

ABSTRACT. We prove some commutation relations for a 3-graded Lie algebra, i.e., a \mathbb{Z} -graded Lie algebra whose nonzero homogeneous elements have degrees -1 , 0 or 1 , over a field K . In particular, we examine the free 3-graded Lie algebra generated by an element of degree -1 and another of degree 1 . We show that if K has characteristic zero, such a Lie algebra can be realized as a Lie algebra of matrices over polynomials in one indeterminate. In the end, we apply the results obtained to derive the classical commutation relations for elements in the universal enveloping algebra of $\mathfrak{sl}_2(K)$.