МАТЕМАТИЧКИ ВЕСНИК
57 (2005), 87–94

UDK 517.984
оригинални научни рад
research paper

THE VORONOVSKAYA THEOREM FOR GENERALIZED BASKAKOV-KANTOROVICH OPERATORS IN POLYNOMIAL WEIGHT SPACES

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Abstract. Recently, K. Bogalska has applied a new technique to establish a Voronovskaya-type theorem. In this paper we will also prove the Voronovskaya Theorem for geenralized Baskakov-Kantorovich-type operators in polynomial weight spaces using the same approach.

1. Introduction

For two times differentiable functions, Voronovskaya [7] was the first to prove a theorem for Bernstein polynomials known as Voronovskaya Theorem. Later on, it was studied by Butzer and Nessel [5], Rempulska and Skorupka [6] for some other linear positive operators.

Recently, K. Bogalska [4] has given a different proof of the Voronovskaya Theorem for Baskakov-Kantorovich operators. In this paper we will also prove a similar theorem for generalized Baskakov-Kantorovich operators defined in [1].

Consider the notation used in M. Becker [3], i.e., $\mathbf{N} = \{1, 2, \dots\}$, $W = \mathbf{N} \cup \{0\}$, and let for a fixed $p \in W$, $x \in [0, \infty)$, weights $w_p(x)$ be given by

$$w_0(x) = 1, \qquad w_p(x) = (1 + x^p)^{-1} \quad \text{if} \quad p \geqslant 1. \tag{1.1}$$

Let $C_p = \{ f \in C[0, \infty) : w_p f$ is uniformly continuous and bounded on $[0, \infty) \}$ and the norm in C_p be defined by the formula

$$\|f\|_p = \sup_{x \geqslant 0} w_p(x) |f(x)|, \tag{1.2}$$

and so $w_p(x)|f(x)| \leqslant \|f\|_p$; let $m \in \mathbf{N}$, $p \in W$ be fixed numbers, and denote

$$C_p^m = \{ f \in C_p : f^{(k)} \in C_p,\ k = 0, 1, 2, \dots, m \}.$$

AMS Subject Classification: 41A36
Keywords and phrases: Voronovskaya Theorem, polynomial weighted spaces.

The Generalized Baskakov-Kantorovich operator $V_n^a(f;x)$[1] is connected with the Taylor's series of functions

$$e^{at}\frac{1}{(1-t)^n}=\left\{\sum_{k=0}^{\infty}\frac{(at)^k}{k!}\right\}\left\{\sum_{k=0}^{\infty}\binom{n-1+k}{k}t^k\right\}$$

for $t\in[0,1)$ and $n=1,2,\dots$, which for $t=\dfrac{x}{1+x}$ and $x\in[0,\infty)$ yields

$$e^{\frac{ax}{1+x}}(1+x)^n=\sum_{k=0}^{\infty}\sum_{i=0}^{k}\binom{k}{i}\frac{(n-1+i)!\,a^{k-i}}{(n-1)!\,k!}\left(\frac{x}{1+x}\right)^k.$$

We define the operator as follows

$$V_n^a(f;x)=n\sum_{k=0}^{n}\left(\int_{k/n}^{(k+1)/n}f(t)\,dt\right)p_{n,k}(x,a), \tag{1.3}$$

where

$$p_{n,k}(x,a)=e^{-\frac{ax}{1+x}}\cdot\frac{p_k(n,a)}{k!}\cdot\frac{x^k}{(1+x)^{n+k}}, \tag{1.4}$$

with

$$p_k(n,a)=\sum_{i=0}^{k}{}^kC_i(n)_i a^{k-i}, \tag{1.5}$$

and $(n)_0=1$, $(n)_i=n(n+1)(n+2)\cdots(n+i-1)$, for $i\geqslant 1$; $a\geqslant 0$ is a fixed absolute constant.

2. Some properties and lemmas

We shall give here some properties of the operator (1.3).

