Journal of Inequalities in Pure and Applied Mathematics

http://jipam.vu.edu.au/

Volume 7, Issue 5, Article 191, 2006

INCLUSION AND NEIGHBORHOOD PROPERTIES OF CERTAIN SUBCLASSES OF ANALYTIC AND MULTIVALENT FUNCTIONS OF COMPLEX ORDER

H.M. SRIVASTAVA, K. SUCHITHRA, B. ADOLF STEPHEN, AND S. SIVASUBRAMANIAN

DEPARTMENT OF MATHEMATICS AND STATISTICS
UNIVERSITY OF VICTORIA
VICTORIA, BRITISH COLUMBIA V8W 3P4, CANADA
harimsri@math.uvic.ca

DEPARTMENT OF APPLIED MATHEMATICS
SRI VENKATESWARA COLLEGE OF ENGINEERING
SRIPERUMBUDUR 602 105
TAMIL NADU, INDIA
suchithrak@svce.ac.in

DEPARTMENT OF MATHEMATICS
MADRAS CHRISTIAN COLLEGE
EAST TAMBARAM 600 059
TAMIL NADU, INDIA
adolfmcc2003@yahoo.co.in

DEPARTMENT OF MATHEMATICS
EASWARI ENGINEERING COLLEGE
RAMAPURAM, CHENNAI 600 089
TAMIL NADU, INDIA
sivasaisastha@rediffmail.com

Received 08 July, 2006; accepted 11 July, 2006
Communicated by Th.M. Rassias

ABSTRACT. In the present paper, the authors prove several inclusion relations associated with the (n, δ)-neighborhoods of certain subclasses of p-valently analytic functions of complex order, which are introduced here by means of a family of extended multiplier transformations. Special cases of some of these inclusion relations are shown to yield known results.

Key words and phrases: Analytic functions, p-valent functions, Coefficient bounds, Multiplier transformations, Neighborhood of analytic functions, Inclusion relations.

2000 *Mathematics Subject Classification.* Primary 30C45.

ISSN (electronic): 1443-5756

The present investigation was supported, in part, by the *Natural Sciences and Engineering Research Council of Canada* under Grant OGP0007353.

187-06

1. INTRODUCTION, DEFINITIONS AND PRELIMINARIES

Let $\mathcal{A}_p(n)$ denote the class of functions $f(z)$ normalized by

$$f(z) = z^p - \sum_{\tau=n+p}^{\infty} a_\tau z^\tau \tag{1.1}$$

$$(a_\tau \geqq 0;\ n, p \in \mathbb{N} := \{1, 2, 3, \dots\}),$$

which are *analytic* and *p-valent* in the open unit disk

$$\mathbb{U} := \{z : z \in \mathbb{C} \quad \text{and} \quad |z| < 1\}.$$

Analogous to the multiplier transformation on $\mathcal{A}$, the operator $I_p(r, \mu)$, given on $\mathcal{A}_p(1)$ by

$$I_p(r, \mu)f(z) := z^p - \sum_{\tau=p+1}^{\infty} \left(\frac{\tau + \mu}{p + \mu}\right)^r a_\tau z^\tau$$

$$\big(\mu \geqq 0;\ r \in \mathbb{Z};\ f \in \mathcal{A}_p(1)\big),$$

was studied by Kumar *et al.* [6]. It is easily verified that

$$(p + \mu)I_p(r + 1, \mu)f(z) = z[I_p(r, \mu)f(z)]' + \mu I_p(r, \mu)f(z).$$

The operator $I_p(r, \mu)$ is closely related to the Sălăgean derivative operator [11]. The operator

$$I_\mu^r := I_1(r, \mu)$$

was studied by Cho and Srivastava [4] and Cho and Kim [3]. Moreover, the operator

$$I_r := I_1(r, 1)$$

was studied earlier by Uraleggadi and Somanatha [13].

