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On the Self Crossing Six Sided Figure Problem

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Abstract. It was shown by Carbery, Christ, and Wright that any measurable set E in the unit square in $\mathbb{R}^2$ not containing the corners of a rectangle with area greater than λ has measure bounded by $O(\sqrt{\lambda \log \frac{1}{\lambda}})$. We remove the log under the additional assumption that the set does not contain the corners of any axis-parallel, possibly self-crossing hexagon with unsigned area bigger than λ. Our proof may be viewed as a bilinearization of Carbery, Christ, and Wright's argument.

Contents

0. Introduction

We say that four points in $\mathbb{R}^2$ are the vertices of an axis parallel rectangle if they are of the form (x_1, y_1), (x_1, y_2), (x_2, y_1), (x_2, y_2) with $x_1 \neq x_2$ and $y_1 \neq y_2$. We say that the area of the rectangle is $|x_1 - x_2||y_1 - y_2|$.

We say that six points in $\mathbb{R}^2$ are the vertices of a right angled, axis parallel hexagon (possibly self-crossing) if they are of the form (x_1, y_2), (x_1, y_3), (x_2, y_1), (x_2, y_3), (x_3, y_1), (x_3, y_2) with x_1, x_2, x_3 pairwise distinct and with y_1, y_2, y_3 pairwise distinct. We say that the area of the hexagon is

$$A = \max(|x_i - x_j|) \min(|y_i - y_j|) + \max(|y_i - y_j|) \min(|x_i - x_j|).$$

While not exactly the geometrical area (by which we mean the absolute area rather than the oriented area), the quantity A is comparable to it.

The purpose of this paper is to prove the following theorem.

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Theorem. *There exists a constant $C > 0$ so that for any number $\mu > 0$, whenever $E \subset [0,1] \times [0,1]$ is a set which does not contain the vertices of any axis parallel rectangle with area greater than μ^2 and does not contain the vertices of any axis parallel hexagon with area greater than μ^2 then $|E| \leq C\mu$.*

We explain the motivation of the theorem, which arose in the study of Fourier integrals. Carbery, Christ, and Wright [CCW] were studying the sublevel sets of phase functions for Fourier integral operators. They were looking for higher dimensional analogues of the statement that a real valued function f on the reals satisfying $f' > \lambda$ has small sublevel sets. (I.e., the set of x for which $|f(x)| \leq 1$ has measure smaller than $\frac{2}{\lambda}$.) This fact is the underlying idea behind Van der Corput's Lemma and the method of stationary phase, (see [S, Chapter 8]).

They observed that a function f on the unit square in $\mathbb{R}^2$ satisfying $\frac{\partial^2 f}{\partial x \partial y} > \lambda$ must satisfy the estimate

$$|\{(x,y) : |f(x,y)| \leq 1\}| \leq C\frac{\sqrt{\log \lambda}}{\sqrt{\lambda}}. \tag{-1}$$

The question arose whether (-1) is sharp. A simple example shows one can find such an f for which the sublevel set is as large as $\frac{1}{\sqrt{\lambda}}$. The question, as is often the case, is whether the log is really there. This question is open.

The question is related to another more geometrical question. Suppose we are given a set E with the property that it does not contain the vertices of any axis parallel rectangle with area larger than μ^2. We then trivially have the estimate

$$|E| \leq C\mu\sqrt{\log\frac{1}{\mu}}. \tag{-2}$$

The question is again whether the log is really there. The estimate (-2) implies the estimate (-1). We may see this by letting E be the sublevel set, by letting $\lambda = \frac{1}{\mu^2}$. We see that E satisfies the geometric condition we have stated by integrating $d\omega$ over any axis parallel rectangle with vertices in the set, where ω is the one-form

$$\omega = \frac{\partial f}{\partial x}dx - \frac{\partial f}{\partial y}dy.$$

The same argument shows that when E is the sublevel set of f, it satisfies more stringent geometrical conditions. Indeed it cannot contain the vertices of large non self-crossing axis parallel hexagons. (Or for that matter $2n$-sided polygons where large depends inversely on n.) One might suppose that with the exclusion of these six sided polygons, one might improve (-2) which would imply an improvement of (-1). This, unfortunately, is also an open problem.