Lemma 2.1. *For $a,x\geqslant 0$, $n\in\mathbf{N}$, the following identities hold:*

$$V_n^a(1;x)=1, \tag{2.1}$$

$$V_n^a(t;x)=x+\frac{ax}{n(1+x)}+\frac{1}{2n}, \tag{2.2}$$

$$V_n^a(t^2;x)=\frac{1+n}{n}x^2+\frac{1}{3n^2}+\frac{2x}{n}+\frac{a^2x^2}{n^2(1+x)^2}+\frac{2ax^2}{n(1+x)}+\frac{2ax}{n^2(1+x)}, \tag{2.3}$$

$$\begin{aligned}V_n^a(t^3;x)={}&\frac{(n+1)(n+2)}{n^2}x^3+\frac{9(n+1)}{2n^2}x^2+\frac{7x}{2n^2}+\frac{1}{4n^3}+\frac{n+1}{n^2}\frac{3ax^3}{n(1+x)}\\&+\frac{3a^2x^3}{n^2(1+x)^2}+\frac{a^3x^3}{n^3(1+x)^3}+\frac{9ax^2}{n^2(1+x)}+\frac{9a^2x^2}{2n^3(1+x)^2}+\frac{7ax}{2n^3(1+x)},\end{aligned} \tag{2.4}$$

$$\begin{aligned}V_n^a(t^4;x)={}&\frac{(n+1)(n+2)(n+3)}{n^3}x^4+\frac{8(n+1)(n+2)}{n^3}x^3+\frac{15(n+1)}{n^3}x^2+\frac{6x}{n^3}\\&+\frac{1}{5n^4}+\frac{(n+1)(n+2)}{n^2}\frac{4ax^4}{n(1+x)}+\frac{n+1}{n^2}\frac{6a^2x^4}{n(1+x)^2}+\frac{4a^3x^4}{n^3(1+x)^3}\end{aligned}$$

$$+\frac{a^4x^4}{n^4(1+x)^4}+\frac{n+1}{n^2}\frac{24ax^3}{n(1+x)}+\frac{24a^2x^3}{n^3(1+x)^2}+\frac{8a^3x^3}{n^4(1+x)^3}$$
$$+\frac{30ax^2}{n^3(1+x)}+\frac{15a^2x^2}{n^4(1+x)^2}+\frac{6ax}{n^4(1+x)}. \tag{2.5}$$

Proof. Identities (2.1), (2.2) and (2.3) are proved in [1]. Identities (2.4) and (2.5) can be calculated similarly using (1.3). ■

LEMMA 2.2. *For $a, x \geqslant 0$, $n \in \mathbf{N}$, we have:*

$$V_n^a((t-x);x)=\frac{1}{n}\left[\frac{ax}{1+x}+\frac{1}{2}\right], \tag{2.6}$$

$$V_n^a((t-x)^2;x)=\frac{x(1+x)}{n}+\frac{1}{3n^2}+\frac{2ax}{n^2(1+x)}+\frac{a^2x^2}{n^2(1+x)^2}, \tag{2.7}$$

$$\begin{aligned}V_n^a((t-x)^4;x)=&\frac{3}{n^2}x^4+\frac{6}{n^3}x^4+\frac{6}{n^2}x^3+\frac{16}{n^3}x^3+\frac{3}{n^2}x^2+\frac{15}{n^3}x^2+\frac{5}{n^3}x+\frac{1}{5n^4}\\&+\frac{1}{n^3}\left[6ax^2+8ax^3+\frac{10ax^3}{1+x}+\frac{6a^2x^3}{1+x}\right]\\&+\frac{1}{n^4}\left[\frac{6ax}{1+x}+\frac{15a^2x^2}{(1+x)^2}+\frac{8a^3x^3}{(1+x)^3}+\frac{a^4x^4}{(1+x)^4}\right].\end{aligned} \tag{2.8}$$

Proof. (2.6) and (2.7) are proved in [1]. For (2.8), lineraity of the operator implies that

$$V_n^a((t-x)^4;x)=V_n^a(t^4;x)-4xV_n^a(t^3;x)+6x^2V_n^a(t^2;x)-4x^3V_n^a(t;x)+V_n^a(1;x).$$

Using Lemma 2.1 we arrive at (2.8). ■

Using the mathematical induction we can prove a formula for $V_n^a(t^q;x)$ similar to the one given by Bogalska [4] for Baskakov-Kantorovich operators.

