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SOME RESULTS ON LOCAL SPECTRAL THEORY OF COMPOSITION OPERATORS ON l^p SPACES

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Abstract. In this paper, we give a condition under which a bounded linear operator on a complex Banach space has Single Valued Extension Property (SVEP) but does not have decomposition property (δ). We also discuss the analytic core, decomposability and SVEP of composition operators C_ϕ on l^p $(1 \le p < \infty)$ spaces. In particular, we prove that if ϕ is onto but not one-one then C_ϕ is not decomposable but has SVEP. Further, it is shown that if ϕ is one-one but not onto then C_ϕ does not have SVEP.

1. Preliminaries

The single valued extension property plays a central role in the local spectral theory. This property was first introduced by N. Dunford [3,4] and subsequently, became an essential tool in determining the decomposability of a bounded linear operator on a Banach space.

Let X be a complex Banach space and $\mathcal{B}(X)$ denote the Banach algebra of bounded linear operators on X. An operator $T \in \mathcal{B}(X)$ is said to have the Single Valued Extension Property (abbreviated as SVEP) if for every open set $G \subseteq \mathbb{C}$, the only analytic solution $f\colon G \to X$, of the equation $(\lambda - T)f(\lambda) = 0$, for all $\lambda \in G$, is the zero function on G. It is clear from the definition that an operator, whose point spectrum has empty interior, has SVEP. However, the converse is not true in general [8, p. 15]. A result by J. K. Finch [6] gives a class of examples of those operators which do not have SVEP.

For $x \in X$, the local resolvent of T at x, denoted by $\rho_T(x)$, is defined as the union of all open subsets G of $\mathbb{C}$ for which there is an analytic function $f\colon G \to X$ satisfying $(\lambda - T)f(\lambda) = x$ for all $\lambda \in G$. The complement of $\rho_T(x)$ is called the local spectrum of T at x and is denoted by $\sigma_T(x)$. If T has the SVEP then $\sigma_T(x) = \emptyset$ if and only if $x = 0$ (for proof see [8, Proposition 1.2.16]). For a subset $F \subseteq \mathbb{C}$, the local spectral subspace of T, denoted by $X_T(F)$, is defined as

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$X_T(F) = \{x \in X : \sigma_T(x) \subseteq F\}$. An operator T is said to have Dunford's property (C) if $X_T(F)$ is closed for every closed subset F of $\mathbb{C}$. T is said to have Bishop's property (β) if for every open subset G of $\mathbb{C}$ and every sequence of analytic functions $f_n \colon G \to X$ with the property that $(\lambda - T)f_n(\lambda) \to 0$ as $n \to \infty$, locally uniformly on G, then $f_n(\lambda) \to 0$ as $n \to \infty$, locally uniformly on G. It is well known that $(\beta) \Rightarrow (C) \Rightarrow$ SVEP.

T is said to be decomposable if for every open cover $\{U, V\}$ of $\mathbb{C}$ there exist T-invariant closed subspaces Y and Z of X such that $\sigma(T|Y) \subseteq U$, $\sigma(T|Z) \subseteq V$ and $X = Y + Z$. T is said to have the decomposition property (δ) if for every open cover $\{U, V\}$ of $\mathbb{C}$, $X = \mathcal{X}_T(\overline{U}) + \mathcal{X}_T(\overline{V})$, where $\mathcal{X}_T(\overline{U})$ is defined as the set of all $x \in X$ such that there is an analytic function $f \colon \mathbb{C} \setminus \overline{U} \to X$ satisfying $(\lambda - T)f(\lambda) = x$. Note that $\mathcal{X}_T(\overline{U})$ is a subspace of X and if T has the SVEP then $\mathcal{X}_T(\overline{U}) = X_T(\overline{U})$. For further reading of local spectral theory we refer to [2, 5, 8].