The theorem we prove says that when we exclude also those axis parallel hexagons which are not polygons, i.e., those which self-cross, one gets the sharp improvement on (-2).

1. Proof of the Theorem

The main point of the proof is the following jigsaw puzzle lemma.

Lemma (jigsaw puzzle)**.** *There exist universal constants $C, \epsilon > 0$ so that for any large N, one has that if $A_1, \dots, A_K, B_1, \dots, B_K \subset [0,1]$ satisfying*

$$\frac{1}{N^{1+\epsilon}} \leq |A_j|, |B_j| \leq \frac{1}{N} \tag{1}$$

and further that if $A_j \cap A_k \neq \emptyset$ then $B_j \cap B_k = \emptyset$ (note this is symmetric in A and B) and further still if $A_j \cap A_k \neq \emptyset$, and $A_k \cap A_l \neq \emptyset$ then $B_j \cap B_l = \emptyset$ and analogously if $B_j \cap B_k \neq \emptyset$ and $B_k \cap B_l \neq \emptyset$ then $A_j \cap A_l = \emptyset$ and finally if we have

$$|A_j \cap A_k|, |B_j \cap B_k| \leq \frac{1}{|j-k|}, \tag{2}$$

then we reach the following conclusion: *We have*

$$K \leq C N^{2-\epsilon}.$$

In proving the jigsaw puzzle lemma, we will frequently make use of the following lemma.

Lemma X. *Let $a_1, \dots, a_N$ be nonnegative real numbers. Suppose that for all j, one has*

$$a_j < M.$$

Suppose further that

$$\sum_{j=1}^{N} a_j > S.$$

Then there exist at least $\frac{S}{2M}$ values of j for which

$$a_j \geq \frac{S}{2N}.$$

Proof. Define $\mathcal{G}$ to be the subset of $\{1, \dots, N\}$ so that for any $j \in \mathcal{G}$, one has $a_j \geq \frac{S}{2N}$. One has

$$\sum_{j \in \mathcal{G}} a_j \geq \frac{S}{2}.$$

But since each a_j is bounded above by M, we have

$$\#(\mathcal{G}) \geq \frac{S}{2M}.$$

□

Now before proving the jigsaw puzzle lemma, we spend a moment explaining the philosophy and the logic of the proof. If we remove the hypothesis (which is what relates to hexagons) that $A_j \cap A_k \neq \emptyset$ and $A_k \cap A_l \neq \emptyset$ imply that $B_j \cap B_l = \emptyset$ and vice versa, K can be as large as N^2. However, all examples for which K is even within a factor of N^ϵ of N^2 (for ϵ sufficiently small) share certain properties which force the hexagon condition to be violated.

The precise logic as far as quantifiers is as follows. We suppose that for any $\epsilon > 0$ a counterexample can be found with C as large as desired, provided N is sufficiently large. (In other words, we choose things so that powers of N which arise always dominate C.) We choose ϵ sufficiently small and reach a contradiction, thus proving the lemma. We abuse the notation slightly, as is the custom in analysis, changing the ϵ in every line with the understanding that it remains small. (This means we must have chosen it very, very small at the beginning.) We also occasionally introduce new constants C which are small compared to any power of N which arises.

Proof of jigsaw puzzle lemma. We suppose the conclusion of the lemma is false. Then there exist sets for which we have $K \geq CN^{2-\epsilon}$ with C arbitrarily large. (By the pairwise disjointness of the product sets $A_j \times B_j$, however, we certainly have $K \leq N^{2+\epsilon}$.) (We have changed the ϵ and simply applied the lower bound on $|A_j|$ and $|B_j|$.)

