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Short Communications

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ON THE SOLVABILITY OF THE PERIODIC TYPE BOUNDARY VALUE PROBLEM FOR LINEAR SYSTEMS OF GENERALIZED ORDINARY DIFFERENTIAL EQUATIONS

Abstract. Effective necessary and sufficient conditions are given for the solvability of periodic type boundary value problems for linear systems of generalized ordinary differential equations.

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Let ω be a positive number. For the linear system of generalized ordinary differential equations

$$dx(t) = dA(t)\cdot x(t) + df(t) \quad \text{for} \quad t \in \mathbb{R} \tag{1}$$

we investigate the periodic boundary value problem

$$x(t+\omega) = x(t) \quad \text{for} \quad t \in \mathbb{R}, \tag{2}$$

where $A = (a_{ik})_{i,k=1}^{n} : \mathbb{R} \to \mathbb{R}^{n\times n}$ and $f = (f_i)_{i=1}^{n} : \mathbb{R} \to \mathbb{R}^n$ are, respectively, matrix- and vector-functions with bounded total variation components on the closed interval $[0,\omega]$, and

$$A(t+\omega) = A(t) + A(\omega) \quad \text{for} \quad t \in \mathbb{R} \tag{3}$$

and

$$f(t+\omega) = f(t) + f(\omega) \quad \text{for} \quad t \in \mathbb{R}. \tag{4}$$

Note that the following ω-type boundary value problem

$$x(t+\omega) = x(t) + c \quad \text{for} \quad t \in \mathbb{R},$$

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where $c \in \mathbb{R}^n$, is reduced to the problem (2) by the transformation

$$y(t) = x(t) - \frac{t}{\omega}\, c \quad \text{for} \quad t \in \mathbb{R},$$

so that we consider only the problem (2).

Along with the system (1) we consider the corresponding homogeneous system

$$dx(t) = dA(t) \cdot x(t). \tag{1_0}$$

In the present paper, we give effective necessary and sufficient conditions for unique solvability of the problem (1), (2). Analogous results for general linear boundary value problems for systems of ordinary differential equations belong to T. Kiguradze [11], and for ω-periodic boundary problem for systems of ordinary differential equations and functional differential equations they belong to I. Kiguradze [8]–[10].

Some questions of the ω-periodic boundary value problem for the system (1) have been considered in [1], [2], [12].

To a considerable extent, the interest to the theory of generalized ordinary differential equations has also been stimulated by the fact that this theory enables one to investigate ordinary differential, impulsive and difference equations from a unified point of view [1]–[7].

In the paper the use will be made of the following notation and definitions.

$\mathbb{R} =]-\infty, +\infty[$, $\mathbb{R}_+ = [0, +\infty[$, $[a, b]$ and $]a, b[$ $(a, b \in \mathbb{R})$ are, respectively, closed and open intervals.

$\mathbb{R}^{n\times m}$ is the space of all real $n \times m$ matrices $X = (x_{ij})_{i,j=1}^{n,m}$ with the norm

$$\|X\| = \max_{j=1,\dots,m} \sum_{i=1}^{n} |x_{ij}|.$$

$O_{n\times m}$ (or O) is the zero $n \times m$ matrix.

If $X = (x_{ij})_{i,j=1}^{n,m} \in \mathbb{R}^{n\times m}$, then

$$|X| = \left(|x_{ij}|\right)_{i,j=1}^{n,m}.$$

$$\mathbb{R}_+^{n\times m} = \left\{(x_{ij})_{i,j=1}^{n,m} : x_{ij} \geq 0 \ \ (i = 1, \dots, n; \ \ j = 1, \dots, m)\right\}.$$

$\mathbb{R}^n = \mathbb{R}^{n\times 1}$ is the space of all real column n-vectors $x = (x_i)_{i=1}^{n}$; $\mathbb{R}_+^n = \mathbb{R}_+^{n\times 1}$.