Let ϕ be a self-map on the set of natural numbers $\mathbb{N}$. Then ϕ induces a linear transformation C_ϕ on the complex vector space V of complex sequences, defined by

$$C_\phi\Big(\sum_{n=1}^{\infty} x_n \chi_n\Big) = \sum_{n=1}^{\infty} x_n \chi_{\phi^{-1}(n)},$$

where χ_n denotes the characteristic function of $\{n\}$. If V is l^p and C_ϕ happens to be bounded, then C_ϕ is called a composition operator on l^p. A necessary and sufficient condition on ϕ to induce a composition operator on $l^p (1 \le p < \infty)$ is that the set $\{|\phi^{-1}(n)| : n \in \mathbb{N}\}$ must be bounded, where $|\cdot|$ denotes the cardinality of the set [10, Theorem 2.1.1]. For further details of the composition operators, we refer to [10]. Throughout the paper ϕ_n denotes the n-th iterate of ϕ.

2. Main results

We begin by proving the following lemma.

Lemma 2.1. *Let X be a complex Banach space and $T \in \mathcal{B}(X)$. Suppose that $\sigma(T)$ is not a singleton and $\bigcap_{x \in X,\, x \neq 0} \sigma_T(x) \neq \emptyset$. Then T has SVEP but T does not have decomposition property (δ) and hence, T is not decomposable.*

Proof. Suppose that T does not have SVEP. Then there is a non-zero $x \in X$ such that $\sigma_T(x) = \emptyset$ [8, Proposition 1.2.16], which implies that $\bigcap_{x \in X,\, x \neq 0} \sigma_T(x) = \emptyset$. Thus

$$\bigcap_{x \in X,\, x \neq 0} \sigma_T(x) \neq \emptyset \Longrightarrow T \text{ has SVEP}.$$

Let $\bigcap_{x \in X,\, x \neq 0} \sigma_T(x) = K \subseteq \sigma(T)$. Now we have the following two cases.

Case I: $K = \sigma(T)$.

In this case $\sigma_T(x) = \sigma(T)$ for all non-zero x in X. Let U be an open set such that $\overline{U}$ and $\mathbb{C} \setminus \overline{U}$ both intersect $\sigma(T)$. Let B be a closed ball inside U with $B^o \cap \sigma(T) \neq \emptyset$, then $\{U, \mathbb{C} \setminus B\}$ is an open covering of $\mathbb{C}$. It is easy to see that $X_T(\overline{U}) = X_T(\overline{\mathbb{C} \setminus B}) = \{0\}$. Since $\mathcal{X}_T(F) = X_T(F)$ for all closed subsets F of $\mathbb{C}$ as T has SVEP, we see that T does not have decomposition property (δ).

Case II: K is a proper subset of $\sigma(T)$.

Again, let U be an open set such that U contains K and $\sigma(T) \cap (\mathbb{C} \setminus \overline{U}) \neq \emptyset$. Let B be a closed ball inside U with $B^o \cap K \neq \emptyset$. Then $\{U, \mathbb{C} \setminus B\}$ is an open covering of $\mathbb{C}$. Now $X_T(\overline{\mathbb{C} \setminus B}) = \{0\}$ since $\sigma_T(x) \supseteq K$ for all non-zero x in X and as T has SVEP so $X_T(\overline{U}) \neq X$. Thus T does not have decomposition property (δ).

Hence from [8, Theorem 1.2.29], T is not decomposable. ∎

The converse of the above lemma is not true. For example, let $T_1 = diag(\alpha_1, \dots, \alpha_n)$ be a diagonal matrix with $|\alpha_i| > 1$, $1 \leq i \leq n$. Then $T_1 \colon \mathbb{C}^n \to \mathbb{C}^n$ is decomposable [8, Proposition 1.4.5]. Let T_2 be the right shift on l^2. Then $T = T_1 \oplus T_2 \in \mathcal{B}(\mathbb{C}^n \oplus l^2)$ has SVEP [2, Proposition 1.3]. Since $\sigma_{ap}(T) = \sigma_{ap}(T_1) \cup \sigma_{ap}(T_2)$ [7, 98], it follows that $\sigma_{ap}(T) = \{\alpha_1, \dots, \alpha_n\} \cup \mathbb{T} \neq \{\alpha_1, \dots, \alpha_n\} \cup \overline{D} = \sigma(T)$, where $\mathbb{T} = \{\lambda : |\lambda| = 1\}$ and $\overline{D} = \{\lambda : |\lambda| \leq 1\}$. Hence from [8, Proposition 1.3.2], it follows that T^* does not have SVEP. Consequently, T does not have decomposition property (δ). Next, let x and y be any non-zero vectors in $\mathbb{C}^n$ and in l^2 respectively. Then from [2, Proposition 1.3], we have

