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The main supergraph of finite groups

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ABSTRACT. Let G be a finite group. The main supergraph $\mathcal{S}(G)$ is a graph with vertex set G in which two vertices x and y are adjacent if and only if $o(x) \mid o(y)$ or $o(y) \mid o(x)$. In this paper, we will show that if $\mathcal{S}(G) \cong \mathcal{S}(S)$, where S belongs to a large class of finite non-solvable groups, then $G \cong S$. This work is an important step in solving Thompson's problem.

CONTENTS

1. Introduction

Let G be a finite group and $x \in G$. The order of x is denoted by $o(x)$. The set of all element orders of G is denoted by $\pi_e(G)$ and the set of all prime factors of $|G|$ is denoted by $\pi(G)$. We set $M_i(G) = |\{g \in G|$ the order of g is $i\}|$. The other notations and terminologies in this paper are standard, and the reader is referred to [14] if necessary.

Define the graph $\mathcal{S}(G)$ with the vertex set G such that two vertices x and y are adjacent if and only if $o(x) \mid o(y)$ or $o(y) \mid o(x)$. This graph is called the main supergraph of $\mathcal{P}(G)$ (power graph of G) and was introduced in [20]. The proper main supergraph $\mathcal{S}^*(G)$ is the graph constructed from $\mathcal{S}(G)$ by removing the identity element of G. We write $x \sim y$ when two vertices x and y are adjacent.

Definition 1.1. *Let G be a finite group. We say that G is recognizable by its main supergraph if for every group H we have $\mathcal{S}(G) \cong \mathcal{S}(H)$, then $G \cong H$.*

Note that not all groups are recognizable by the main supergraph. For example, we have $\mathcal{S}(\mathbb{Z}_4) \cong \mathcal{S}(\mathbb{Z}_2 \times \mathbb{Z}_2)$, but $\mathbb{Z}_4$ is not isomorphic to $\mathbb{Z}_2 \times \mathbb{Z}_2$.

Definition 1.2. *Two finite groups G_1 and G_2 are called of the same order type if and only if $M_t(G_1) = M_t(G_2)$ for all t.*

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In 1987, J. G. Thompson [37, Problem 12.37] posed the following problem:
Thompson's Problem. Suppose that G_1 and G_2 are two finite groups of the same order type. If G_1 is solvable, is it true that G_2 is also necessarily solvable?

Another form of this problem can be stated as follows.

Thompson's Problem. Suppose that G_1 and G_2 are two finite groups of the same order type. If G_1 is non-solvable, is it true that G_2 is also necessarily non-solvable?

By definition of the main supergraph, it is clear that if G_1 and G_2 are groups with the same order type, then $\mathcal{S}(G_1) \cong \mathcal{S}(G_2)$. So, if a finite group G is recognizable by the main supergraph, then for G Thompson's problem is true.

In [3], the authors of this paper proved that alternating groups of degree p, $p+1$, $p+2$ and symmetric groups of degree p are recognizable by their main supergraph. Also, in [6], [4], [2] and [5], it is proved that the groups $L_2(p)$, $\mathrm{PGL}_2(p)$, where p is prime, all sporadic simple groups, $L_2(q)$, small Ree groups ${}^2G_2(3^{2n+1})$, where n is a natural number and Suzuki Ree groups are recognizable by their main supergraph.

The prime graph of G, is denoted by $\Gamma(G)$ and is defined as follows. The vertex set of $\Gamma(G)$ is $\pi(G)$ and two distinct vertices p and q are adjacent if and only if G contains an element of order pq. Let $t(G)$ be the number of connected components of $\Gamma(G)$ and $\pi_i = \pi_i(G)$, $1 \leq i \leq t(G)$ be the connected components of $\Gamma(G)$. For a group of even order we let $2 \in \pi_1(G)$. Then the order of G can be expressed as the product of $m_1, m_2, \ldots, m_{t(G)}$, where m_i ($1 \leq i \leq t(G)$) are positive integers with $\pi(m_i) = \pi_i$. These m_i ($1 \leq i \leq t(G)$) are called the order components of G. We write $OC(G) = \{m_1, m_2, \ldots, m_{t(G)}\}$ and call it the set of order components of G (see [9]).

In this paper, first we prove if G and S are two arbitrary groups such that $\mathcal{S}(G) \cong \mathcal{S}(S)$, then their order components are equal. Then, we conclude that if S is one of the nonabelian simple groups

- A_p, where p and $p-2$ are primes,
- A_n, where $n = p, p+1, p+2$,
- $L_n(q)$, where $n = 2, 3, 5$,
- $U_p(q), L_{p+1}(q)$,
- $L_{p+1}(2)$,
- $U_{p+1}(q)$,
- $C_n(q)$, where q is an even prime power,
- $C_2(q)$, where $q > 5$,
- $E_8(q)$,
- $F_4(q)$, where $q = 2^n > 2$,
- $G_2(q)$ $(3 \mid q)$,
- $G_2(q)$ $(2 < q \equiv 1 \pmod 3)$,
- ${}^2G_2(q)$,
- ${}^3D_4(q)$,
- ${}^2D_n(3)$, where $9 \leq n = 2^m + 1$ not a prime,
- ${}^2D_{p+1}(q)$, where $5 < p \neq 2^m - 1$,

