1 + B_2 - B_1^2\frac{(\alpha+1-2\mu)(2\beta+\alpha)}{2(\beta+\alpha)^2}\right]|a_2|^2 \leq -\frac{B_1}{(2\beta+\alpha)}.$$

3. APPLICATIONS TO FUNCTIONS DEFINED BY FRACTIONAL DERIVATIVES

In order to introduce the class $N^\lambda_{\alpha,\beta}(\phi)$, we need the following:

Definition 3.1 (see [5, 6]; see also [12, 13])**.** Let f be analytic in a simply connected region of the z-plane containing the origin. The *fractional derivative* of f of order λ is defined by

$$D_z^\lambda f(z) := \frac{1}{\Gamma(1-\lambda)}\frac{d}{dz}\int_0^z \frac{f(\zeta)}{(z-\zeta)^\lambda}d\zeta \quad (0 \leq \lambda < 1)$$

where the multiplicity of $(z-\zeta)^\lambda$ is removed by requiring that $\log(z-\zeta)$ is real for $z-\zeta > 0$.

Using the above Definition 3.1 and its known extensions involving fractional derivatives and fractional integrals, Owa and Srivastava [5] introduced the operator $\Omega^\lambda : \mathcal{A} \to \mathcal{A}$ defined by

$$(\Omega^\lambda f)(z) = \Gamma(2-\lambda)z^\lambda D_z^\lambda f(z), \quad (\lambda \neq 2, 3, 4, \ldots).$$

The class $N^\lambda_{\alpha,\beta}(\phi)$ consists of functions $f \in \mathcal{A}$ for which $\Omega^\lambda f \in N_{\alpha,\beta}(\phi)$. Note that $N^\lambda_{\alpha,\beta}(\phi)$ is the special case of the class $N^g_{\alpha,\beta}(\phi)$ when

$$g(z) = z + \sum_{n=2}^\infty \frac{\Gamma(n+1)\Gamma(2-\lambda)}{\Gamma(n+1-\lambda)}z^n. \tag{3.1}$$

Let $g(z) = z + \sum_{n=2}^{\infty} g_n z^n$ ($g_n > 0$). Since $f(z) = z + \sum_{n=2}^{\infty} a_n z^n \in N^g_{\alpha,\beta}(\phi)$ if and only if $(f * g) = z + \sum_{n=2}^{\infty} g_n a_n z^n \in N_{\alpha,\beta}(\phi)$, we obtain the coefficient estimate for functions in the class $N^g_{\alpha,\beta}(\phi)$, from the corresponding estimate for functions in the class $N_{\alpha,\beta}(\phi)$. Applying Theorem 2.1 for the function $(f * g)(z) = z + g_2 a_2 z^2 + g_3 a_3 z^3 + \cdots$, we get the following Theorem 3.1 after an obvious change of the parameter μ:

Theorem 3.1. *Let the function ϕ be given by $\phi(z) = 1 + B_1 z + B_2 z^2 + B_3 z^3 + \cdots$. If f given by (1.1) belongs to $N^g_{\alpha,\beta}(\phi)$, then*

$$|a_3 - \mu a_2^2| \leq \begin{cases} \frac{1}{g_3}\left\{-\frac{B_2}{(\alpha+2\beta)} - \frac{\mu g_3 B_1^2}{g_2^2 2(\alpha+\beta)^2} + \frac{(1+\alpha)}{2(\alpha+\beta)^2} B_1^2\right\} & \text{if} \quad \mu \leq \sigma_1; \\ -\frac{1}{g_3}\frac{B_1}{(\alpha+2\beta)} & \text{if} \quad \sigma_1 \leq \mu \leq \sigma_2; \\ \frac{1}{g_3}\left\{\frac{B_2}{(\alpha+2\beta)} + \frac{\mu g_3 B_1^2}{2(\alpha+\beta)^2 g_2^2} - \frac{(1+\alpha)}{2(\alpha+\beta)^2} B_1^2\right\} & \text{if} \quad \mu \geq \sigma_2, \end{cases}$$

where

$$\sigma_1 := \frac{g_3}{g_2^2}\frac{(1+\alpha)(2\beta+\alpha)B_1^2 - 2(B_2 - B_1)(\beta+\alpha)^2}{2(2\beta+\alpha)B_1^2}$$

$$\sigma_2 := \frac{g_3}{g_2^2}\frac{(1+\alpha)(2\beta+\alpha)B_1^2 - 2(B_2 + B_1)(\beta+\alpha)^2}{2(2\beta+\alpha)B_1^2}.$$

The result is sharp.

Since

$$(\Omega^\lambda f)(z) = z + \sum_{n=2}^{\infty} \frac{\Gamma(n+1)\Gamma(2-\lambda)}{\Gamma(n+1-\lambda)} a_n z^n,$$

we have

$$g_2 := \frac{\Gamma(3)\Gamma(2-\lambda)}{\Gamma(3-\lambda)} = \frac{2}{2-\lambda} \tag{3.2}$$

and

$$g_3 := \frac{\Gamma(4)\Gamma(2-\lambda)}{\Gamma(4-\lambda)} = \frac{6}{(2-\lambda)(3-\lambda)}. \tag{3.3}$$

For g_2 and g_3 given by (3.2) and (3.3), Theorem 3.1 reduces to the following:

Theorem 3.2. *Let the function ϕ be given by $\phi(z) = 1 + B_1 z + B_2 z^2 + B_3 z^3 + \cdots$. If f given by (1.1) belongs to $N^g_{\alpha,\beta}(\phi)$, then*

$$|a_3 - \mu a_2^2| \leq \begin{cases} \frac{(2-\lambda)(3-\lambda)}{6}\left\{-\frac{B_2}{(\alpha+2\beta)} - \frac{\mu g_3 B_1^2}{g_2^2 2(\alpha+\beta)^2} + \frac{(1+\alpha)}{2(\alpha+\beta)^2} B_1^2\right\} & \text{if} \quad \mu \leq \sigma_1; \\ -\frac{(2-\lambda)(3-\lambda)}{6}\frac{B_1}{(\alpha+2\beta)} & \text{if} \quad \sigma_1 \leq \mu \leq \sigma_2; \\ \frac{(2-\lambda)(3-\lambda)}{6}\left\{\frac{B_2}{(\alpha+2\beta)} + \frac{\mu g_3 B_1^2}{2(\alpha+\beta)^2 g_2^2} - \frac{(1+\alpha)}{2(\alpha+\beta)^2} B_1^2\right\} & \text{if} \quad \mu \geq \sigma_2, \end{cases}$$

where

$$\sigma_1 := \frac{2(3-\lambda)}{3(2-\lambda)}\frac{(1+\alpha)(2\beta+\alpha)B_1^2 - 2(B_2 - B_1)(\beta+\alpha)^2}{2(2\beta+\alpha)B_1^2},$$

$$\sigma_2 := \frac{2(3-\lambda)}{3(2-\lambda)}\frac{(1+\alpha)(2\beta+\alpha)B_1^2 - 2(B_2 + B_1)(\beta+\alpha)^2}{2(2\beta+\alpha)B_1^2}.$$

The result is sharp.

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