If $X \in \mathbb{R}^{n\times n}$, then X^{-1}, $\det X$ and $r(X)$ are, respectively, the matrix inverse to X, the determinant of X and the spectral radius of X; I_n is the identity $n \times n$-matrix.

The inequalities between the matrices are understood componentwise.

A matrix-function is said to be continuous, integrable, nondecreasing, etc., if each of its component is such.

If $X : \mathbb{R} \to \mathbb{R}^{n\times m}$ is a matrix-function, then $\overset{b}{\underset{a}{\mathrm{V}}}(X)$ is the sum of total variations on $[a, b]$ of its components x_{ij} $(i = 1, \dots, n;\ j = 1, \dots, m)$;

$V(X)(t) = (v(x_{ij})(t))_{i,j=1}^{n,m}$, where $v(x_{ij})(0) = 0$, $v(x_{ij})(t) = \overset{t}{\underset{0}{\mathrm{V}}}(x_{ij})$ for $t > 0$, $v(x_{ij})(t) = -\overset{0}{\underset{t}{\mathrm{V}}}(x_{ij})$ for $t < 0$;

$X(t-)$ and $X(t+)$ are, respectively, the left and the right limits of the matrix-function $X : \mathbb{R} \to \mathbb{R}^{n\times m}$ at the point t;

$$d_1X(t) = X(t) - X(t-), \quad d_2X(t) = X(t+) - X(t);$$

$$\|X\|_s = \sup\left\{\|X(t)\| : t \in [0,\omega]\right\}, \quad |X|_s = (\|x_{ij}\|_s)_{i,j=1}^{n,m}.$$

$\mathrm{BV}([a,b],\mathbb{R}^{n\times m})$ is the normed space of all bounded variation matrix-functions $X : [a,b] \to \mathbb{R}^{n\times m}$ (i.e., such that $\overset{b}{\underset{a}{\mathrm{V}}}(X) < \infty$) with the norm $\|X\|_s$.

$\mathrm{BV}_\omega(\mathbb{R},\mathbb{R}^{n\times m})$ is the set of all matrix-functions $X : \mathbb{R} \to \mathbb{R}^{n\times m}$ such that

$$X(t+\omega) = X(t) + X(\omega) \quad \text{for} \quad t \in \mathbb{R}$$

and its restriction on $[0,\omega]$ belongs to $\mathrm{BV}([0,\omega],\mathbb{R}^{n\times m})$.

$s_j : \mathrm{BV}_\omega(\mathbb{R},\mathbb{R}) \to \mathrm{BV}_\omega(\mathbb{R},\mathbb{R})$ $(j = 0,1,2)$ are the operators defined, respectively, by

$$s_1(x)(a) = s_2(x)(a) = 0,$$

$$s_1(x)(t) = \sum_{0<\tau\le t} d_1x(\tau) \quad \text{and} \quad s_2(x)(t) = \sum_{0\le\tau<t} d_2x(\tau) \quad \text{for} \quad t > 0,$$

$$s_1(x)(t) = -\sum_{t<\tau\le 0} d_1x(\tau) \quad \text{and} \quad s_2(x)(t) = -\sum_{t\le\tau<0} d_2x(\tau) \quad \text{for} \quad t < 0,$$

and

$$s_0(x)(t) = x(t) - s_1(x)(t) - s_2(x)(t) \quad \text{for} \quad t \in \mathbb{R}.$$

If $g : [a,b] \to \mathbb{R}$ is a nondecreasing function, $x : [a,b] \to \mathbb{R}$ and $a \le s < t \le b$, then

$$\int_s^t x(\tau)\,dg(\tau) = \int_{]s,t[} x(\tau)\,ds_0(g)(\tau) + \sum_{s<\tau\le t} x(\tau)d_1g(\tau) + \sum_{s\le\tau<t} x(\tau)d_2g(\tau),$$

where $\int_{]s,t[} x(\tau)\,ds_0(g)(\tau)$ is the Lebesgue–Stieltjes integral over the open interval $]s,t[$ with respect to the measure $\mu_0(s_0(g))$ corresponding to the function $s_0(g)$.