Here, in our present investigation, we define the operator $I_p(r, \mu)$ on $\mathcal{A}_p(n)$ by

$$I_p(r, \mu)f(z) := z^p - \sum_{\tau=n+p}^{\infty} \left(\frac{\tau + \mu}{p + \mu}\right)^r a_\tau z^\tau \tag{1.2}$$

$$(\mu \geqq 0;\ p \in \mathbb{N};\ r \in \mathbb{Z}).$$

By using the operator $I_p(r, \mu)f(z)$ given by (1.2), we introduce a subclass $\mathcal{S}_{n,m}^p(\mu, r, \lambda, b)$ of the p-valently analytic function class $\mathcal{A}_p(n)$, which consists of functions $f(z)$ satisfying the following inequality:

$$\left|\frac{1}{b}\left(\frac{z[I_p(r, \mu)f(z)]^{(m+1)} + \lambda z^2[I_p(r, \mu)f(z)]^{(m+2)}}{\lambda z[I_p(r, \mu)f(z)]^{(m+1)} + (1 - \lambda)[I_p(r, \mu)f(z)]^{(m)}} - (p - m)\right)\right| < 1 \tag{1.3}$$

$$\big(z \in \mathbb{U};\ p \in \mathbb{N};\ m \in \mathbb{N}_0;\ r \in \mathbb{Z};\ \mu \geqq 0;\ \lambda \geqq 0;\ p > \max(m, -\mu);\ b \in \mathbb{C} \setminus \{0\}\big).$$

Next, following the earlier investigations by Goodman [5], Ruscheweyh [10] and Altintas *et al.* [2] (see also [1], [7] and [12]), we define the (n, δ)-neighborhood of a function $f(z) \in \mathcal{A}_p(n)$ by (see, for details, [2, p. 1668])

$$N_{n,\delta}(f) := \left\{ g \in \mathcal{A}_p(n) : g(z) = z^p - \sum_{\tau=n+p}^{\infty} b_\tau z^\tau \text{ and } \sum_{\tau=n+p}^{\infty} \tau|a_\tau - b_\tau| \leqq \delta \right\}. \tag{1.4}$$

It follows from (1.4) that, if

$$h(z) = z^p \ \ (p \in \mathbb{N}), \tag{1.5}$$

then

$$N_{n,\delta}(h) := \left\{ g \in \mathcal{A}_p(n) : g(z) = z^p - \sum_{\tau=n+p}^{\infty} b_\tau z^\tau \text{ and } \sum_{\tau=n+p}^{\infty} \tau |b_\tau| \leqq \delta \right\}. \tag{1.6}$$

Finally, we denote by $\mathcal{R}^p_{n,m}(\mu, r, \lambda, b)$ the subclass of $\mathcal{A}_p(n)$ consisting of functions $f(z)$ which satisfy the inequality (1.7) below:

$$\left| \frac{1}{b} \left\{ [1 - \lambda(p - m - 1)][I_p(r,\mu)f(z)]^{(m+1)} + \lambda z [I_p(r,\mu)f(z)]^{(m+2)} - (p - m) \right\} \right| < p - m \tag{1.7}$$

$$\big(z \in \mathbb{U};\ p \in \mathbb{N};\ m \in \mathbb{N}_0;\ r \in \mathbb{Z};\ \mu \geqq 0;\ \lambda \geqq 0;\ p > \max(m, -\mu);\ b \in \mathbb{C} \setminus \{0\}\big).$$

The object of the present paper is to investigate the various properties and characteristics of analytic p-valent functions belonging to the subclasses

$$\mathcal{S}^p_{n,m}(\mu, r, \lambda, b) \quad \text{and} \quad \mathcal{R}^p_{n,m}(\mu, r, \lambda, b),$$

which we have defined here. Apart from deriving a set of coefficient bounds for each of these function classes, we establish several inclusion relationships involving the (n, δ)-neighborhoods of analytic p-valent functions (with negative and missing coefficients) belonging to these subclasses.

