Abstract. For any field k and any integers m, n with $0 \leqslant 2m \leqslant n+1$, let W_n be the k-vector space of sequences $(x_0, \ldots, x_n)$, and let $H_m \subseteq W_n$ be the subset of sequences satisfying a degree-m linear recursion — that is, for which there exist $a_0, \ldots, a_m \in k$, not all zero, such that

$$\sum_{i=0}^{m} a_i x_{i+j} = 0$$

holds for each $j = 0, 1, \ldots, n-m$. Equivalently, H_m is the set of $(x_0, \ldots, x_n)$ such that the $(m+1) \times (n-m+1)$ matrix with (i,j) entry x_{i+j} ($0 \leqslant i \leqslant m$, $0 \leqslant j \leqslant n-m$) has rank at most m. We use elementary linear and polynomial algebra to study these sets H_m. In particular, when k is a finite field of q elements, we write the characteristic function of H_m as a linear combination of characteristic functions of linear subspaces of dimensions m and $m+1$ in W_n. We deduce a formula for the discrete Fourier transform of this characteristic function, and obtain some consequences. For instance, if the $2m+1$ entries of a square Hankel matrix of order $m+1$ are chosen independently from a fixed but not necessarily uniform distribution μ on k, then as $m \to \infty$ the matrix is singular with probability approaching $1/q$ provided $\|\widehat{\mu}\|_1 < q^{1/2}$. This bound $q^{1/2}$ is best possible if q is a square.