PUBLICATIONS DE L'INSTITUT MATHÉMATIQUE
Nouvelle série, tome 58 (72), 1995, 35–42
Slobodan Aljančić memorial volume

ON $\mathcal{M}$-HARMONIC SPACE $\mathcal{B}_p^s$

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Dedicated to the memory of Professor Slobodan Aljančić

Abstract. We give several characterizations of the Besov space $\mathcal{B}_p^s$ of $\mathcal{M}$-harmonic functions in the open unit ball in $\mathbb{C}^n$.

1. Introduction and results

In [**4**] Hahn and Youssfi considered the boundary behavior in the Besov spaces $\mathcal{B}_p^s$ of $\mathcal{M}$-harmonic functions in the open unit ball B in $\mathbb{C}^n$. In this paper we deal with several characterizations of the spaces $\mathcal{B}_p^s$. As a consequence we have:

1) If $s > n$, then $\mathcal{B}_p^s = \mathcal{A}_p^s$, where $\mathcal{A}_p^s$ is the weighted Bergman space.

2) If $s = n$, the spaces $\mathcal{B}_p^n$ are closely related to the Hardy spaces $\mathcal{H}^p$ of $\mathcal{M}$-harmonic functions in B.

3) For $0 \leq s < n$, $\mathcal{B}_p^s$ are Besov spaces ($\mathcal{B}_p^0$ is the diagonal Besov space).

4) For $-p < s < 0$ the functions in the space $\mathcal{B}_p^s$ have Lipschitz continuity of order $-s/p$ and thus extend continuously to the closed unit ball (see also Theorem 1.4 of [**4**]).

5) If $s \leq -p$ then $\mathcal{B}_p^s$ ={constants}.

Let $B = B_n$ be the open unit ball in $\mathbb{C}^n$ and $S = \partial B$ the unit sphere in $\mathbb{C}^n$. We denote by ν the normalized Lebesgue measure on B and by σ the rotation invariant probability measure on S.

Let $\widetilde{\Delta}$ be the invariant Laplacian on B. That is, $\widetilde{\Delta} f(z) = \Delta(f \circ \varphi_z)(0)$, $f \in C^2(B)$, where Δ is the ordinary Laplacian and φ_z the standard automorphism of B, $\varphi_z \in \mathrm{Aut}(B)$, taking 0 to z (see [**9**]).

The C^2-functions f that are anihilated by $\widetilde{\Delta}$ are called $\mathcal{M}$-harmonic ($f \in \mathcal{M}$).

AMS Subject Classification (1991): Primary 32A37

Definition 1.1. For $0 < p < \infty$, and $s \in \mathbb{R}$, the weighted Bergman space $\mathcal{A}_p^s$ is defined as the space of $\mathcal{M}$-harmonic functions f on B for which

$$\|f\|_{\mathcal{A}_p^s} = \left[\int_B (1-|z|^2)^s |f(z)|^p d\lambda(z)\right]^{1/p} < \infty,$$

where $d\lambda(z) = (1-|z|^2)^{-n-1} d\nu(z)$ is the measure on B that is invariant under the group $\mathrm{Aut}(B)$.

For $f \in C^1(B)$, $Df = (\frac{\partial f}{\partial z_1}, \dots, \frac{\partial f}{\partial z_n})$, denotes the complex gradient of f, $\nabla f = (\frac{\partial f}{\partial x_1}, \dots, \frac{\partial f}{\partial x_{2n}})$, $z_k = x_{2k-1} + i x_{2k}$, $k = 1, 2, \dots, n$, denotes the real gradient of f.

For $f \in C^1(B)$ let $\widetilde{D} f(z) = D(f \circ \varphi_z)(0)$, $z \in B$, and $\widetilde{\nabla} f(z) = \nabla(f \circ \varphi_z)(0)$, $z \in B$, be the invariant complex gradient of f and the invariant real gradient of f respectively.

Definition 1.2. For $0 < p < \infty$, and $s \in \mathbb{R}$, the $\mathcal{M}$-harmonic Dirishlet space $\mathcal{D}_p^s$ is defined as the space of $\mathcal{M}$-harmonic functions f on B for which

$$\int_B |\widetilde{\nabla} f(z)|^p (1-|z|^2)^s d\lambda(z) < \infty.$$

The (differential) Bergman metric $b : B \times \mathbb{C}^n \mapsto \mathbb{R}$ is defined by

$$b(z,\xi) = \left(\frac{(1-|z|^2)|\xi|^2 + |\langle z,\xi\rangle|^2}{(1-|z|^2)^2}\right)^{1/2}.$$

For $f \in C^1(B)$, define the functional quantity

$$Qf(z) = \sup_{|\xi|=1} \frac{|\nabla f(z)\cdot\xi|}{b(z,\xi)} = \sup_{|\xi|=1} \frac{|\langle Df(z),\xi\rangle + \overline{\langle D\overline{f}(z),\xi\rangle}|}{b(z,\xi)},\ z \in B.$$

This quantity is invariant under $\mathrm{Aut}(B)$, that is $Q(f \circ \varphi) = Q(f) \circ \varphi$, for all C^1-functions f in B and $\varphi \in \mathrm{Aut}(B)$ (see [**5**, **6**]).