Our definitions of the function classes

$$\mathcal{S}^p_{n,m}(\mu, r, \lambda, b) \quad \text{and} \quad \mathcal{R}^p_{n,m}(\mu, r, \lambda, b)$$

are motivated essentially by the earlier investigations of Orhan and Kamali [8], and of Raina and Srivastava [9], in each of which further details and closely-related subclasses can be found. In particular, in our definition of the function classes

$$\mathcal{S}^p_{n,m}(\mu, r, \lambda, b) \quad \text{and} \quad \mathcal{R}^p_{n,m}(\mu, r, \lambda, b)$$

involving the inequalities (1.3) and (1.7), we have relaxed the parametric constraint

$$0 \leqq \lambda \leqq 1,$$

which was imposed earlier by Orhan and Kamali [8, p. 57, Equations (1.10) and (1.11)] (see also Remark 3 below).

2. A Set of Coefficient Bounds

In this section, we prove the following results which yield the coefficient inequalities for functions in the subclasses

$$\mathcal{S}^p_{n,m}(\mu, r, \lambda, b) \quad \text{and} \quad \mathcal{R}^p_{n,m}(\mu, r, \lambda, b).$$

Theorem 1. *Let* $f(z) \in \mathcal{A}_p(n)$ *be given by* (1.1). *Then* $f(z) \in \mathcal{S}^p_{n,m}(\mu, r, \lambda, b)$ *if and only if*

$$\sum_{\tau=n+p}^{\infty} \left(\frac{\tau + \mu}{p + \mu} \right)^r \binom{\tau}{m} [1 + \lambda(\tau - m - 1)](\tau - p + |b|) a_\tau \leqq |b| \left\{ \binom{p}{m} [1 + \lambda(p - m - 1)] \right\}, \tag{2.1}$$

where

$$\binom{\tau}{m} = \frac{\tau(\tau - 1) \cdots (\tau - m + 1)}{m!}.$$

Proof. Let a function $f(z)$ of the form (1.1) belong to the class $\mathcal{S}_{n,m}^{p}(\mu, r, \lambda, b)$. Then, in view of (1.2) and (1.3), we have the following inequality:

$$\Re\left(\frac{-\sum_{\tau=n+p}^{\infty}\left(\frac{\tau+\mu}{p+\mu}\right)^{r}\binom{\tau}{m}(\tau-p)[1+\lambda(\tau-m-1)]a_{\tau}z^{\tau-m}}{\binom{p}{m}[1+\lambda(p-m-1)]z^{p-m}-\sum_{\tau=n+p}^{\infty}\left(\frac{\tau+\mu}{p+\mu}\right)^{r}\binom{\tau}{m}[1+\lambda(\tau-m-1)]a_{\tau}z^{\tau-m}}\right) > -|b| \qquad (z \in \mathbb{U}). \tag{2.2}$$

Putting $z = r_1$ $(0 \leqq r_1 < 1)$ in (2.2), we observe that the expression in the denominator on the left-hand side of (2.2) is positive for $r_1 = 0$ and also for all r_1 $(0 < r_1 < 1)$. Thus, by letting $r_1 \to 1-$ through real values, (2.2) leads us to the desired assertion (2.1) of Theorem 1.

Conversely, by applying (2.1) and setting $|z| = 1$, we find by using (1.2) that

$$\begin{aligned}
&\left|\frac{z[I_p(r,\mu)f(z)]^{(m+1)}+\lambda z^2[I_p(r,\mu)f(z)]^{(m+2)}}{\lambda z[I_p(r,\mu)f(z)]^{(m+1)}+(1-\lambda)[I_p(r,\mu)f(z)]^{(m)}}-(p-m)\right| \\
&\quad = \left|\frac{\sum_{\tau=n+p}^{\infty}\left(\frac{\tau+\mu}{p+\mu}\right)^{r}\binom{\tau}{m}[1+\lambda(\tau-m-1)](\tau-p)a_{\tau}}{\binom{p}{m}[1+\lambda(p-m-1)]-\sum_{\tau=n+p}^{\infty}\left(\frac{\tau+\mu}{p+\mu}\right)^{r}\binom{\tau}{m}[1+\lambda(\tau-m-1)]a_{\tau}}\right| \\
&\quad \leqq \frac{|b|\left[\binom{p}{m}[1+\lambda(p-m-1)]-\sum_{\tau=n+p}^{\infty}\left(\frac{\tau+\mu}{p+\mu}\right)^{r}\binom{\tau}{m}[1+\lambda(\tau-m-1)]a_{\tau}\right]}{\binom{p}{m}[1+\lambda(p-m-1)]-\sum_{\tau=n+p}^{\infty}\left(\frac{\tau+\mu}{p+\mu}\right)^{r}\binom{\tau}{m}[1+\lambda(\tau-m-1)]a_{\tau}} \\
&\quad = |b|.
\end{aligned}$$