$$\sigma_T(x \oplus 0) = \sigma_{T_1}(x) \cup \sigma_{T_2}(0) \subseteq \{\alpha_1, \dots, \alpha_n\}$$

and

$$\sigma_T(0 \oplus y) = \sigma_{T_1}(0) \cup \sigma_{T_2}(y) \subseteq \overline{D}.$$

Since $\overline{D} \cap \{\alpha_1, \dots, \alpha_n\} = \emptyset$ therefore, $\bigcap_{x \in \mathbb{C}^n \oplus l^2,\, x \neq 0} \sigma_T(x) = \emptyset$.

Let T be a bounded linear operator on a complex Banach space X. Then

$$\kappa(T) = \inf\{\|Tx\| : x \in X \quad \text{with } \|x\| = 1\}$$

denotes the lower bound of T. Now define

$$i(T) = \lim_{n \to \infty} \kappa(T^n)^{1/n}.$$

It is clear that $i(T) \leq r(T) = \lim_{n \to \infty} \|T^n\|^{1/n}$. Also, define the hyperrange of T as $T^\infty(X) = \bigcap_{n=1}^{\infty} T^n(X)$.

Remark 2.1. Combining [8, Theorem 1.6.3] and the above lemma, it can be easily shown that if the hyperrange of T is $\{0\}$ and $\sigma(T)$ is not a singleton, then T does not have decomposition property (δ).

Definition 2.1. Let X be a complex Banach space and $T \in \mathcal{B}(X)$. The analytic core of T is the set $K(T)$ of all $x \in X$ such that there exists a sequence $(y_n) \subset X$ and $\delta > 0$ for which:

(1) $x = y_0$, and $Ty_{n+1} = y_n$ for every $n \in \mathbb{N}$.

(2) $\|y_n\| \leq \delta^n \|x\|$ for every $n \in \mathbb{N}$.

It is easy to see that $K(T)$ is a subspace of X which is not necessarily closed. Also, $K(T) \subseteq T^\infty(X)$. In Proposition 2.1 below we prove that, in the case of a composition operator C_ϕ on $l^p (1 \leq p < \infty)$, the analytic core of C_ϕ is always closed and coincides with the hyperrange of C_ϕ.

Proposition 2.1. *Let $\phi\colon \mathbb{N} \to \mathbb{N}$ induce a composition operator C_ϕ on $l^p(1 \leq p < \infty)$. Then $K(C_\phi) = C_\phi^\infty(l^p)$.*

Proof. It is clear that $K(C_\phi) \subseteq C_\phi^\infty(l^p)$. Since from [9, Theorem 2.1.3],

$$C_\phi^k(l^p) = \{x \in l^p : x|\phi_k^{-1}(n) \text{ is constant for each } n \geq 1\}$$

therefore,

$$\begin{aligned} C_\phi^\infty(l^p) &= \bigcap_{k=1}^{\infty} C_\phi^k(l^p) \\ &= \{x \in l^p : x|\phi_k^{-1}(n) \text{ is constant for each } n \geq 1 \text{ and for each } k \geq 1\}. \end{aligned}$$

Let $x \in C_\phi^\infty(l^p)$. Then for each $n \in \mathbb{N}$, define

$$y_n(m) = x|\phi_n^{-1}(m) \text{ for all } m \geq 1.$$

Since $x|\phi_k^{-1}(n)$ is constant for each $n \geq 1$ and for each $k \geq 1$, therefore each $y_n(m)$ is well-defined and $y_n \in l^p$ for each $n \geq 1$. Also

$$(C_\phi y_{n+1})(m) = y_{n+1}(\phi(m)) = x|\phi_{n+1}^{-1}(\phi(m)).$$

Since $\phi_n^{-1}(m) \subseteq \phi_{n+1}^{-1}(\phi(m))$, therefore $x|\phi_{n+1}^{-1}(\phi(m)) = x|\phi_n^{-1}(m)$. Hence

$$(C_\phi y_{n+1})(m) = x|\phi_n^{-1}(m) = y_n(m) \ \forall m \geq 1.$$

That is, $C_\phi y_{n+1} = y_n \ \forall n \geq 0$, where $y_0 = x$.

