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Erratum to "Higher-rank graph C^*-algebras"

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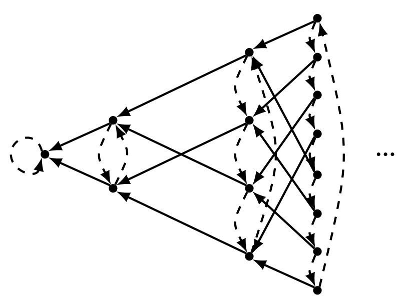
Abstract. We fix a longstanding error in [KP, Proposition 4.9] and provide a correct version of the result in the original generality.

1. A counterexample, the correct definition and the correct arguments

Some time ago the first two authors received the following advice from Aidan Sims: Consider the 2-graph from [PRRS, Figure 4] whose 1-skeleton determines its commuting squares



The 2-graph satisfies the hypothesis of [KP, Proposition 4.9], which would then say that the C^*-algebra of this graph is purely infinite. Yet the C^*-algebra of the above graph is Morita-Rieffel equivalent to the Bunce-Dendens algebra of type 2^∞ which is an $A\mathbb{T}$-algebra and hence not purely infinite, [PRRS, Example 6.1]. Hence this graph is a counterexample to [KP, Proposition 4.9]. This is due to an incorrect definition of loop with an entrance given in the statement.

The correct definition of loop with an entrance is to be found in [S, Definition 8.7] and is given below.

Definition 1. Let Λ be a locally convex, row-finite k-graph. A *loop with an entrance* is an element $\mu \in \Lambda$ such that $r(\mu) = s(\mu)$ such that there exists $\alpha \in s(\mu)\Lambda$ such that $d(\mu) \geq d(\alpha)$ and $\mu(0, d(\alpha)) \neq \alpha$.

If the above definition had been used, then the proof in [KP, Proposition 4.9], using the results of [A-D], would have been correct. The condition originally used does not imply the groupoid is locally contracting as stated in the first sentence.

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A correct published version of the result, is to be found in [S, Proposition 8.8]. The proof follows the one given in [BPRSz].

Theorem 2. *Let Λ be an aperiodic, row-finite k-graph with no sources, such that every vertex can be reached from a loop with an entrance. Then every hereditary subalgebra has an infinite projection. Hence, if Λ is cofinal then $C^*(\Lambda)$ is purely infinite.*

Remark 3. The condition (C) used in [S, Proposition 8.8], is a version of aperiodicity for k-graphs which are not necessarily row-finite. We briefly show that condition (C) reduces to condition (A) described in Definition [KP, Definition 4.3] under the hypotheses used in [KP, Proposition 4.9] and completes the description of the relationship between between the two results.

As Λ is row-finite with no sinks many of the hypotheses in condition (C) are trivial: Λ is finitely aligned, $FE(\Lambda) = \{v\Lambda^n : v \in \Lambda^0 \text{ and } n \in \mathbb{N}^k\}$, and is equal to the satiation of this set in the sense of [S0, Definition 4.1], so

- $\partial(\Lambda; FE(\Lambda)) = \partial(\Lambda)$ where $\partial(\Lambda; FE(\Lambda))$ is defined in [S0, Definition 4.3] and $\partial\Lambda$ is defined in [FMY, Definition 5.10].
- $\partial\Lambda = \Lambda^{\leq\infty}$ where $\Lambda^{\leq\infty}$ is defined in [RSY, Definition 2.8].
- $\Lambda^{\leq\infty} = \Lambda^\infty$ where Λ^∞ is defined in [KP, Definitions 2.1].

By [LS, Proposition 3.6] one may then see that condition (C) reduces to condition (A) in [KP].

References

[A-D] ANANTHARAMAN-DELAROCHE, C. Purely infinite C^*-algebras arising from dynamical systems, *Bull. Soc. Math. France* **125** (1997), 199–225. MR1478030, Zbl 0896.46044. 1029

[BPRSz] BATES, T., PASK, D., RAEBURN, I., AND SZYMAŃSKI, W., The C^*-algebras of row-finite graphs, *New York J. Math.* **6** (2000), 307–324. MR1478030, Zbl 0976.46041. 1030

[FMY] FARTHING, C., MUHLY, P.S., AND YEEND, T., Higher-rank graph C^*-algebras: An inverse semigroup and groupoid approach, *Semigroup Forum* **71** (2005), 159–187. MR2184052, Zbl 1099.46036. 1030

[KP] KUMJIAN, A. AND PASK, D., Higher rank graph C^*-algebras, *New York J. Math.* **6** (2000), 1–20. MR1745529, Zbl 0946.46044. 1029, 1030

[LS] LEWIN, P. AND SIMS, A., Aperiodicity and cofinality for finitely aligned higher-rank graphs, *Math. Proc. Cambridge Philos. Soc.* **149** (2010), 333–350. MR2670219, Zbl 1213.46047. 1030

[PRRS] PASK, D., RAEBURN, I., RØRDAM, M., AND SIMS, A., Rank-Two Graphs whose C^*-algebras are direct limits of Circle Algebras, *J. Funct. Anal.* **239** (2006), 137–178. MR2258220, Zbl 1112.46042. 1029

[RSY] RAEBURN, I., SIMS, A., AND YEEND, T., The C*-algebras of finitely aligned higher rank graphs, *J. Funct. Anal.* **213** (2004), 206—240. MR2069786, Zbl 1063.46041. 1030

[S0] SIMS, A., Relative Cuntz-Krieger algebras of finitely aligned higher-rank graphs, *Indiana Univ. Math. J.* **55** (2006), 849–868. MR2225454. 1030

[S] SIMS, A., Gauge-invariant ideals in the C^*-algebras of finitely aligned higher-rank graphs, *Canad. J. Math.* **58** (2006), 1268–1290. MR2270926, Zbl 1115.46050. 1029, 1030

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