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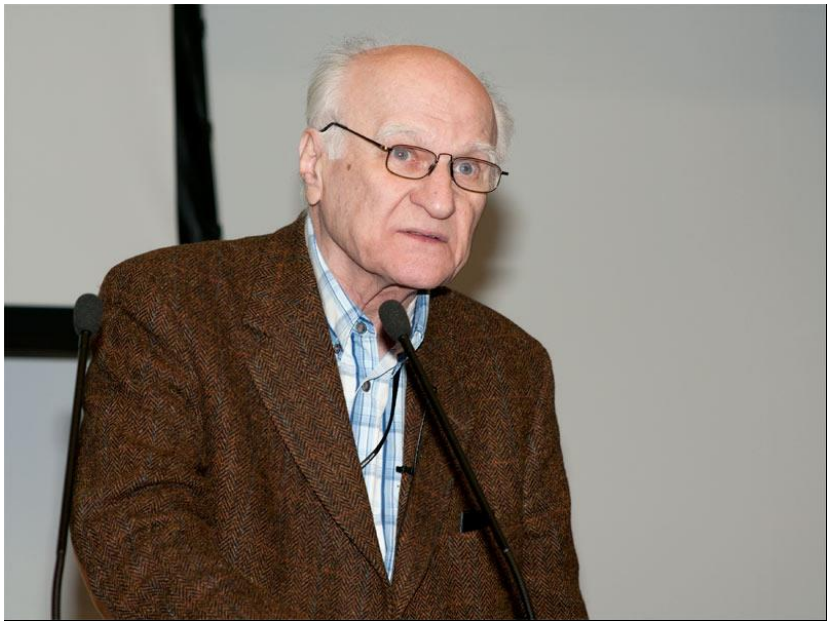
Revaz Gamkrelidze – 90



Revaz Gamkrelidze, the well-known Georgian mathematician, Academician of the Georgian National Academy of Sciences and the Russian Academy of Sciences, was born on February 4, 1927 in Kutaisi (Georgia). His father, Valerian Gamkrelidze, was one of the noted figures of Publishing house in Georgia. Upon graduation from the secondary school, R. Gamkrelidze continued his education at the Mechanics and Mathematics department of Tbilisi State University, where he won reputation of a talented and successful student. All the aforesaid gives every reason to send R. Gamkrelidze to Lomonosov Moscow State University for further studies. Here, for the first time, he met one of the leading figures in mathematical thought, the great Russian mathematician, Academician Lev Pontryagin, who has left an ineffaceable mark on further creation of scientific interests and demonstration of unique talent in young R. Gamkrelidze.

The first scientific result obtained by Revaz Gamkrelidze deals with Chern's cycles of complex algebraic manifolds, and his next scientific activity refers to the nonclassical variational calculus, i.e., the optimal control theory. R. Gamkrelidze, together with L. Pontryagin, V. Boltyanskii and E. Mishchenko, laid the foundation of the Mathematical Theory of Optimal Processes. He was the first who has proved the maximum principle, the necessary optimality condition of control for the linear time-optimal control problem. He studied the question of the existence of optimal control, introduced the notion of a general state of the system and showed that the maximum principle for such systems is likewise a sufficient optimality condition. Notable is the fact that the maximum principle, as a hypothesis, has been stated by Pontryagin, and in scientific literature is known as Pontryagin's maximum principle. R. Gamkrelidze's fundamental researches dealing with the optimal problems with bounded phase coordinates belong to the same period. Scientific achievements obtained by Pontryagin's school were published in 1961 as a monograph under the title "The mathematical theory of optimal processes" (Russian): L. S. Pontryagin, V. G. Boltyanskii, R. V. Gamkrelidze and E. F. Mishchenko, *Gosudarstv. Izdat. Fiz.-Mat. Lit., Moscow*, 1961. The work was awarded with the State Lenin Prize in 1962. It

has been translated into many languages and made a great contribution to the development of the theory of optimal control in many countries. Later on, R. Gamkrelidze discovered and studied sliding optimal modes.

Investigations of these objects took him to the concepts of a quasi-convex set and a quasi-convex filter in vector topological spaces, which laid the foundation for creation of a general theory of extremal problems. In this theory, R. Gamkrelidze was the first who has proved maximum principles in the integral form. Besides, he has studied quasi-linear differential games and identified an escape strategy for them. Later, he has constructed exponential representation of flows and the chronological calculus. Fundamental results in geometrical theory of control also belong to him.

R. Gamkrelidze is occupied with profound scientific-research and pedagogical activities in different scientific centers of the world (USA, Germany, France, England, etc.). He takes part in many International scientific congresses and conferences. He is a member of editorial boards of many scientific journals. Among R. Gamkrelidze's pupils are many famous scientists all over the world.

Over a long period of time, R. Gamkrelidze headed the Department of Theory of Control at Tbilisi State University (TSU) and the Department of Differential Equations at Steklov Institute of Mathematics. R. Gamkrelidze, together with his pupil G. Kharatishvili, has made a great contribution to the development of the optimal control theory in Georgia, he is deemed a successor of A. Razmadze, the first Georgian mathematician. A course of lectures delivered by R. Gamkrelidze in TSU laid the foundation for writing the monograph, which was translated into Russian and published by Publishing house of Tbilisi State University. For his book, R. Gamkrelidze was endowed with A. Razmadze's Prize of the Georgian National Academy of Sciences in 1975. Later on, the monograph has been translated into English and published in the USA: R. V. Gamkrelidze, "Principles of Optimal Control Theory", Mathematical Concepts and Methods in Science and Engineering, Vol. 7. *Plenum Press, New York–London*, 1978.

Along with the fertile scientific-pedagogical activities, R. Gamkrelidze is engaged with editorial activity. He is editor-in-chief of many scientific publications, including abstract journal *"Matematika"*, serial publications: *"Contemporary Mathematics and Its Applications"*, *"Contemporary Mathematics, Fundamental Directions"*, *"Fundamental and Applied Mathematics"*.

The honored and famous scientist Revaz Gamkrelidze meets his ninetieth anniversary with inexhaustible endeavor and achievements, with unshakable faith in the future. Wish him a long life and creative success.

Tamaz Tadumadze

Scientific Papers of Revaz V. Gamkrelidze

(i) Monographs

1. The mathematical theory of optimal processes (with L. S. Pontrjagin, V. G. Boltjanskiĭ and E. F. Mishchenko). (Russian) *Gosudarstv. Izdat. Fiz.-Mat. Lit., Moscow*, 1961.
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