Now, we have that

$$\sum_{j=1}^{K} |A_j| \geq \frac{K}{N^{1+\epsilon}} \geq N^{1-\epsilon},$$

but

$$\begin{aligned}\sum_{j=1}^{K} |A_j| &= \sum_{j=1}^{K} \int_0^1 \chi_{A_j}(x)dx \\ &= \int_0^1 (\sum_{j=1}^{K} \chi_{A_j}(x))dx \\ &\leq (\int_0^1 (\sum_{j=1}^{K} \chi_{A_j}(x))^2 dx)^{\frac{1}{2}} \\ &\leq (\sum_{j=1}^{K}\sum_{k=1}^{K} |A_j \cap A_k|)^{\frac{1}{2}}.\end{aligned}$$

We conclude

$$\sum_{j=1}^{K}\sum_{k=1}^{K} |A_j \cap A_k| \geq K^2 \frac{1}{N^{2+\epsilon}}. \tag{3}$$

Fixing j arbitrary, we observe that

$$\sum_{k=1}^{K} |A_j \cap A_k| \leq \sum_{k=1}^{K} \frac{1}{|j-k|} \leq C \log N \leq C^{\frac{1}{2}} N^{\epsilon}. \tag{4}$$

Now combining (3), (4), and Lemma X, we conclude there are at least $\frac{K^2}{2C^{\frac{1}{2}}N^{2+\epsilon}}$ values of j for which

$$\sum_{k=1}^{K} |A_j \cap A_k| \geq \frac{K}{2N^{2+\epsilon}}. \tag{5}$$

We refer to the set of j's for which (5) holds as $\mathcal{G}_1$, the first good set.

Using our estimate on K, we rewrite (5) as

$$\sum_{k=1}^{K} |A_j \cap A_k| \geq \frac{C}{2N^{\epsilon}}. \tag{6}$$

Now we make a couple of observations. First, whenever we have distinct k_1, k_2 with $A_j \cap A_{k_1} \neq \emptyset$ and $A_j \cap A_{k_2} \neq \emptyset$, by assumption, we must have $B_{k_1} \cap B_{k_2} = \emptyset$. But by the lower bound on the measures of the B_j's, we conclude that

$$\#(\{k : A_j \cap A_k \neq \emptyset\}) \leq N^{1+\epsilon}. \tag{7}$$

Furthermore, by the upper bound on the measure of A_j, we have automatically that

$$|A_j \cap A_k| \leq \frac{1}{N}, \tag{8}$$

for every value of k. Combining (6), (7), (8), and Lemma X, we conclude that for every $j \in \mathcal{G}_1$ there must be at least $\frac{CN^{1-\epsilon}}{2}$ values of k with

$$|A_j \cap A_k| \geq \frac{C}{2N^{1+\epsilon}}. \tag{9}$$

Now we repeat the same argument for the B_j's indexed by $j \in \mathcal{G}_1$. We observe that

$$\sum_{j \in \mathcal{G}_1} |B_j| \geq \#(\mathcal{G}_1) \frac{1}{N^{1+\epsilon}}, \tag{10}$$

and also that

$$\begin{aligned} \sum_{j \in \mathcal{G}_1} |B_j| &= \int_0^1 \sum_{j \in \mathcal{G}_1} \chi_{B_j}(x) dx \\ &\leq (\int_0^1 (\sum_{j \in \mathcal{G}_1} \chi_{B_j}(x))^2 dx)^{\frac{1}{2}} \\ &\leq (\int_0^1 \sum_{j \in \mathcal{G}_1} \sum_{k \in \mathcal{G}_1} \chi_{B_j}(x) \chi_{B_k}(x) dx)^{\frac{1}{2}} \\ &= (\sum_{j \in \mathcal{G}_1} \sum_{k \in \mathcal{G}_1} |B_j \cap B_k|)^{\frac{1}{2}} \end{aligned} \tag{11}$$

and of course, that for any fixed j,

$$\sum_{k \in \mathcal{G}_1} |B_j \cap B_k| \leq C^{\frac{1}{2}} N^{\epsilon}. \tag{12}$$

Now combining (10), (11), (12), and Lemma X, we obtain as usual that there are at least $CN^{2-\epsilon}$ values of j for which

$$\sum_{k \in \mathcal{G}_1} |B_j \cap B_k| \geq \frac{CN^{-\epsilon}}{4}. \tag{13}$$

(Roughly speaking, we have shown that the sum whose square root appears in (11) is almost as large as N^2 by combining (11) and (10). The inequality (12) says that no summand is much larger than 1. Since the sum is over N^2 terms, there must be many almost as large as 1.) We refer to those j's for which (13) is satisfied as the elements of $\mathcal{G}_2$, the second good set.