Further, for any $n \geq 0$,

$$\|y_n\|^p = \sum_{k=1}^{\infty} |y_n(k)|^p = \sum_{k=1}^{\infty} |x|\phi_n^{-1}(k)|^p \leq \|x\|^p.$$

Thus, letting $\delta = 1$, we get the required sequence $(y_n)_{n=1}^\infty$ in l^p which satisfies all the conditions of $K(C_\phi)$. Hence $C_\phi^\infty(l^p) \subseteq K(C_\phi)$. Therefore $K(C_\phi) = C_\phi^\infty(l^p)$. ∎

Corollary 2.1. *The analytic core of a composition operator on l^p $(1 \leq p < \infty)$ is closed.*

Proof. The proof follows from the above lemma and the fact that the range of a composition operator on l^p $(1 \leq p < \infty)$ is closed [9, Theorem 2.1.4]. ∎

We now prove the following proposition which is not true in general but is true for composition operators on l^p $(1 \leq p < \infty)$ spaces.

Proposition 2.2. *Suppose that $\phi\colon \mathbb{N} \to \mathbb{N}$ induces a composition operator C_ϕ on l^p $(1 \leq p < \infty)$. If hyperrange of C_ϕ is $\{0\}$, then C_ϕ does not have decomposition property (δ).*

Proof. If ϕ is injective then C_ϕ is surjective and hence hyperrange of C_ϕ is l^p. Since ϕ not injective implies C_ϕ not onto implies $0 \in \sigma(C_\phi)$ and since there are no quasinilpotent composition operators C_ϕ on l^p, $r(C_\phi) > 0$ and from Lemma 2.1, $C_\phi^\infty(l^p) = \{0\}$ imply C_ϕ does not satisfy decomposition property (δ). ∎

The following examples show the various possibilities for hyperrange and $i(C_\phi)$ of C_ϕ.

EXAMPLE 2.1. Let $\phi\colon \mathbb{N} \to \mathbb{N}$ be defined as

$$\phi(1) = \phi(2) = \phi(3) = 2 \quad \text{and} \quad \phi(n) = n-1 \ \forall n \geq 4.$$

Then for each $n \geq 1$, $C_\phi^n \chi_1 = 0$. Therefore, $\kappa(C_\phi^n) = 0$, $\forall n \geq 1$ and hence, $i(C_\phi) = 0$. Further, as $\lim_{k\to\infty} \phi_k^{-1}(2) = \mathbb{N}$, it follows that $C_\phi^\infty(l^p) = \{0\}$ and hence by Proposition 2.1 C_ϕ does not have decomposition property (δ).

EXAMPLE 2.2. Let $\phi\colon \mathbb{N} \to \mathbb{N}$ be defined as

$$\phi(1) = 1 \text{ and } \phi(n) = n-1 \ \forall n \geq 2.$$

Since $\lim_{k\to\infty} \phi_k^{-1}(1) = \mathbb{N}$ therefore, $C_\phi^\infty(l^p) = \{0\}$. Thus C_ϕ does not have decomposition property (δ).

EXAMPLE 2.3. Let $\phi\colon \mathbb{N} \to \mathbb{N}$ be defined as

$$\phi(1) = 1,\ \phi(2) = 2, \quad \phi(2n+1) = n+1 \ \forall n \geq 1 \quad \text{and} \quad \phi(2n) = n+1 \ \forall n \geq 2.$$

Then $\lim_{k\to\infty} |\phi_k^{-1}(n)| = \infty \ \forall n \in \mathbb{N} \setminus \{1\}$ and therefore, $K(C_\phi) = C_\phi^\infty(l^p) = [\chi_1]$, a one-dimensional subspace.