Hence, by the maximum modulus principle, we infer that $f(z) \in \mathcal{S}_{n,m}^{p}(\mu, r, \lambda, b)$, which completes the proof of Theorem 1. ☐

Remark 1. In the special case when

$$\begin{aligned} &m = 0,\ \ p = 1,\ \ b = \beta\gamma\ \ (0 < \beta \leqq 1;\ \ \gamma \in \mathbb{C} \setminus \{0\}), \\ &r = \Omega\ \ (\Omega \in \mathbb{N}_0 := \mathbb{N} \cup \{0\}),\ \ \tau = k+1,\ \ \text{and}\ \ \mu = 0, \end{aligned} \tag{2.3}$$

Theorem 1 corresponds to a result given earlier by Orhan and Kamali [8, p. 57, Lemma 1].

By using the same arguments as in the proof of Theorem 1, we can establish Theorem 2 below.

Theorem 2. *Let* $f(z) \in \mathcal{A}_p(n)$ *be given by* (1.1). *Then* $f(z) \in \mathcal{R}_{n,m}^{p}(\mu, r, \lambda, b)$ *if and only if*

$$\sum_{\tau=n+p}^{\infty}\left(\frac{\tau+\mu}{p+\mu}\right)^{r}\binom{\tau}{m}(\tau-m)[1+\lambda(\tau-p)]a_{\tau} \leqq (p-m)\left[\frac{|b|-1}{m!}+\binom{p}{m}\right]. \tag{2.4}$$

Remark 2. Making use of the same parametric substitutions as mentioned above in (2.3), Theorem 2 yields another known result due to Orhan and Kamali [8, p. 58, Lemma 2].

3. INCLUSION RELATIONSHIPS INVOLVING THE (n, δ)-NEIGHBORHOODS

In this section, we establish several inclusion relationships for the function classes

$$\mathcal{S}_{n,m}^{p}(\mu, r, \lambda, b) \quad \text{and} \quad \mathcal{R}_{n,m}^{p}(\mu, r, \lambda, b)$$

involving the (n, δ)-neighborhood defined by (1.6).

Theorem 3. *If*

$$\delta := \frac{|b|(n+p)\binom{p}{m}[1+\lambda(p-m-1)]}{(n+|b|)\left(\frac{n+p+\mu}{p+\mu}\right)^{r}\binom{n+p}{m}[1+\lambda(n+p-m-1)]} \quad (p > |b|), \tag{3.1}$$

then

$$\mathcal{S}_{n,m}^{p}(\mu, r, \lambda, b) \subset N_{n,\delta}(h). \tag{3.2}$$

Proof. Let $f(z) \in \mathcal{S}_{n,m}^{p}(\mu, r, \lambda, b)$. Then, in view of the assertion (2.1) of Theorem 1, we have

$$(n+|b|)\left(\frac{n+p+\mu}{p+\mu}\right)^{r}\binom{n+p}{m}[1+\lambda(n+p-m-1)]\sum_{\tau=n+p}^{\infty} a_{\tau} \leqq |b|\binom{p}{m}[1+\lambda(p-m-1)],$$

which yields

$$\sum_{\tau=n+p}^{\infty} a_{\tau} \leqq \frac{|b|\binom{p}{m}[1+\lambda(p-m-1)]}{(n+|b|)\left(\frac{n+p+\mu}{p+\mu}\right)^{r}\binom{n+p}{m}[1+\lambda(n+p-m-1)]}. \tag{3.3}$$