- ${}^2D_p(3)$, where $p \geq 5$ is a prime number not of the form $2^m + 1$,
- ${}^2D_n(2)$, where $n = 2^m + 1 \geq 5$,
- $D_{p+1}(2)$,
- $D_{p+1}(3)$,
- ${}^2D_n(2)$, where $n = 2^m$,
- Suzuki Ree groups,
- Sporadic simple groups,
- ${}^2E_6(q)$,
- $L_p(q)$,
- $U_n(q)$, where $n = 3, 5, 11$,
- almost sporadic simple groups, except $\mathrm{Aut}(McL)$ and $\mathrm{Aut}(J_2)$,
- S_n, for $n = p, p+1$, where $p \geq 3$ is a prime number,

then S is recognizable by the main supergraph.

Main Theorem. Let G and S be two arbitrary groups such that $\mathcal{S}(G) \cong \mathcal{S}(S)$. Then $OC(G) = OC(S)$.

To get the main result of this paper, we need to the following lemma.

Lemma 1.3. *Let $OC(G) = OC(S)$, where S is one of the nonabelian simple groups listed before the main theorem, then $G \cong S$.*

Proof. See [1, 7, 8, 10, 11, 12, 13, 17, 15, 18, 16, 21, 24, 25, 26, 27, 23, 22, 28, 29, 31, 32, 36, 34, 35, 33, 30, 38, 39]. □

Corollary 1.4. *Let G be a finite group listed in the above lemma. Then G is recognizable by the main supergraph.*

Corollary 1.5. *Let G be a finite group listed in the above lemma. Then Thompson's problem is true for G.*

According to research conducted on non-solvable groups of order less than 2000 by using GAP and Corollary 1.4, we pose the following two conjectures:

Conjecture 1.6. *Let S be a finite simple group and G be an arbitrary finite group such that $\mathcal{S}(G) \cong \mathcal{S}(S)$. Then $G \cong S$.*

Conjecture 1.7. *Let S be a finite non-solvable group and G be an arbitrary finite group such that $\mathcal{S}(G) \cong \mathcal{S}(S)$. Then G is a non-solvable group.*

It is clear that if Conjecture 1.7 is true, then Thompson's problem is also true. Note that there exist non-solvable groups S and G such that $\mathcal{S}(G) \cong \mathcal{S}(S)$, but G and S are not isomorphic. For example, if $G = \mathbb{Z}_4 \times A_5$ and $S = < a, b >$, where

$$a = (1, 20, 17, 5, 12)(2, 3, 9, 19, 10)(4, 14, 22, 11, 6)(7, 8, 15, 13, 16)$$

$$b = (2, 18)(5, 11)(6, 21)(7, 24)(9, 17)(10, 16)(12, 23)(13, 20)(14, 19)(15, 22)$$

(in fact S has structure description $\mathrm{SL}(2, 5) : \mathbb{Z}_2$), then $\mathcal{S}(G) \cong \mathcal{S}(S)$, but G and S are not isomorphic.

The next lemma is used in the proof of the main theorem.

Lemma 1.8. [40, Theorem 3] *Let G be a finite group. Then the number of elements whose order is a multiple of n is either zero, or a multiple of the greatest divisor of $|G|$ that is prime to n.*

2. Proof of main theorem

By definition of the main supergraph and our assumption, we have $|G| = |S|$ and $\mathcal{S}^*(S) \cong \mathcal{S}^*(G)$. First, let $\mathcal{S}^*(G)$ be a connected graph. We show that $\Gamma(G)$ and $\Gamma(S)$ are connected. If $\pi(G) = \{p\}$, then $\Gamma(G) = \Gamma(S)$ has one vertex. So, $\Gamma(G)$ and $\Gamma(S)$ are connected. Let $|\pi(G)| \geq 2$. If $\Gamma(G)$ or $\Gamma(S)$ are disconnected, then $\Gamma(G)$ or $\Gamma(S)$ have two or more connected components. By definition of the main supergraph $\mathcal{S}^*(G)$ is a disconnected graph, which is a contradiction.

Now, let $\mathcal{S}^*(G)$ be a disconnected graph. Then it has two or more connected components. Suppose that K_1 and K_2 are two connected components of $\mathcal{S}^*(G)$. We show that if x, y are two arbitrary vertices of K_1 and K_2, respectively such that $o(x) = r$ and $o(y) = s$, where r and s are primes, then r and s are not joined by an edge in the prime graph of G. Assume that r and s are joined by an edge in the prime graph of G. Then $rs \in \pi_e(G)$. So, there exists an element of order rs in G. Assume $z \in G$ and $o(z) = rs$. By definition of the main supergraph $x \sim z$ and $y \sim z$. Thus K_1 and K_2 are connected, which is a contradiction.