EXAMPLE 2.4. Let $\phi\colon \mathbb{N} \to \mathbb{N}$ be defined as

$$\phi(1) = \phi(3) = 1,\ \phi(4) = 2, \quad \phi(2n+1) = 2n-1 \ \forall n \geq 2,$$
$$\phi(n) = n+4,\ \forall n \in \{2, 6, 10, 14, \dots\} \quad \text{and} \quad \phi(n) = n-4,\ \forall n \in \{8, 12, 16, \dots\}.$$

Then $\lim_{k\to\infty} \phi_k^{-1}(1) = \mathbb{N} \setminus \{2n : n \in \mathbb{N}\}$ and therefore, $K(C_\phi) = C_\phi^\infty(l^p) = \{x \in l^p : x|(\mathbb{N} \setminus \{2n : n \in \mathbb{N}\}) = 0\}$, an infinite-dimensional subspace.

In Examples 2.2, 2.3 and 2.4 above, since ϕ is surjective, therefore $i(C_\phi) \geq 1$. Further, the following theorem implies that the composition operators in Examples 2.3 and 2.4 do not have decomposition property (δ).

THEOREM 2.1. *If $\phi\colon \mathbb{N} \to \mathbb{N}$ is onto but not one-one then $C_\phi\colon l^p \to l^p$ $(1 \leq p < \infty)$ has SVEP but does not have decomposition property (δ). Hence C_ϕ is not decomposable.*

Proof. Let $f(\lambda) = (x_1(\lambda), x_2(\lambda), \dots)$ be an analytic function defined on an open subset U of $\mathbb{C}$ into l^p, satisfying

$$(\lambda - C_\phi) f(\lambda) = 0 \quad \text{for each } \lambda \in U. \tag{2.1}$$

Suppose that $f(\lambda)$ is non-zero. Without loss of generality, we may assume that f is never zero on U. Choose $\lambda_0 \in U \setminus (\mathbb{T} \cup \{0\})$, where $\mathbb{T} = \{\lambda : |\lambda| = 1\}$. Since $f(\lambda_0) \neq 0$ therefore there is a natural number n_0 such that $x_{n_0}(\lambda_0) \neq 0$. For each $k \geq 0$, put $n_k = \phi_k(n_0)$, where ϕ_k denotes the k-th iterate of ϕ.

CLAIM: All n_k's are distinct.

On contrary, suppose that $n_i = n_j$ for some i, j with $i < j$. Then $j = i + k$, for some $k > 0$. Hence equation (2.1) gives

$$\lambda_0 x_{n_i}(\lambda_0) = x_{n_{i+1}}(\lambda_0)$$

$$\begin{aligned}
\lambda_0 x_{n_{i+1}}(\lambda_0) &= x_{n_{i+2}}(\lambda_0) \\
&\cdots \\
\lambda_0 x_{n_{i+k-1}}(\lambda_0) &= x_{n_{i+k}}(\lambda_0) \\
&= x_{n_j}(\lambda_0) \\
&= x_{n_i}(\lambda_0).
\end{aligned}$$

Thus $(\lambda_0)^k x_{n_i}(\lambda_0) = x_{n_i}(\lambda_0)$. This implies that $x_{n_i}(\lambda_0) = 0$.

Further, equation (2.1) implies

$$\begin{aligned}
\lambda_0 x_{n_0}(\lambda_0) &= x_{n_1}(\lambda_0) \\
&\cdots \\
\lambda_0 x_{n_{i-1}}(\lambda_0) &= x_{n_i}(\lambda_0).
\end{aligned}$$

Therefore, $(\lambda_0)^i x_{n_0}(\lambda_0) = x_{n_i}(\lambda_0) = 0$, which implies that $x_{n_0}(\lambda_0) = 0$, a contradiction. This proves our claim.

For $k \geq 1$, let n_{-k} be any element chosen from the set $\phi_k^{-1}(n_0) = \{n : \phi_k(n) = n_0\}$. Using the same arguments as above one can easily show that n_{-k}'s are all distinct and $\{n_k : k \geq 0\} \cap \{n_{-k} : k \geq 1\} = \emptyset$.

