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ON OSCILLATORY SOLUTIONS OF NONLINEAR DIFFERENTIAL EQUATIONS WITH ADVANCED ARGUMENTS

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We consider the differential equation with power nonlinearities

$$u^{(n)}(t) = (-1)^k \sum_{i=1}^{m} p_i(t)|u(\tau_i(t))|^{\lambda_i} \operatorname{sgn} u(\tau_i(t)), \tag{1_k}$$

where $n \geq 2$, $m \geq 2$, $k \in \{1, 2\}$, $\lambda_m > \cdots > \lambda_1 > 0$, $p_i : [0, +\infty[\to [0, +\infty[$ $(i = 1, \dots, m)$ are locally Lebesgue integrable functions, and $\tau_i : [0, +\infty[\to [0, +\infty[$ $(i = 1, \dots, m)$ are continuous functions such that

$$\tau_i(t) \geq t \quad \text{for} \quad t \geq 0 \ \ (i = 1, \dots, m).$$

A solution u of the equation (1_k), defined on some interval $[a, +\infty[\subset [0, +\infty[$, is said to be *proper* if it is not identically zero in any neighborhood of $+\infty$.

A proper solution of the equation (1_k) is said to be *oscillatory* if it has a sequence of zeros converging to $+\infty$; it is said to be *nonoscillatory* otherwise.

According to [1] and [6], we say that the equation (1_k) has *Property A* if every proper solution of this equation for n even is oscillatory and for n odd either is oscillatory or satisfies the condition

$$\lim_{t \to +\infty} u^{(i)}(t) = 0 \quad (i = 0, \dots, n-1). \tag{2}$$

Equation (1_k) has *Property B* if every proper solution of this equation for n even either is oscillatory or satisfies (2) or satisfies the condition

$$\lim_{t \to +\infty} |u^{(i)}(t)| = +\infty \quad (i = 0, \dots, n-1), \tag{3}$$

and for n odd either is oscillatory or satisfies (3).

In [3], there are obtained necessary and sufficient conditions for the equation (1_k) to have properties A and B in the case $\lambda_m < 1$. In the present paper, the case is considered where $\lambda_m > 1$. The results given below are new not only for $\tau_i(t) \not\equiv t$ $(i = 1, \dots, m)$, but also for $\tau_i(t) \equiv t$ $(i = 1, \dots, m)$, i.e., for the case where the equation (1_k) has the form

$$u^{(n)}(t) = (-1)^k \sum_{i=1}^{m} p_i(t)|u(t)|^{\lambda_i} \operatorname{sgn} u(t) \tag{4_k}$$

(compare with results from [1]–[5], [7]–[9]).

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Theorem. *Let* $m_0 \in \{1, \dots, m-1\}$*,* $\lambda_{m_0+1} > 1$*, n be odd (even), and*

$$\int_0^{+\infty} t^{n-2}\Big(\sum_{i=m_0+1}^{m} [\tau_i(t)]^{\lambda_i} p_i(t)\Big)\,dt = +\infty.$$

Then the condition

$$\int_0^{+\infty} t^{n-1}\Big(\sum_{i=1}^{m_0} p_i(t)\Big)\,dt = +\infty \tag{5}$$

is sufficient and, if

$$\int_0^{+\infty} t^{n-1}\Big(\sum_{i=m_0+1}^{m} p_i(t)\Big)\,dt < +\infty, \tag{6}$$

also necessary for the equation (1_1) *(equation* (1_2)*) to have property* A *(property* B*).*

Corollary. *Let* $m_0 \in \{1, \dots, m-1\}$*,* $\lambda_{m_0+1} > 1$*, n be odd (even), and*

$$\int_0^{+\infty} \Big(\sum_{i=m_0+1}^{m} t^{n-2+\lambda_i} p_i(t)\Big)\,dt = +\infty.$$

Then the condition (5) *is sufficient and, if (6) is fulfilled, also necessary for the equation* (4_1) *(equation* (4_2)*) to have property* A *(property* B*).*

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