If $a = b$, then we assume

$$\int_a^b x(t)\,dg(t) = 0,$$

and if $a > b$, then we assume

$$\int_a^b x(t)\,dg(t) = -\int_b^a x(t)\,dg(t).$$

If $g(t) \equiv g_1(t) - g_2(t)$, where g_1 and g_2 are nondecreasing functions, then

$$\int_s^t x(\tau)\,dg(\tau) = \int_s^t x(\tau)\,dg_1(\tau) - \int_s^t x(\tau)\,dg_2(\tau) \quad \text{for} \quad s,\ t \in \mathbb{R}.$$

If $G = (g_{ik})_{i,k=1}^{l,n} : [a,b] \to \mathbb{R}^{l\times n}$ is a nondecreasing matrix-function and $X = (x_{kj})_{k,j=1}^{n,m} : [a,b] \to \mathbb{R}^{n\times m}$, then

$$\int_s^t dG(\tau)\cdot X(\tau) = \bigg(\sum_{k=1}^{n} \int_s^t x_{kj}(\tau) dg_{ik}(\tau)\bigg)_{i,j=1}^{l,m} \quad \text{for} \quad a \le s \le t \le b,$$

$$S_j(G)(t) \equiv \big(s_j(g_{ik})(t)\big)_{i,k=1}^{l,n} \quad (j = 0,1,2).$$

If $G_j : [a,b] \to \mathbb{R}^{l\times n}$ $(j = 1,2)$ are nondecreasing matrix-functions, $G = G_1 - G_2$ and $X : [a,b] \to \mathbb{R}^{n\times m}$, then

$$\int_s^t dG(\tau)\cdot X(\tau) = \int_s^t dG_1(\tau)\cdot X(\tau) - \int_s^t dG_2(\tau)\cdot X(\tau) \ \text{ for } \ s,\ t \in \mathbb{R},$$

$$S_k(G) = S_k(G_1) - S_k(G_2) \quad (k = 0,1,2).$$

$\mathcal{A} : \mathrm{BV}_\omega(\mathbb{R},\mathbb{R}^{n\times n}) \times \mathrm{BV}_\omega(\mathbb{R},\mathbb{R}^{n\times m}) \to \mathrm{BV}_\omega(\mathbb{R},\mathbb{R}^{n\times m})$ is the operator defined by

$$\mathcal{A}(X,Y)(t+\omega) \equiv \mathcal{A}(X,Y)(t) + \mathcal{A}(X,Y)(\omega),$$

where

$$\mathcal{A}(X,Y)(0) = 0,$$

$$\mathcal{A}(X,Y)(t) = Y(t) + \sum_{0<\tau\le t} d_1X(\tau)\cdot\big(1 - d_1X(\tau)\big)^{-1} d_1Y(\tau) - \\ - \sum_{0\le\tau<t} d_2X(\tau)\cdot\big(1 + d_2X(\tau)\big)^{-1} d_2Y(\tau) \quad (0 < t \le \omega)$$

for every $X \in \mathrm{BV}_\omega(\mathbb{R},\mathbb{R}^{n\times n})$ such that

$$1 + (-1)^j d_jX(t) \ne 0 \ \text{ for } \ t \in [0,\omega] \quad (j = 1,2).$$

For every matrix-function $X \in \mathrm{BV}([0,\omega],\mathbb{R}^{n\times n})$ such that $\det(I_n - d_1X(t)) \neq 0$ for $t \in [0,\omega]$ we put