We observe, as before, that

$$\#(\{k : B_j \cap B_k \neq \emptyset\}) \leq N^{1+\epsilon}, \tag{14}$$

and for any k one has

$$|B_j \cap B_k| \leq \frac{1}{N}. \tag{15}$$

Combining (13), (14), (15) and Lemma X, we obtain that whenever $j \in \mathcal{G}_2$ there are at least $CN^{1-\epsilon}$ values of k for which

$$|B_j \cap B_k| \geq \frac{C}{N^{1+\epsilon}}. \tag{16}$$

Now we are in a position to reach a contradiction. We pick a fixed $j \in \mathcal{G}_2$. Then we know there are $CN^{1-\epsilon}$ values of $k \in \mathcal{G}_1$ for which (16) holds. Denote this set of k by $\mathcal{P}_1$. Now (16) implies that for each $k \in \mathcal{P}_1$ one has

$$|j - k| \leq \frac{N^{1+\epsilon}}{C}.$$

Further, the A_k's indexed by $\mathcal{P}_1$ are pairwise disjoint. Finally, since $\mathcal{P}_1$ is a subset of $\mathcal{G}_1$, for each $k \in \mathcal{P}_1$, one has $\frac{CN^{1-\epsilon}}{2}$ values of l for which $|A_k \cap A_l| \geq \frac{C}{2N^{1+\epsilon}}$. We denote the set of all these l's by $\mathcal{P}_2$. Since the A_k's are pairwise disjoint and the A_l's have measure at most $\frac{1}{N}$, each l may occur for at most $\frac{2N^{\epsilon}}{C}$ different values of k. Thus we have

$$\#(\mathcal{P}_2) \geq CN^{2-\epsilon}. \tag{17}$$

But, fixing k, we have that for each l corresponding to that k, one has

$$|k - l| \leq \frac{2N^{1+\epsilon}}{C},$$

so that for every $l \in \mathcal{P}_2$, one has

$$|j - l| \leq \frac{N^{1+\epsilon}}{C} + \frac{2N^{1+\epsilon}}{C}. \tag{18}$$

However, (17) and (18) imply a contradiction, for otherwise there would be a set of $N^{2-\epsilon}$ distinct l's at a distance of $N^{1+\epsilon}$ from j. $\square$

In what remains, we make use of the jigsaw puzzle lemma to prove the theorem. Suppose $\eta > 0$ is a number. We define $K(\eta)$ to be the smallest number so that if E is any set satisfying the hypotheses of the theorem for parameter $\mu > \eta$, one always has

$$|E| \leq K(\eta)\mu.$$

By a well known method of extremal graph theory (essentially the application of Cauchy Schwartz in (11) again), one can show a priori that $K(\eta) \leq C(1 + \log(|\frac{1}{\eta}|))$. We will proceed by proving a bootstrapping inequality for $K(\eta)$.

First, we need a lemma about rescaling.

Lemma (rescaling)**.** *Let E be any set satisfying the hypotheses of the theorem with parameter μ. Suppose that the x-projection of the set, $\Pi_x E$ satisfies*

$$|\Pi_x E| \leq c,$$

then one has

$$|E| \leq c^{\frac{1}{2}} K(\mu)\mu.$$

The main tool in proving the rescaling lemma will be the following lemma of one dimensional measure theory.

Lemma (compression)**.** *Let $A \subset [0,1]$ be a measurable set with $|A| = c$. Then there is a map Q from a full measured subset of A one-to-one and onto a full measured subset of $(0,c)$ which is measure preserving and monotone and so that one always has*

$$|Q(y) - Q(x)| \leq |y - x|.$$

Proof of the rescaling lemma. We apply the compression lemma to the x-projection of E. By applying the map Q tensored with the identity to the set E, one obtains a new set F with $|F| = |E|$, with F satisfying the hypotheses of the theorem and with $F \subset [0,c] \times [0,1]$. Denote by G, the set F dilated by $\frac{1}{c}$. Then $G \subset [0,1] \times [0,1]$ and G satisfies the hypotheses of the theorem with parameter $\frac{1}{\sqrt{c}}\mu$ which is larger than μ. Thus