Applying the assertion (2.1) of Theorem 1 again, in conjunction with (3.3), we obtain

$$\begin{aligned}
&\left(\frac{n+p+\mu}{p+\mu}\right)^{r}\binom{n+p}{m}[1+\lambda(n+p-m-1)]\sum_{\tau=n+p}^{\infty} \tau a_{\tau} \\
&\leqq |b|\binom{p}{m}[1+\lambda(p-m-1)] + (p-|b|)\left(\frac{n+p+\mu}{p+\mu}\right)^{r} \\
&\qquad \cdot \binom{n+p}{m}[1+\lambda(n+p-m-1)]\sum_{\tau=n+p}^{\infty} a_{\tau} \\
&\leqq |b|\binom{p}{m}[1+\lambda(p-m-1)] + (p-|b|)\left(\frac{n+p+\mu}{p+\mu}\right)^{r} \\
&\qquad \cdot \binom{n+p}{m}[1+\lambda(n+p-m-1)]\,\frac{|b|\binom{p}{m}[1+\lambda(p-m-1)]}{(n+|b|)\left(\frac{n+p+\mu}{p+\mu}\right)^{r}\binom{n+p}{m}[1+\lambda(n+p-m-1)]} \\
&= |b|\binom{p}{m}[1+\lambda(p-m-1)]\left(\frac{n+p}{n+|b|}\right).
\end{aligned}$$

Hence

$$\sum_{\tau=n+p} \tau a_{\tau} \leqq \frac{|b|(n+p)\binom{p}{m}[1+\lambda(p-m-1)]}{(n+|b|)\left(\frac{n+p+\mu}{p+\mu}\right)^{r}\binom{n+p}{m}[1+\lambda(n+p-m-1)]} =: \delta \quad (p > |b|), \tag{3.4}$$

which, by virtue of (1.6), establishes the inclusion relation (3.2) of Theorem 3. □

Analogously, by applying the assertion (2.4) of Theorem 2 instead of the assertion (2.1) of Theorem 1 to functions in the class $\mathcal{R}^p_{n,m}(\mu, r, \lambda, b)$, we can prove the following inclusion relationship.

Theorem 4. *If*

$$\delta = \frac{(p-m)\left[\frac{|b|-1}{m!} + \binom{p}{m}\right]}{\left(\frac{n+p+\mu}{p+\mu}\right)^r \binom{n+p-1}{m}(1+\lambda n)} \qquad \left(\lambda > \frac{1}{p}\right), \tag{3.5}$$

then

$$\mathcal{R}^p_{n,m}(\mu, r, \lambda, b) \subset N_{n,\delta}(h). \tag{3.6}$$

Remark 3. Applying the parametric substitutions listed in (2.3), Theorem 3 and Theorem 4 would yield the known results due to Orhan and Kamali [8, p. 58, Theorem 1; p. 59, Theorem 2]. Incidentally, just as we indicated in Section 1 above, the condition $\lambda > 1$ is needed in the proof of one of these known results [8, p. 59, Theorem 2]. This implies that the constraint $0 \leqq \lambda \leqq 1$ in [8, p. 57, Equations (1.10) and (1.11)] should be replaced by the less stringent constraint $\lambda \geqq 0$.

4. FURTHER NEIGHBORHOOD PROPERTIES

In this last section, we determine the neighborhood properties for each of the following (slightly modified) function classes:

$$\mathcal{S}^{p,\alpha}_{n,m}(\mu, r, \lambda, b) \quad \text{and} \quad \mathcal{R}^{p,\alpha}_{n,m}(\mu, r, \lambda, b).$$

Here the class $\mathcal{S}^{p,\alpha}_{n,m}(\mu, r, \lambda, b)$ consists of functions $f(z) \in \mathcal{A}_p(n)$ for which there exists another function $g(z) \in \mathcal{S}^p_{n,m}(\mu, r, \lambda, b)$ such that

$$\left|\frac{f(z)}{g(z)} - 1\right| < p - \alpha \qquad (z \in \mathbb{U};\ 0 \leqq \alpha < p) \tag{4.1}$$

Analogously, the class $\mathcal{R}^{p,\alpha}_{n,m}(\mu, r, \lambda, b)$ consists of functions $f(z) \in \mathcal{A}_p(n)$ for which there exists another function $g(z) \in \mathcal{R}^p_{n,m}(\mu, r, \lambda, b)$ satisfying the inequality (4.1).