$$[X(t)]_0 = (I_n - d_1X(t))^{-1},$$
$$[X(t)]_i = (I_n - d_1X(t))^{-1} \int\limits_0^t dX_-(\tau)\cdot [X(\tau)]_{i-1}$$
$$\text{for} \quad t\in[0,\omega] \quad (i=1,2,\dots), \tag{5_1}$$
$$(X(t))_0 = O_{n\times n}, \quad (X(t))_1 = X(t) - X(0),$$
$$(X(t))_{i+1} = \int\limits_0^t dX_-(\tau)\cdot (X(\tau))_i \quad \text{for} \quad t\in[0,\omega] \quad (i=1,2,\dots) \tag{6_1}$$

and

$$V_1(X)(t) = |(I_n - d_1X(t))^{-1}|V(X_-)(t),$$
$$V_{i+1}(X)(t) = |(I_n - d_1X(t))^{-1}| \int\limits_0^t dV(X_-)(\tau)\cdot V_i(X)(\tau)$$
$$\text{for} \;\; t\in[0,\omega] \;\; (i=1,2,\dots), \tag{7_1}$$

where $X_-(t) = X(t-)$ for $0 < t \leq \omega$ $(X_-(0) = X(0))$; and for every $X \in \mathrm{BV}([0,\omega],\mathbb{R}^{n\times n})$ such that $\det(I_n + d_2X(t)) \neq 0$ for $t\in[0,\omega]$ we put

$$[X(t)]_0 = (I_n + d_2X(t))^{-1},$$
$$[X(t)]_i = (I_n + d_2X(t))^{-1} \int\limits_\omega^t dX_+(\tau)\cdot [X(\tau)]_{i-1}$$
$$\text{for} \;\; t\in[0,\omega] \quad (i=1,2,\dots), \tag{5_2}$$
$$(X(t))_0 = O_{n\times n}, \quad (X(t))_1 = X(t) - X(\omega),$$
$$(X(t))_{i+1} = \int\limits_0^t dX_+(\tau)\cdot (X(\tau))_i \quad \text{for} \quad t\in[0,\omega] \quad (i=1,2,\dots) \tag{6_2}$$

and

$$V_1(X)(t) = |(I_n + d_2X(t))^{-1}|(V(X_+)(t)(\omega) - V(X_+)(t)|,$$
$$V_{i+1}(X)(t) = |(I_n + d_2X(t))^{-1}| \Big| \int\limits_\omega^t dV(X_+)(\tau)\cdot V_i(X)(\tau) \Big|$$
$$\text{for} \;\; t\in[0,\omega] \;\; (i=1,2,\dots), \tag{7_2}$$

where $X_+(t) = X(t+)$ for $0 \leq t < \omega$ $(X_+(\omega) = X(\omega))$.

A vector-function $x:\mathbb{R}\to\mathbb{R}^n$ is said to be a solution of the system (1) if $x\in \mathrm{BV}([s,t],\mathbb{R}^n)$ and

$$x(t)=x(s)+\int\limits_s^t dA(\tau)\cdot x(\tau)+f(t)-f(s) \quad \text{for} \ \ s\le t.$$

We assume that $A(0)=O_{n\times n}$, $f(0)=0$ and

$$\det(I_n+(-1)^j d_jA(t))\neq 0 \quad \text{for} \quad t\in[0,\omega] \quad (j=1,2). \tag{8}$$

The above inequalities guarantee the unique solvability of the Cauchy problem for the corresponding systems (see [12, Theorem III.1.4]).

Theorem 1. *Let the conditions* (3), (4) *and* (8) *hold. Then the system* (1) *has a unique ω-periodic solution if and only if the corresponding homogeneous problem* (1_0) *has only the trivial ω-periodic solution, i.e.,*

$$\det(Y(0)-Y(\omega))\neq 0, \tag{9}$$

where Y is a fundamental matrix of the system (1_0).