LEMMA 2.3. *For $q \in W$, $n \in \mathbf{N}$, $x \in [0,\infty)$ there exist positive coefficients $\xi_{q,n,j}$ and $\delta_{j,n,i}$, $0 \leqslant j \leqslant q$, $1 \leqslant i \leqslant j$ depending only on q, j, n, i and bounded for n and x such that*

$$V_n^a(t^q;x)=\sum_{j=0}^{q}\xi_{q,n,j}x^jn^{j-q}+\sum_{j=1}^{q}n^{j-q-1}\left\{\sum_{i=1}^{j}\delta_{j,n,i}\left(\frac{a}{1+x}\right)^i\right\}x^j, \tag{2.9}$$

where $1 \leqslant \xi_{q,n,j} \leqslant q!$.

LEMMA 2.4. *For every fixed $p \in W$, there exists a positive constant $M_1(p,a)$ depending only on p and a such that for all $n \in \mathbf{N}$, we have*

$$\left\|V_n^a\left(\frac{1}{w_p(t)};x\right)\right\|_p \leqslant M_1(p,a). \tag{2.10}$$

Proof. For $p = 0$, (2.10) is obvious. For $p \geqslant 1$, we have

$$w_p(x)V_n^a\left(\frac{1}{w_p(t)}\,;x\right) = w_p(x)V_n^a(1+t^p;x) = w_p(x)[V_n^a(1;x)+V_n^a(t^p;x)]$$
$$= w_p(x)\left[1+\sum_{j=0}^{p}\xi_{p,n,j}x^jn^{j-p}+\sum_{j=1}^{p}n^{j-p-i}\left\{\sum_{i=1}^{j}\delta_{j,n,i}\left(\frac{a}{1+x}\right)^i\right\}x^j\right]$$
$$\leqslant \frac{1+p!\,x^p}{1+x^p}+\sum_{j=o}^{p-1}\xi_{p,n,j}n^{j-p}\frac{x^j}{1+x^p}+\sum_{j=1}^{p}n^{j-p-1}\left\{\sum_{i=1}^{j}\delta_{j,n,i}\left(\frac{a}{1+x}\right)^i\right\}\frac{x^j}{1+x^p}.$$

But $0 \leqslant \dfrac{x^j}{1+x^p} \leqslant 1$ for $x \in [0,\infty)$ and $1 \leqslant j \leqslant p-1$. Also $\dfrac{1+p!\,x^p}{1+x^p} \leqslant p!$ and $\left(\dfrac{a}{1+x}\right)^i \leqslant a^i$. Therefore,

$$w_p(x)V_n^a\left(\frac{1}{w_p(t)}\,;x\right) \leqslant p!+\sum_{j=0}^{p-1}\xi_{p,n,j}n^{j-p}+\sum_{j=1}^{p}n^{j-p-1}\left\{\sum_{i=1}^{j}\delta_{j,n,i}a^i\right\}$$
$$\leqslant M_1(p,a). \quad \blacksquare$$

Using Lemma 2.4, we shall prove the following lemma.

LEMMA 2.5. *For every fixed $p \in W$, there exists a positive constant $M_1(p,a)$ depending only on p and a such that for any $f \in C_p$,*

$$\|V_n^a(f;x)\|_p \leqslant \|f\|_pM_1(p,a), \qquad n \in \mathbf{N}. \tag{2.11}$$

Proof. From (1.2) we have $\|V_n^a(f;x)\|_p = \sup_{x\geqslant 0} w_p(x)|V_n^a(f;x)|$. But

$$w_p(x)|V_n^a(f;x)| \leqslant w_p(x)n\sum_{k=0}^{\infty}\left(\int_{k/n}^{(k+1)/n} f(t)w_p(t)\frac{1}{w_p(t)}\,dt\right)p_{n,k}(x,a)$$
$$\leqslant \|f\|_pw_p(x)V_n^a\left(\frac{1}{w_p(t)};x\right);$$

hence, from Lemma 2.4 we obtain $\|V_n^a(f;x)\|_p \leqslant \|f\|_pM_1(p,a)$. $\blacksquare$