Hence from equation 2.1 we get

$$x_{n_k}(\lambda_0) = (\lambda_0)^k x_{n_0}(\lambda_0) \ \forall k \in \mathbb{Z}.$$

Thus

$$\begin{aligned}
\|f(\lambda_0)\|^p &\geq \sum_{k=-\infty}^{\infty} |x_{n_k}(\lambda_0)|^p \\
&= |x_{n_0}(\lambda_0)|^p (1 + |\lambda_0|^p + |\lambda_0|^{2p} + \cdots) + |x_{n_0}(\lambda_0)|^p \left(\frac{1}{|\lambda_0|^p} + \frac{1}{|\lambda_0|^{2p}} + \cdots\right) \\
&= \infty.
\end{aligned}$$

This is a contradiction. Hence $f(\lambda) = 0 \ \forall \lambda \in U$. Therefore, C_ϕ has SVEP.

Next, since ϕ is onto implies C_ϕ is injective, C_ϕ has SVEP (at 0). Again, since C_ϕ has closed range, C_ϕ^* is onto. C_ϕ^* cannot, however, have SVEP. For if it does, then it is injective [6], and we already know that C_ϕ is not onto. ∎

THEOREM 2.2. *If $\phi \colon \mathbb{N} \to \mathbb{N}$ is one-one but not onto then $C_\phi \colon l^p \to l^p$ $(1 \leq p < \infty)$ does not have SVEP.*

Proof. Let $n_0 \in \mathbb{N}$ be such that $n_0 \notin \text{range}(\phi)$. For each $k \geq 0$ put $n_k = \phi_k(n_0)$, where ϕ_k denotes the k-th iterate of ϕ. Since ϕ is one-one, we see that all n_k's are distinct. Now set $x_{n_k}(\lambda) = \lambda^k$, $k \geq 0$, $|\lambda| < 1$ and $x_j(\lambda) = 0$, if $j \notin \{n_0, n_1, \dots\}$. Then $f(\lambda) = (x_1(\lambda), x_2(\lambda), \dots)$ is a non-zero analytic map from open unit disk D into l^p satisfying $(\lambda - C_\phi) f(\lambda) = 0 \ \forall \lambda \in D$. Thus C_ϕ does not have SVEP. ∎

THEOREM 2.3. *If $\phi \colon \mathbb{N} \to \mathbb{N}$ is a bijection then $C_\phi \colon l^p \to l^p$ $(1 \leq p < \infty)$ is decomposable.*

Proof. If $\phi\colon \mathbb{N} \to \mathbb{N}$ is a bijective map then C_ϕ is an invertible isometry [10, p. 20], and hence, decomposable [8, Proposition 1.6.7]. ■

The following two examples show that if $\phi\colon \mathbb{N} \to \mathbb{N}$ is neither one-one nor onto then C_ϕ may or may not be decomposable.

EXAMPLE 2.5. Let $\phi\colon \mathbb{N} \to \mathbb{N}$ be defined as

$$\phi(n) = \begin{cases} n+1, & n \text{ is odd}, \\ n, & n \text{ is even}. \end{cases}$$

Then ϕ is neither one-one nor onto. Since $\phi_2 = \phi$ therefore, C_ϕ is a projection and hence, is decomposable.

EXAMPLE 2.6. Let $\phi\colon \mathbb{N} \to \mathbb{N}$ be defined as

$$\phi(2n-1) = \phi(2n) = 2n+1, \ \forall n \geq 1.$$

Then ϕ is neither one-one nor onto. Now define a map f from the open unit disk D into l^p as

$$f(\lambda) = (1, 1, \lambda, \lambda, \lambda^2, \lambda^2, \lambda^3, \lambda^3, \dots) \ \forall \lambda \in D.$$

Then f is a non-zero analytic map satisfying $(\lambda - C_\phi)f(\lambda) = 0$ for each $\lambda \in D$. Thus C_ϕ does not have SVEP and hence, is not decomposable.

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