Corollary 1. *Let the conditions* (3), (4) *and* (8) *hold. Let, in addition, the matrix-function A be such that the matrices $S_0(A)(t)$, $S_1(A)(t)$ and $S_2(A)(t)$ are pairwise permutable for every $t\in[0,\omega]$ and there exists $t_0\in[0,\omega]$ such that*

$$\int\limits_{t_0}^t S_0(A)(\tau)\,dS_0(A)(\tau)=\int\limits_{t_0}^t dS_0(A)(\tau)\cdot S_0(A)(\tau) \quad \textit{for} \quad t\in[0,\omega].$$

Then the system (1) *has a unique ω-periodic solution if and only if*

$$\begin{aligned}\det\Bigg(&\exp\big(S_0(A)(\omega)-S_0(A)(t_0)\big)\prod_{t_0\le\tau<\omega}(I_n+d_2A(\tau))\times\\ &\times\prod_{t_0<\tau\le\omega}(I_n-d_1A(\tau))^{-1}-\exp\big(S_0(A)(0)-S_0(A)(t_0)\big)\times\\ &\times\prod_{0\le\tau<t_0}(I_n+d_2A(\tau))^{-1}\prod_{0<\tau\le t_0}(I_n-d_1A(\tau))\Bigg)\neq 0.\end{aligned}$$

Remark 1. If the homogeneous system (1_0) has a nontrivial ω-periodic solution, then for every $f\in \mathrm{BV}_\omega(\mathbb{R},\mathbb{R}^n)$ there exists a vector $c\in\mathbb{R}^n$ such that the system

$$dx(t)=dA(t)\cdot x(t)+d(f(t)-ct) \quad \text{for} \quad t\in\mathbb{R}$$

has no ω-periodic solution.

Definition 1. A matrix-function $G_\omega:\mathbb{R}\times\mathbb{R}\to\mathbb{R}^{n\times n}$ is said to be the Green matrix of the problem (1_0),(2) if

$$G_\omega(t+\omega,\tau+\omega)=G_\omega(t,\tau), \quad G_\omega(t,t+\omega)-G_\omega(t,t)=I_n \quad \text{for} \quad t \ \text{and} \ \tau\in\mathbb{R},$$

and the matrix-function $G_\omega(\cdot,\tau):\mathbb{R}\to\mathbb{R}^{n\times n}$ is a fundamental matrix of the system (1_0).

If the condition (3) holds and the problem $(1_0),(2)$ has only the trivial solution, then there exists a unique Green matrix and it admits the following representation

$$G_\omega(t,\tau)=Y(t)\big(Y^{-1}(\omega)Y(0)-I_n\big)^{-1}Y^{-1}(\tau) \quad \text{for} \quad t \quad \text{and} \quad \tau\in\mathbb{R},$$

where Y is a fundamental matrix of the system (1_0).

Theorem 2. *Let the conditions* (3), (4) *and* (8) *hold. Then the system* (1) *has a unique ω-periodic solution if and only if the corresponding homogeneous problem* (1_0) *has only the trivial ω-periodic solution. If the latter condition holds, then the ω-periodic solution x of the system* (1) *admits the representation*

$$x(t)=\int\limits_t^{t+\omega} G_\omega(t,s)d\mathcal{A}(A,f)(s) \quad \textit{for} \quad t\in\mathbb{R},$$

where G_ω is the Green matrix of the problem (1_0), (2).

The last representation can be rewritten in the form

$$x(t)=\int\limits_t^{t+\omega} d_sG_\omega(t,s)\cdot f(s)-\sum_{t<s\le t+\omega} d_1G_\omega(t,s)\cdot d_1f(s)$$
$$+\sum_{t\le s<t+\omega} d_2G_\omega(t,s)\cdot d_2f(s) \quad \text{for} \quad t\in\mathbb{R}.$$

In general, it is quite difficult to verify the condition (9) directly even in the case where one is able to write out the fundamental matrix of the system (1_0) explicitly. Therefore it is important to seek for effective conditions which would guarantee the absence of nontrivial solutions of the homogeneous problem (1_0), (2).