LEMMA 2.6 *For some $x_0 \in [0,\infty)$ there exists a positive constant $M_2(x_0,a)$ depending only on x_0 and a such that for all $n \in \mathbf{N}$ we have*

$$V_n^a((t-x)^4;x) \leqslant M_2(x_0,a)n^{-2}. \tag{2.12}$$

Proof. From (2.8) we get

$$\begin{aligned}
V_n((t-x_0)^4;x_0) &= \frac{3}{n^2}x_0^4 + \frac{6}{n^3}x_0^4 + \frac{6}{n^2}x_0^3 + \frac{16}{n^3}x_0^3 + \frac{3}{n^2}x_0^2 + \frac{15}{n^3}x_0^2 + \frac{5}{n^3}x_0 + \frac{1}{5n^4} \\
&\quad + \frac{1}{n^3}\left[6ax_0^2 + 8ax_0^3 + \frac{10ax_0^3}{1+x_0} + \frac{6a^2x_0^3}{1+x_0}\right] \\
&\quad + \frac{1}{n^4}\left[\frac{6ax_0}{1+x_0} + \frac{15a^2x_0^2}{(1+x_0)^2} + \frac{8a^3x_0^3}{(1+x_0)^3} + \frac{a^4x_0^4}{(1+x_0)^4}\right], \\
V_n((t-x_0)^4;x_0) &\leqslant (9x_0^4 + 22x_0^3 + 18x_0^2 + 5x_0 + 0.2)n^{-2} + (6ax_0 + 24ax_0^2 \\
&\quad + 8ax_0^3 + 6a^2x_0 + 6a^2x_0^2 + 6a + 15a^2 + 8a^3 + a^4)n^{-2} \\
&\leqslant M_2(x_0,a)n^{-2}. \quad \blacksquare
\end{aligned}$$

Lemma 2.7. *For every $x_0 \in [0,\infty)$ there hold*

$$\operatorname*{Lt}_{n\to\infty} nV_n^a((t-x_0);x_0) = \frac{1}{2} + \frac{ax_0}{1+x_0} \tag{2.13}$$

and

$$\operatorname*{Lt}_{n\to\infty} nV_n^a((t-x_0)^2;x_0) = x_0(1+x_0). \tag{2.14}$$

Proof. From Lemma 2.2 we know that for $a, x_0 \geqslant 0$,

$$\begin{aligned}
V_n^a((t-x_0);x_0) &= \frac{1}{2n} + \frac{ax_0}{n(1+x_0)}, \\
V_n^a((t-x_0)^2;x_0) &= \frac{x_0(1+x_0)}{n} + \frac{1}{3n^2} + \frac{2ax_0}{n^2(1+x_0)} + \frac{a^2x_0^2}{n^2(1+x_0)^2},
\end{aligned}$$

which yield (2.13) and (2.14). $\blacksquare$

The following lemma [2] is needed for further estimations.

Lemma 2.8. *For each $p \in W$ there exists a constant $M_3(p,a)$ such that for all $n \in \mathbf{N}$,*

$$w_p(x)V_n^a\left(\frac{(t-x)^2}{w_p(t)};x\right) \leqslant M_3(p,a)\frac{x(1+x)}{n}.$$