Definition 1.3. For $0 < p < \infty$, $s \in \mathbb{R}$, let $\mathcal{B}_p^s$ be the space of $\mathcal{M}$-harmonic functions f on B such that

$$\|f\|_{p,s} = \left(\int_B (Qf)^p(z)(1-|z|^2)^s d\lambda(z)\right)^{1/p} < \infty.$$

THEOREM 1.4. *Let $0 < p < \infty$, $s > n - p/2$ and $f \in \mathcal{M}$. Then the following statements are equivalent:*

(i) $f \in \mathcal{D}_p^s$, (ii) $f \in \mathcal{B}_p^s$, (iii) $\int_B |\nabla f(z)|^p (1-|z|^2)^{s+p} d\lambda(z) < \infty$,

(iv) $\int_B (1-|z|^2)^{s+p}(|Rf(z)| + |\overline{R}f(z)|)^p d\lambda(z) < \infty$.

As usual, $Rf(z) = \sum_{j=1}^{n} z_j \dfrac{\partial f}{\partial z_j}$*, is the radial derivative of* f *and* $\overline{R}f(z) = \sum_{j=1}^{n} \overline{z}_j \dfrac{\partial f}{\partial \overline{z}_j}$.

2. Proof of Theorem

If $0 < r < 1$, we set $E_r(z) = \{w \in B : |\varphi_z(w)| < r\} = \varphi_z(rB)$. It is easy to see that $E_r(z)$ is an ellipsoid and its volume is given by $\nu(E_r(z)) = \dfrac{r^{2n}(1-|z|^2)^{n+1}}{(1-r|z|)^{n+1}}$ (see [**9**, p. 30]). We set $|E_r(z)| = \nu(E_r(z))$.

For the proof of Theorem 1.4 the following lemmas will be needed.

LEMMA 2.1. [**7**] *Let* $0 < r < 1$*. There is a constanat* $C > 0$ *such that if* $f \in \mathcal{M}$ *then*

(i) $$|T_{ij}Rf(w)| \le C(1-|w|^2)^{-1/2} \int_{E_r(w)} |Rf(z)|d\lambda(z), \quad w \in B,$$

(ii) $$|T_{ij}\overline{R}f(w)| \le C(1-|w|^2)^{-1/2} \int_{E_r(w)} |\overline{R}f(z)|d\lambda(z), \quad w \in B,$$

(ii) $$|T_{ij}f(w)| \le C(1-|w|^2)^{-1/2} \int_{E_r(w)} |f(z)|d\lambda(z), \quad w \in B.$$

Here, as usual, $T_{ij} = \overline{z}_i \dfrac{\partial}{\partial z_j} - \overline{z}_j \dfrac{\partial}{\partial z_i}$, $\overline{T}_{ij} = z_i \dfrac{\partial}{\partial \overline{z}_j} - z_j \dfrac{\partial}{\partial \overline{z}_i}$*, are tangential derivatives.*

Here and elsewhere constants are denoted by C which may indicate a different constant from one occurrence to the next.

LEMMA 2.2. *If* $s > 1$*, then*

$$\int_0^1 \frac{dt}{|1 - t\langle z, w\rangle|^s} \le \frac{C}{|1 - \langle z, w\rangle|^{s-1}}, \quad z, w \in B.$$

LEMMA 2.3. [**9**, P. 17] *If* $\alpha > 0$*, then*

$$\int_S \frac{d\sigma(\xi)}{|1 - \langle \xi, z\rangle|^{n+\alpha}} = O\Big(\frac{1}{(1-|z|)^{\alpha}}\Big), \quad z \in B.$$

It is easy to see that $|\widetilde{\nabla} f(z)| = Qf(z)$. Hence, $\mathcal{D}_p^s = \mathcal{B}_p^s$, for all $0 < p < \infty$ and $s \in \mathbb{R}$.

From the inequality $Qf(z) \ge (1-|z|^2)|\nabla f(z)|$ (see [**4**, p. 221]) it follows that (ii) $\Longrightarrow$ (iii).

(iii) $\Longrightarrow$ (iv) It is easy to see that if (iii) holds then

$$\int_B (1-|z|^2)^{s+p} \Big|\frac{\partial f}{\partial z_j}(z)\Big|^p d\lambda(z) < \infty, \ 1 \le j \le n,$$

$$\int_B (1-|z|^2)^{s+p} \Big|\frac{\partial f}{\partial \overline{z}_j}(z)\Big|^p d\lambda(z) < \infty, \ 1 \le j \le n,$$

which in turn implies that

$$\int_B (1-|z|^2)^{s+p}|Rf(z)|^p d\lambda(z) < \infty$$

$$\int_B (1-|z|^2)^{s+p}|\overline{R}f(z)|^p d\lambda(z) < \infty.$$

Thus, (iii) $\implies$ (iv).