Theorem 5. *Let $g(z) \in \mathcal{S}^p_{n,m}(\mu, r, \lambda, b)$. Suppose also that*

$$\alpha = p - \frac{\delta}{n+p} \cdot \left[\frac{(n+|b|)\left(\frac{n+p+\mu}{p+\mu}\right)^r \binom{n+p}{m}[1+\lambda(n+p-m-1)]}{(n+|b|)\left(\frac{n+p+\mu}{p+\mu}\right)^r \binom{n+p}{m}[1+\lambda(n+p-m-1)] - |b|\binom{p}{m}[1+\lambda(p-m-1)]}\right]. \tag{4.2}$$

Then

$$N_{n,\delta}(g) \subset \mathcal{S}^{p,\alpha}_{n,m}(\mu, r, \lambda, b). \tag{4.3}$$

Proof. Suppose that $f(z) \in N_{n,\delta}(g)$. We then find from (1.4) that

$$\sum_{\tau=n+p}^{\infty} \tau |a_\tau - b_\tau| \leqq \delta, \tag{4.4}$$

which readily implies the following coefficient inequality:

$$\sum_{\tau=n+p}^{\infty} |a_\tau - b_\tau| \leqq \frac{\delta}{n+p} \quad (n \in \mathbb{N}). \tag{4.5}$$

Next, since $g \in \mathcal{S}^{p}_{n,m}(\mu, r, \lambda, b)$, we have

$$\sum_{\tau=n+p}^{\infty} b_\tau \leqq \frac{|b|\binom{p}{m}[1+\lambda(p-m-1)]}{(n+|b|)\left(\frac{n+p+\mu}{p+\mu}\right)^r \binom{n+p}{m}[1+\lambda(n+p-m-1)]}, \tag{4.6}$$

so that

$$\begin{aligned}
\left|\frac{f(z)}{g(z)} - 1\right| &< \frac{\sum_{\tau=n+p}^{\infty}|a_\tau - b_\tau|}{1-\sum_{\tau=n+p}^{\infty} b_\tau} \\
&\leqq \frac{\delta}{n+p}\left[1 - \frac{|b|\binom{p}{m}[1+\lambda(p-m-1)]}{(n+|b|)\left(\frac{n+p+\mu}{p+\mu}\right)^r \binom{n+p}{m}[1+\lambda(n+p-m-1)]}\right]^{-1} \\
&= \frac{\delta}{n+p}\left[\frac{(n+|b|)\left(\frac{n+p+\mu}{p+\mu}\right)^r \binom{n+p}{m}[1+\lambda(n+p-m-1)]}{(n+|b|)\left(\frac{n+p+\mu}{p+\mu}\right)^r \binom{n+p}{m}[1+\lambda(n+p-m-1)] - |b|\binom{p}{m}[1+\lambda(p-m-1)]}\right] \\
&= p - \alpha,
\end{aligned}$$

provided that α is given precisely by (4.2). Thus, by definition, $f \in \mathcal{S}^{p,\alpha}_{n,m}(\mu, r, \lambda, b)$ for α given by (4.2). This evidently completes the proof of Theorem 5. □

The proof of Theorem 6 below is much similar to that of Theorem 5; hence the proof of Theorem 6 is being omitted.