Lemma 2.9. *Let $x_0 \in [0,\infty)$ be a fixed point and let $g(t;x_0) \in C_p$ with $p \in W$ such that*

$$\operatorname*{Lt}_{t\to x_0} g(t;x_0) = 0. \tag{2.15}$$

Then

$$\operatorname*{Lt}_{t\to x_0} V_n^a(g(t;x_0);x_0) = 0. \tag{2.16}$$

Proof. Let $\varepsilon > 0$ and the constant $M_1(p,a)$ be as in Lemma 2.4. Then from (2.15) and properties of $g(t;x_0)$ there exist positive constants $\delta = \delta(\varepsilon, M)$ and M_4 such that

$$w_p(t)|g(t;x_0)| < \frac{\varepsilon}{2M_1(p,a)}, \quad \text{for } |t-x_0| < \delta \tag{2.17}$$

and

$$w_p(t)|g(t;x_0)| < M_4, \quad \text{for all } t \in [0,\infty). \tag{2.18}$$

Then for all $n \in \mathbf{N}$ we have

$$\begin{aligned}
w_p(x_0)|V_n^a(g(t;x_0);x_0)| &\leqslant w_p(x_0) n \sum_{k=0}^{n} \left(\int_{k/n}^{(k+1)/n} |g(t;x_0)|\, dt \right) p_{n,k}(x_0,a) \\
&= w_p(x_0) n \sum_{\left|\frac{k}{n}-x_0\right|<\delta} \left(\int_{k/n}^{(k+1)/n} |g(t;x_0)|\, dt \right) p_{n,k}(x_0,a) \\
&\quad + w_p(x_0) n \sum_{\left|\frac{k}{n}-x_0\right|\geqslant\delta} \left(\int_{k/n}^{(k+1)/n} |g(t;x_0)|\, dt \right) p_{n,k}(x_0,a).
\end{aligned}$$

From 2.17 and Lemma 2.4, we obtain

$$\begin{aligned}
S_1 &\leqslant \frac{\varepsilon}{2M_1(p,a)} w_p(x_0) n \sum_{\left|\frac{k}{n}-x_0\right|<\delta} \left(\int_{k/n}^{(k+1)/n} \frac{1}{w_p(t)}\, dt \right) p_{n,k}(x_0,a) \\
&\leqslant \frac{\varepsilon}{2M_1(p,a)} w_p(x_0) V_n^a\left(\frac{1}{w_p(t)}; x_0 \right) \leqslant \frac{\varepsilon}{2M_1(p,a)} M_1(p,a) = \frac{\varepsilon}{2}.
\end{aligned}$$

Moreover, by (2.18) we get

$$S_2 \leqslant M_4 w_p(x_0) n \sum_{\left|\frac{k}{n}-x_0\right|\geqslant\delta} \left(\int_{k/n}^{(k+1)/n} \frac{1}{w_p(t)}\, dt \right) p_{n,k}(x_0,a).$$

But from [4] we know that

$$\int_{k/n}^{(k+1)/n} \frac{1}{w_p(t)}\, dt \leqslant \frac{1}{n w_p\left(\frac{k+1}{n}\right)} \leqslant \frac{2^{p+1}}{n} \frac{1}{w_p\left(\frac{k}{n}\right)},$$

therefore,

$$\begin{aligned}
S_2 &\leqslant 2^{p+1} M_4 w_p(x_0) \sum_{\left|\frac{k}{n}-x_0\right|\geqslant\delta} \frac{1}{w_p\left(\frac{k}{n}\right)} p_{n,k}(x_0,a) \\
&\leqslant 2^{p+1} M_4 w_p(x_0) \sum_{\left|\frac{k}{n}-x_0\right|\geqslant\delta} \frac{1}{\delta^2} \left(\frac{k}{n} - x_0 \right)^2 \frac{1}{w_p\left(\frac{k}{n}\right)} p_{n,k}(x_0,a) \\
&\leqslant M_4 \frac{2^{p+1}}{\delta^2} w_p(x_0) \sum_{k=0}^{\infty} \left(\frac{k}{n} - x_0 \right)^2 \frac{1}{w_p\left(\frac{k}{n}\right)} p_{n,k}(x_0,a) \\
&\leqslant M_4 \frac{2^{p+1}}{\delta^2} w_p(x_0) V_n^a\left(\frac{(t-x_0)^2}{w_p(t)}; x_0 \right).
\end{aligned}$$

From [2], we have

$$S_2 \leqslant M_4 \frac{2^{p+1}}{\delta^2} M_3(p,a) \frac{x_0(1+x_0)}{n} \equiv M_5(p,a) \frac{x_0(1+x_0)}{n\delta^2}.$$