Theorem 3. *Let the conditions* (3), (4) *and* (8) *hold. Then the system* (1) *has a unique ω-periodic solution if and only if there exist natural numbers k and m such that the matrix*

$$M_k=(-1)^l\Bigg(I_n-\sum_{i=0}^{k-1}[A(c_l)]_i\Bigg)$$

is nonsingular for some $l\in\{1,2\}$ and

$$r(M_{k,m})<1, \tag{10}$$

where

$$M_{k,m}=V_m(A)(c_l)+\Bigg(\sum_{i=0}^{m-1}\big||A(\cdot)|_i\big|_s\Bigg)\cdot|M_k^{-1}|V_k(A)(c_l),$$

$c_l = \omega(2-l)$, and $[A(c_l)]_i$ and $V_i(A)(c_l)$ are defined, respectively, by (5_l) and (7_l).

Theorem 4. *Let the conditions (3), (4) and (8) hold. Let, moreover, there exist natural numbers k and m such that the matrix*

$$M_k = (-1)^l \Big(I_n - \sum_{i=0}^{k-1} [A(c_l)]_i \Big)$$

is nonsingular for some $l \in \{1,2\}$ and the condition (10) holds, where

$$M_{k,m} = \big(V(A)(c_l)\big)_m + \Big(I_n + \sum_{i=0}^{m-1} \big||A(\cdot)|_i\big|_s \Big) \cdot |M_k^{-1}| \big(V(A)(c_l)\big)_k,$$

$c_l = \omega(2-l)$, and $\big(A(c_l)\big)_i$ and $\big(V(A)(c_l)\big)_i$ are defined by (6_l). Then the system (1) has one and only one ω-periodic solution.

Corollary 2. *Let the conditions (3), (4) and (8) hold and there exist a natural number k such that*

$$\big(A(c_l)\big)_i = 0 \quad (i = 0, \dots, k-1)$$

and

$$\det\big(\big(A(c_l)\big)_k\big) \neq 0$$

for some $l \in \{1,2\}$, where $c_l = \omega(2-l)$ and $\big(A(c_l)\big)_i$ is defined by (6_l). Then there exists $\varepsilon_0 > 0$ such that the system

$$dx(t) = \varepsilon dA(t) \cdot x(t) + df(t) \quad \textit{for} \quad t \in \mathbb{R}$$

has one and only one ω-periodic solution for every $\varepsilon \in]0, \varepsilon_0[$.

Corollary 3. *Let*

$$\det\big(A(\omega)\big) \neq 0.$$

Then the conclusion of Corollary 2 is true.

Theorem 5. *Let a matrix-function $A_0 \in \mathrm{BV}_\omega(\mathbb{R}, \mathbb{R}^{n\times n})$ be such that*

$$\det(I_n + (-1)^j d_j A_0(t)) \neq 0 \quad \textit{for} \quad t \in \mathbb{R} \quad (j = 1,2)$$

and the homogeneous system

$$dx(t) = dA_0(t) \cdot x(t) \tag{11}$$

has only the trivial ω-periodic solution. Let, moreover, the matrix-function $A \in \mathrm{BV}_\omega(\mathbb{R}, \mathbb{R}^{n\times n})$ admit the estimate

$$\int_t^{t+\omega} |G_{0\omega}0(t,\tau)| dV(S_0(A-A_0))(\tau) + \sum_{t<\tau\le t+\omega} |G_{0\omega}(t,\tau-) \cdot d_1(A(\tau)-A_0(\tau))|$$
$$+ \sum_{t\le\tau<t+\omega} |G_{0\omega}(t,\tau+) \cdot d_2(A(\tau)-A_0(\tau))| \le M \quad \textit{for} \quad t \in \mathbb{R},$$

where $G_{0\omega}$ *is the Green matrix of the problem* (11), (2), *and* $M \in \mathbb{R}_+^{n\times n}$ *is a constant matrix such that*

$$r(M) < 1.$$

Then the system (1) *has one and only one* ω*-periodic solution.*

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