Theorem 6. *Let* $g(z) \in \mathcal{R}^{p,\alpha}_{n,m}(\mu, r, \lambda, b)$. *Suppose also that*

$$\alpha = p - \frac{\delta}{n+p}\left[\frac{\left(\frac{n+p+\mu}{p+\mu}\right)^r \binom{n+p}{m}(n+p-m)(1+\lambda n)}{\left(\frac{n+p+\mu}{p+\mu}\right)^r \binom{n+p}{m}(n+p-m)(1+\lambda n) - (p-m)\left[\frac{|b|-1}{m!} + \binom{p}{m}\right]}\right]. \tag{4.7}$$

Then

$$N_{n,\delta}(g) \subset \mathcal{R}^{p,\alpha}_{n,m}(\mu, r, \lambda, b). \tag{4.8}$$

Remark 4. Applying the parametric substitutions listed in (2.3), Theorem 5 and Theorem 6 would yield the known results due to Orhan and Kamali [8, p. 60, Theorem 3; p. 61, Theorem 4].

REFERENCES

[1] O. ALTINTAŞ, Ö. ÖZKAN AND H.M. SRIVASTAVA, Neighborhoods of a class of analytic functions with negative coefficients, *Appl. Math. Lett.*, **13**(3) (2000), 63–67.

[2] O. ALTINTAŞ, Ö. ÖZKAN AND H.M. SRIVASTAVA, Neighborhoods of a certain family of multivalent functions with negative coefficients, *Comput. Math. Appl.*, **47** (2004), 1667–1672.

[3] N.E. CHO AND T.H. KIM, Multiplier transformations and strongly close-to-convex functions, *Bull. Korean Math. Soc.*, **40** (2003), 399–410.

[4] N.E. CHO AND H.M. SRIVASTAVA, Argument estimates of certain analytic functions defined by a class of multiplier transformations, *Math. Comput. Modelling*, **37** (2003), 39–49.

[5] A.W. GOODMAN, Univalent functions and nonanalytic curves, *Proc. Amer. Math. Soc.*, **8** (1957), 598–601.

[6] S.S. KUMAR, H.C. TANEJA AND V. RAVICHANDRAN, Classes of multivalent functions defined by Dziok-Srivastava linear operator and multiplier transformation, *Kyungpook Math. J.*, **46** (2006), 97–109.

[7] G. MURUGUSUNDARAMOORTHY AND H.M. SRIVASTAVA, Neighborhoods of certain classes of analytic functions of complex order, *J. Inequal. Pure. Appl. Math.*, **5**(2) (2004), Art. 24. [ONLINE: `http://jipam.vu.edu.au/article.php?sid=374`].

[8] H. ORHAN AND M. KAMALI, Neighborhoods of a class of analytic functions with negative coefficients, *Acta Math. Acad. Paedagog. Nyházi. (N.S.)*, **21**(1) (2005), 55–61 (electronic).

[9] R. K. RAINA AND H. M. SRIVASTAVA, Inclusion and neighborhood properties of some analytic and multivalent functions, *J. Inequal. Pure. Appl. Math.*, **7**(1) (2006), Art.5. [ONLINE: `http://jipam.vu.edu.au/article.php?sid=640`].

[10] S. RUSCHEWEYH, Neighborhoods of univalent functions, *Proc. Amer. Math. Soc.*, **81** (1981), 521–527.

[11] G.Ş. SĂLĂGEAN, Subclasses of univalent functions, *Complex Analysis*: *Fifth Romanian-Finnish Seminar. Part* 1 (Bucharest, 1981), pp. 362–372, Lecture Notes in Mathematics, No. 1013, Springer-Verlag, Berlin, 1983.

[12] H.M. SRIVASTAVA AND S. OWA (Eds.), *Current Topics in Analytic Function Theory*, World Scientific Publishing Company, Singapore, New Jersey, London and Hong Kong, 1992.

[13] B.A. URALEGADDI AND C. SOMANATHA, Certain classes of univalent functions, in *Current Topics in Analytic Function Theory* (H. M. Srivastava and S. Owa, Eds.), pp. 371–374, World Scientific Publishing Company, Singapore, New Jersey, London and Hong Kong, 1992.