For the fixed positive numbers ε, δ, M_5 and $x_0 \geqslant 0$ there exists a natural number n_1, depending only on ε, δ and M_5, such that $M_5(p,a)\frac{x_0(1+x_0)}{n\delta^2} < \frac{\varepsilon}{2}$ for all $n > n_1$. Thus $S_2 < \frac{\varepsilon}{2}$ and hence

$$w_p(x_0)|V_n^a(g(t;x_0);x_0)| < \varepsilon \quad \text{for all } n > n_1,$$

i.e. $\mathrm{Lt}_{n\to\infty} V_n^a(g(t;x_0);x_0) = 0$. Above estimation and Definition 1.1 imply (2.16). ■

3. The Voronovskaya Theorem

The main theroem is as follows.

THEOREM 3.1. *Let $f \in C_p^2$ with some $p \in W$. Then for every $x \in [0,\infty)$ we have*

$$\underset{n\to\infty}{\mathrm{Lt}} \{V_n^a(f;x) - f(x)\} = \left\{\frac{1}{2} + \frac{ax}{1+x}\right\} f'(x) + \frac{x(1+x)}{2} f''(x). \tag{3.1}$$

Proof. Take a fixed point $x_0 \in [0,\infty)$. Then for $f \in C_p^2$, $t \in [0,\infty)$ we have by Taylor's formula

$$f(t) = f(x_0) + f'(x_0)(t-x_0) + \frac{1}{2} f''(x_0)(t-x_0)^2 + \varphi(t;x_0)(t-x_0)^2,$$

where $\varphi(t;x_0) \in C_p$ and $\mathrm{Lt}_{t\to x_0} \varphi(t;x_0) = 0$. Multiplying both sides by $np_{n,k}(x,a)$, integrating with respect to t between the limits $\frac{k}{n}$ and $\frac{k+1}{n}$, then summing over k, we get

$$V_n^a(f;x_0) = f(x_0)V_n^a(1;x_0) + f'(x_0)V_n^a((t-x_0);x_0) + \frac{1}{2} f''(x_0) V_n^a((t-x_0)^2;x_0) + V_n^a(\varphi(t;x_0)(t-x_0)^2;x_0).$$

Using Lemma 2.7 we obtain

$$\underset{n\to\infty}{\mathrm{Lt}}\, n\{V_n^a(f;x_0) - f(x_0)\} = \left\{\frac{1}{2} + \frac{ax_0}{1+x_0}\right\} f'(x_0) + \frac{x_0(1+x_0)}{2} f''(x_0) + \underset{n\to\infty}{\mathrm{Lt}}\, nV_n^a(\varphi(t;x_0)(t-x_0)^2;x_0). \tag{3.2}$$

From Hölder's inequality and Lemma 2.6, we have

$$\begin{aligned} V_n^a(\varphi(t;x_0)(t-x_0)^2;x_0) &\leqslant \{V_n^a(\varphi^2(t;x_0);x_0)\}^{\frac{1}{2}} \{V_n^a((t-x_0)^4;x_0)\}^{\frac{1}{2}} \\ &\leqslant \{V_n^a(\varphi^2(t;x_0);x_0)\}^{\frac{1}{2}} \{M_2(x_0,a)n^{-2}\}^{\frac{1}{2}}. \end{aligned} \tag{3.3}$$

The properties of the function $\varphi(t;x_0)$ imply that for $\psi(t;x_0) \equiv \varphi^2(t;x_0)$ we have $\mathrm{Lt}_{t\to x_0} \psi(t;x_0) = 0$. By Lemma 2.9 we get

$$\operatorname*{Lt}_{n\to\infty} V_n^a(\psi(t;x_0);x_0) = 0. \tag{3.4}$$

Combining (3.2), (3.3) and (3.4), we arrive at (3.1). ∎

Remark. When $a = 0$, (3.1) reduces to Bogalska estimate [4].

Acknowledgement. The authors are thankful to the referee for his valuable suggestions and comments.

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(received 24.09.2003, in revised form 27.07.2005)

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