$$|G| \leq K(\mu)\frac{\mu}{\sqrt{c}}.$$

But

$$|E| = |F| = c|G|,$$

so that

$$|E| \leq K(\mu)\sqrt{c}\mu,$$

which was to be shown. □

The rescaling lemma easily implies the following

Lemma (distribution)**.** *Let E be a set satisfying the hypotheses of the theorem with parameter μ. Define*

$$H = \{(x,y) \in E : \int \chi_E(x,z)dz \geq K(\mu)^3\mu\}.$$

Then

$$|H| \leq \mu.$$

Proof. By the definition of $K(\mu)$, one has

$$|\Pi_x H| \leq \frac{1}{K(\mu)^2}.$$

Applying the rescaling lemma, we obtain

$$|H| \leq \mu.$$

□

Observe that the same is true if we replace the x direction by the y direction.

The theorem will now be implied by the following bootstrapping lemma:

Lemma (bootstrap)**.** *Let E be a set satisfying the hypotheses of the theorem, so that it is true for no value of x that*

$$\int \chi_E(x,z)dz \geq K(\mu)^3\mu,$$

and for no value of y is it true that

$$\int \chi_E(z,y)dz \geq K(\mu)^3\mu.$$

Then

$$|E| \leq C(\sqrt{1+\log K(\mu)})\mu.$$

The bootstrap lemma would imply together with the distribution lemma that

$$K(\mu) \leq 2 + C\sqrt{1+\log K(\mu)},$$

which in turn implies that $K(\mu)$ is bounded by a universal constant.

We will prove the bootstrap lemma by reducing it to well known methods of extremal graph theory combined with the jigsaw puzzle lemma. We introduce some notation.

We define

$$E_x = \{y \in [0,1] : (x,y) \in E\},$$

and

$$E^y = \{x \in [0,1] : (x,y) \in E\}.$$

Now we are ready to proceed.

We observe as usual that

$$\begin{aligned}
|E| &= \int\int \chi_E(x,y)dxdy \\
&= \int |E_x|dx \\
&\leq (\int |E_x|^2 dx)^{\frac{1}{2}} \\
&\leq (\int\int |E^y \cap E^z|)^{\frac{1}{2}} \\
&\leq \left(\sum_j \int_{\{(\frac{4}{3})^j\mu \leq |y-z| \leq (\frac{4}{3})^{j+1}\mu\}} |E^y \cap E^z|\right)^{\frac{1}{2}}.
\end{aligned}$$

Now, from the hypotheses of the theorem, we have the estimate

$$|E^y \cap E^z| \leq \frac{\mu^2}{|y-z|}, \tag{19}$$

by the hypotheses of the bootstrap lemma, we have

$$|E^y \cap E^z| \leq |E^y| \leq K(\mu)^3\mu. \tag{20}$$

Define the constant C sufficiently large so that $j \geq C\log K(\mu)$ implies that

$$\left(\frac{4}{3}\right)^j \geq K(\mu)^{500}.$$

Then it is immediate from (19) and (20) that

$$\sum_{j \leq C\log K(\mu)} \int_{(\frac{4}{3})^j\mu \leq |y-z| \leq (\frac{4}{3})^{j+1}\mu} |E^y \cap E^z| \leq C\log K(\mu).$$

We need only consider the sum over large j.

Our strategy from this point on will be to show that there are constants $C > 0$ and $\nu > 0$ so that whenever

$$R = SK(\mu)^{500}\mu,$$

with $S > 1$, one has that

$$\int_{\frac{3R}{2} \le |y-z| \le R} |E^y \cap E^z| \le CS^{-\nu}\mu^2. \tag{21}$$

Summing the geometric series will prove the theorem.

