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Professor Nikolai Viktorovich Azbelev is 80

A well-known Russian mathematician Nikolai Viktorovich Azbelev was born on April 15, 1922 in selo (small village) Bazlovo, Pskov Region, Russia, in the family of a physician. His study at the Mechanics and Mathematics Faculty of M. V. Lomonosov Moscow State University was interrupted because of his military service in the Soviet Army (World War II). In 1945 he entered Moscow State Aviation Institute, which he graduated in 1949 with engineer's qualification and started work at the Design Bureau directed by academician A. A. Mikulin. In 1951–1954 he was a post-graduate student at the Chair of Higher Mathematics of Moscow Machine and Instruments Institute where his supervisor was Professor Bentsion I. Segal. In 1954 Azbelev defended his Candidate's Thesis "On the boundaries of feasibility of Chaplygin's Theorem on differential inequalities" at Moscow State University. The same year Azbelev left Moscow for Izhevsk to work as the head of the Chair of Higher Mathematics at the Izhevsk Mechanical Institute (IMI). In Izhevsk N.V.Azbelev wrote his Doctor's Thesis "On the Chaplygin problem", which he defended in 1962 at the Kazan State University. In 1963 he was granted the title of professor. In the period 1966–75 Azbelev was the head of the Chair of Mathematics of the Tambov Institute for Chemical Engineering. In 1975 he moved to the Perm Polytechnic Institute (PPI) where he founded the Chair of Mathematical Analysis. Azbelev's scientific expertise and leadership contributed immensely to the development of his chair to the extent that it has become one of the well known mathematical centers and the kernel of the Perm Seminar on Functional Differential Equations. Since 1994 N. Azbelev has been the head of the Research Center for Functional Differential Equations at the Perm State Technical University (former PPI).

Nikolai Azbelev's research works cover differential and functional differential equations and inequalities, numerical methods, stability theory, boundary value problems and calculus of variations. He is one of the founders of the Russian scientific school on differential and integral inequalities. In his first papers N. Azbelev gave a solution to the Chaplygin problem on the boundaries of feasibility of the differential inequality theorem. These works essentially expanded the area of applications of differential inequalities covering, in particular, some topics in the stability theory, numerical methods, various estimates of solutions.

An invariable feature of Azbelev's activity is his ability to unite around himself colleagues and students talented in mathematics. So one of the first things N. Azbelev did on arriving at IMI was to found the Izhevsk Mathematical Seminar. This became the central meeting point for mathematicians and engineers in the Izhevsk area. Azbelev's warmth and sensitiveness were tremendously important to the creation of the mathematical community around IMI. The works of the participants of this seminar concerning the theory of integral, differential and difference inequalities allowed to answer a number of questions on existence, uniqueness and asymptotic behavior of solutions to differential equations. Most of those results are based on Azbelev's "fork principle" which is very useful to find invariant sets of operators. Other works of the Izhevsk Seminar are devoted to effective conditions and criteria of the unique solvability for boundary value problems for ordinary differential equations and investigation of properties of the Green function to those problems. Since 1961 the major attention of N. Azbelev and his seminar has been focused on the problems of the general theory of equations with discontinuous operators and, some later, on differential equations with deviating arguments. These equations and their wide generalization, the so called functional differential equations (FDE's), became the central subject of investigations for N. Azbelev and the participants of the Tambov Seminar (in

the period 1966–1975) and the Perm Seminar (since 1975). There was suggested and systematically worked out a new approach different from the traditional one. This new approach offered a direct way to use the theory of linear equations in Banach spaces. It can be said that the contemporary theory of FDE's, treated thoroughly in the monographs "Introduction to the theory of functional differential equations" (translated into English and published in the United States) and "Methods of the contemporary theory of linear functional differential equations", exists thanks to new concepts and approaches worked by Azbelev jointly with L. F. Rakhmatullina. Nowadays this theory covers many classes of equations containing ordinary derivatives of a sought-for function. Let us point out some contributions of Azbelev to the theory of FDE's: the results on the reducibility of the equation (the property of the equation to allow a kind of regularization); the results on boundary value problems concerning the solvability, representation of solutions, estimates and properties of solutions; the results on the stability of solutions, in particular, the famous W-method and its wide development. Of special importance are the contributions of Professor Azbelev to the creation and the development of the theory of abstract functional differential equations (AFDE's). Such a class of equations in Banach spaces is an essential further generalization of equations with ordinary derivatives. n-th order FDE's, systems with impulses, equations with quasi-derivatives, and some classes of singular equations may be considered in the context of the Theory of AFDE. Note also that this theory has become a very useful tool for solving some variational problems, especially in the cases where the problem of minimization of a functional is unsolvable within the framework of the classical calculus of variations, as well as for the study of boundary value problems with an arbitrary finite number of boundary conditions in the form of equalities and inequalities.

Azbelev's influence is not limited to the original and fundamental contributions he has made to the theory of integral and functional differential equations. He has had significant influence on the education of young mathematicians, as indicated by his supervising over 60 Candidates as well as 10 Doctors of Sciences. Azbelev's joy in doing mathematics both individually and with other people had a profound positive impact on the lives of many of his friends and pupils. To the post-graduate students and colleagues who work with him, he gives much more than intellectual stimulus. Until now, he is tireless in his support for young scientists of any promise. Also, it should be pointed out that early mathematical schools for children talented in Mathematics were founded in the 60's at Izhevsk, Russia, thanks to the efforts and the enthusiasm of Professor Azbelev.

Apart from his research and teaching work, Nikolai Azbelev is also concerned with publishing matters. During more than 10 years he was the editor-in-chief of the "Boundary Value Problems" and the "Functional Differential Equations" (Perm), more than 25 years he was a member of editorial board of "Differentsial'nye Uravneniya". Nowadays he is a member of editorial boards of "Memoirs on Differential Equations and Mathematical Physics" (Georgia), "Functional Differential Equations" (Israel) and some others.

Professor Azbelev has received many honors and awards over the course of his career. He is the holder of the Badge of Honor, he has been recognized as a Meritorius Science Worker of the Russian Federation, and he is an Honorary member of the Academy of Nonlinear Sciences. In recognition and appreciation of outstanding contributions to the world of science and education, Nikolai Viktorovich was awarded the Grant of the Russian Federation President for Leading Scientists and selected by the International Soros Science Education Program as a George Soros Emeritus Professor.

On his 80th birthday Nikolai Viktorovich Azbelev is, as always, full of intentions, ideas and energy. We wish him good health, long years of further successful activity and new talented pupils.

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List of Main Publications of N. Azbelev

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3. On the limits of applicability of S. A. Chaplygin's theorem. (Russian) *Dokl. Akad. Nauk SSSR***89**(1953), No. 4, 589–591.
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