Suppose that K_1, K_2,..., and K_n are all connected components of $\mathcal{S}^*(G)$. Let $\pi(K)$ be all prime numbers that divide the order of vertices of K, where K is one of the connected components of $\mathcal{S}^*(G)$. We claim that $\pi(K_i)$ for every $1 \leq i \leq n$ is the set of vertices of one of the connected components of $\Gamma(G)$. Assume that T_i is a component in the prime graph G such that the vertices of T_i are subset of $\pi(K_i)$. Thus, $rs \notin \pi_e(G)$ for every $r \in V(T_i)$ and $s \in \pi(K_i) \backslash V(T_i)$. By definition of the main supergraph, we can conclude that K_i is not connected, a contradiction. Therefore, there exists a one-to-one correspondence between connected components of $\mathcal{S}^*(G)$ and $\Gamma(G)$. It follows that $\pi(K_i)$ for every $1 \leq i \leq n$ is the set of vertices of one connected component of $\Gamma(G)$.

Let K be one of the connected components of $\mathcal{S}^*(G)$. The vertices of K are elements of G and their orders are divided by some prime numbers. We will show how to find these prime numbers.

Suppose that K_1, K_2 are two arbitrary connected components of $\mathcal{S}^*(G)$. Since K_1 and K_2 are isolated, we have $rs \notin \pi_e(G)$, where $r \in \pi(K_1)$ and $s \in \pi(K_2)$.

Let $p \in \pi(K_1)$ be arbitrary. If $\pi(K_2) = \{p_1\}$, then considering $n = p_1$ in Lemma 1.8, $|P| \mid \sum_{t \text{ is a multiple of } p_1} M_t = (M_{p_1} + M_{p_1{}^2} + \cdots + M_{p_1^k}) = |K_2|$ $(p_1^k \in \pi_e(G))$, where P is a Sylow p-subgroup of G. Assume that $\pi(K_2) = \{p_1, p_2\}$. Considering $n = p_1 p_2$ in Lemma 1.8, we have $|P| \mid \sum_{t \text{ is a multiple of } p_1 p_2} M_t$. On the other hand, considering $n = p_1$ in Lemma 1.8, $|P| \mid \sum_{t \text{ is a multiple of } p_1} M_t = (\sum_{t \text{ is a multiple of } p_1 p_2} M_t) + (M_{p_1} + M_{p_1{}^2} + \cdots + M_{p_1^k})$. It follows that $|P| \mid (M_{p_1} + M_{p_1{}^2} + \cdots + M_{p_1^k})$. Similarly, $|P| \mid (M_{p_2} + M_{p_2{}^2} + \cdots + M_{p_2^e})$ $(p_2^e \in \pi_e(G))$. Therefore, $|P| \mid (M_{p_1} + M_{p_1{}^2} + \cdots + M_{p_1^k}) + (M_{p_2} + M_{p_2{}^2} + \cdots + M_{p_2^e}) + (\sum_{t \text{ is a multiple of } p_1 p_2} M_t) = |K_2|$. If $\pi(K_2) = \{p_1, p_2, p_3\}$, then considering $n = p_1 p_2 p_3$, $p_1 p_2$, $p_1 p_3$ and p_1

in Lemma 1.8, $|P| \mid \sum_{t \text{ is a multiple of } p_1p_2p_3} M_t$, $|P| \mid \sum_{t \text{ is a multiple of } p_1p_2} M_t$, $|P| \mid \sum_{t \text{ is a multiple of } p_1p_3} M_t$ and $|P| \mid \sum_{t \text{ is a multiple of } p_1} M_t$. Thus, $|P| \mid (M_{p_1}+M_{{p_1}^2}+\cdots+M_{p_1^k})$. Similarly, $|P| \mid (M_{p_2}+M_{{p_2}^2}+\cdots+M_{p_2^e})$ and $|P| \mid (M_{p_3}+M_{{p_3}^2}+\cdots+M_{p_3^l})$. Therefore, $|P| \mid |K_2|$.

Arguing as above, if $\pi(K_2) = \{p_1, p_2, .., p_r\}$, then $|P| \mid |K_2|$. Also, $|P| \mid |K_i|$ for every connected component K_i $(i \neq 1)$ of $\mathcal{S}^*(G)$. Since $|P| \mid |G| = (1 + |K_1| + |K_2| + \cdots + |K_n|)$, we have $|P| \nmid |K_1|$. Now, let K be one of the connected components of $\mathcal{S}^*(G)$. If $p \in \pi(G)$ is such that $|P| \nmid |K|$, then $p \in \pi(K)$.

Since there exists a one-to-one correspondence between connected components of $\mathcal{S}^*(G)$ and $\Gamma(G)$ and $|G| = |S|$, we have $OC(G) = OC(S)$. This completes the proof of the main theorem.

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