We now make a trivial remark about the set of pairs (y, z) with

$$\frac{3R}{2} \le |y-z| \le 2R.$$

We cover [0,1] by disjoint intervals of width R in two ways. We define

$$\mathcal{D}_R = \{[0,R],[R,2R],\dots,[R[\frac{1}{R}],(R+1)[\frac{1}{R}]]\} = \{I_0, I_1, \dots, I_{[\frac{1}{R}]}\},$$

and

$$\mathcal{D}'_R = \{[\frac{-R}{2},\frac{R}{2}],\dots,[(R+\frac{1}{2})[\frac{1}{R}],(R+\frac{3}{2})[\frac{1}{R}]]\} = \{J_{-1}, J_0, \dots, J_{[\frac{1}{R}]}\}.$$

Now, any pair y, z having the desired distance with $y < z$ will have for some integer j, either $y \in I_j$ and $z \in I_{j+2}$ or $y \in J_j$ and $z \in J_{j+2}$. Thus we obtain

$$\int_{\frac{3R}{2} \le |y-z| \le 2R} |E^y \cap E^z| \le \sum_j \int |E_x \cap I_j||E_x \cap I_{j+2}|dx + \sum_j \int |E_x \cap J_j||E_x \cap J_{j+2}|dx.$$

Thus it suffices to show for fixed j that

$$\int |E_x \cap I_j||E_x \cap I_{j+2}| \le CS^{-\nu}R\mu^2. \tag{22}$$

This is exactly what we shall do.

We divide up $[0,1]$ into the set of intervals $\mathcal{D}_{\frac{\mu^2}{R}}$ which are dual to $\mathcal{D}_R$ and we denote these K_k's. For each K_k, we choose an x_k for which $|E_{x_k} \cap I_j||E_{x_k} \cap I_{j+2}|$ is at least half as big as the supremum over $x \in K_k$. It suffices to estimate

$$\sum_k |E_{x_k} \cap I_j||E_{x_k} \cap I_{j+2}| \frac{\mu^2}{R} \le C\mu^2 S^{-\nu} R \tag{23}$$

[We consider only k odd (and separately consider only k even).] Rescaling respectively I_j and I_{j+2} into $[0,1]$ and renaming the rescaled $E_{x_k} \cap I_j$ and $E_{x_k} \cap I_{j+2}$ as A_k and B_k respectively, we see we need only show

$$\sum_k |A_k||B_k| \le CS^{-\nu}. \tag{24}$$

It is easy to see from the hypotheses of the theorem that the A's and B's satisfy all the hypotheses of the jigsaw puzzle lemma except for those regarding the measure of the A_j's or B_j's. We do know from the hypotheses of the bootstrapping lemma that these are less than $\frac{1}{S}$. We divide the k's into the classes $\mathcal{C}_{A,l}$ and $\mathcal{C}_{B,l}$ by letting $k \in \mathcal{C}_{A,l}$ if $|A_k| > |B_k|$ and $2^{-l}\frac{1}{S} > |A_k| \ge 2^{-l-1}\frac{1}{S}$ and $k \in \mathcal{C}_{B,l}$ if $|B_k| \ge |A_k|$ and $2^{-l}\frac{1}{S} > |B_k| \ge 2^{-l-1}\frac{1}{S}$. It suffices to show

$$\sum_{k \in \mathcal{C}_{A,l}} |A_k||B_k| \le CS^{-\nu}2^{-l\nu}. \tag{25}$$

We let $\frac{1}{N} = 2^{-l}\frac{1}{S}$. We divide $\mathcal{C}_{A,l} = C_g \cup C_b$, where $k \in C_b$ if $|B_k| \le \frac{1}{N^{1+\epsilon}}$.

We claim that

$$\#(C_b) \le CN^2 \log N \le CN^{2+\frac{\epsilon}{2}}. \tag{26}$$

This is because

$$\frac{1}{N}\#(C_b) \sim \sum_{j\in C_b} |A_j| \le \sqrt{\sum_{j\in C_b}\sum_{k\in C_b} \frac{1}{|j-k|}}.$$

Thus

$$\sum_{k\in C_b} |A_k||B_k| \le CN^{2+\frac{\epsilon}{2}}\frac{1}{N}\frac{1}{N^{1+\epsilon}} \le CN^{\frac{-\epsilon}{2}}. \tag{27}$$

On the other hand, C_g satisfies the hypotheses of the jigsaw puzzle lemma, so that applying the lemma we get

$$\#(C_g) \le CN^{2-\epsilon},$$

so that

$$\sum_{j\in C_g} |A_j||B_j| \le \frac{1}{N^2}CN^{2-\epsilon} \le CN^{-\epsilon}. \tag{28}$$

The inequalities (27) and (28) together imply (25) and hence the theorem.

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