(iv) $\implies$ (i) Assume now that

$$\int_B (1-|z|^2)^{s+p}(|Rf(z)|+|\overline{R}f(z)|)^p d\lambda(z) < \infty.$$

It is easy to check that $|z|^2|Df(z)|^2 = |Rf(z)|^2 + \sum_{i<j}|T_{ij}f(z)|^2$. Using this and the equality

$$\begin{aligned}|\widetilde{\nabla} f(z)|^2 &= 2(|\widetilde{D}f(z)|^2 + |\widetilde{D}\overline{f}(z)|^2)\\ &= 2(1-|z|^2)(|Df(z)|^2 - |Rf(z)|^2 + |D\overline{f}(z)|^2 - |R\overline{f}(z)|^2)\end{aligned}$$

(see [**8**]) we find that

$$\begin{aligned}&|z|^2|\widetilde{\nabla} f(z)|^2 =\\ &2(1-|z|^2)\left[(1-|z|^2)(|Rf(z)|^2 + |R\overline{f}(z)|^2) + \sum_{i<j}|T_{ij}f(z)|^2 + \sum_{i<j}|T_{ij}\overline{f}(z)|^2\right].\end{aligned}$$

Hence, to show that $f \in \mathcal{D}_p^s$ it is sufficient to show that

$$\int_B (1-|z|^2)^{s+p/2}\big(|T_{ij}f(z)|^p + |T_{ij}\overline{f}(z)|^p\big)d\lambda(z) < \infty,\ 1 \le i < j \le n.$$

Integration by parts shows that

$$f(z) = \int_0^1 \big[Rf(tz) + \overline{R}f(tz) + f(tz)\big]dt.$$

From this we conclude that it is sufficient to prove that

$$\int_B (1-|z|^2)^{s+p/2}\left(\int_0^1 |T_{ij}u(tz)|dt\right)^p d\lambda(z) < \infty,\ 1 \le i < j \le n,$$

where u is Rf or $\overline{R}f$ or $R\overline{f}$ or $\overline{Rf}$ or f.

We will show that, for fixed $1 \le i < j \le n$,

$$I = \int_B (1-|z|^2)^{s+p/2}\left(\int_0^1 |T_{ij}Rf(tz)|dt\right)^p d\lambda(z) < \infty.$$

The remaining cases may be treated analogously.

Using Lemma 2.1, Fubini's theorem and Lemma 2.2 we find that for any $a > 0$

$$\begin{aligned}\int_0^1 |T_{ij}Rf(tz)|dt &\le C\int_0^1\left(\int_{E_r(tz)}\frac{|Rf(w)|(1-|w|^2)^a}{|1-t\langle z,w\rangle|^{n+a+3/2}}d\nu(w)\right)dt\\ &\le C\int_0^1\left(\int_B\frac{|Rf(w)|(1-|w|^2)^a d\nu(w)}{|1-t\langle z,w\rangle|^{n+a+3/2}}\right)dt\\ &= C\int_B|Rf(w)|(1-|w|^2)^a\left(\int_0^1\frac{dt}{|1-t\langle z,w\rangle|^{n+a+3/2}}\right)d\nu(w)\\ &\le C\int_B\frac{|Rf(w)|(1-|w|^2)^a}{|1-\langle z,w\rangle|^{n+a+1/2}}d\nu(w).\end{aligned}$$

Assume now $1 < p < \infty$. Applying the continuous form of Minkowski's inequality we obtain

$$\begin{aligned}I \le& C\int_0^1(1-r)^{s+p/2-n-1}\\ &\cdot\left(\int_0^1\left(\int_S\left(\int_S\frac{|Rf(\rho\xi)|(1-\rho)^a d\sigma(\xi)}{|1-\langle r\zeta,\rho\xi\rangle|^{n+a+1/2}}\right)^p d\sigma(\zeta)\right)^{1/p}d\rho\right)^p dr.\end{aligned}\tag{2.1}$$

By Hölder's inequality

$$\begin{aligned}&\int_S\frac{|Rf(\rho\xi)|d\sigma(\xi)}{|1-\langle r\zeta,\rho\xi\rangle|^{n+a+1/2}}\\ &\le\left(\int_S\frac{|Rf(\rho\xi)|^p d\sigma(\xi)}{|1-\langle r\zeta,\rho\xi\rangle|^{n+a+1/2}}\right)^{1/p}\left(\int_S\frac{d\sigma(\xi)}{|1-\langle r\zeta,\rho\xi\rangle|^{n+a+1/2}}\right)^{1/p'}\\ &\le\frac{C}{(1-r\rho)^{(a+1/2)/p'}}\left(\int_S\frac{|Rf(\rho\xi)|^p d\sigma(\xi)}{|1-\langle r\zeta,\rho\xi\rangle|^{n+a+1/2}}\right)^{1/p},\ \text{by Lemma 2.3.}\end{aligned}\tag{2.2}$$

(Here $1/p + 1/p' = 1$).

Now we substitute (2.2) into (2.1) and use Fubini's theorem and Lemma 2.3 to get

$$\begin{aligned}I \le& C\int_0^1(1-r)^{s+p/2-n-1}\left(\int_0^1\frac{(1-\rho)^a}{(1-r\rho)^{(a+1/2)/p'}}\right.\\ &\left.\cdot\left(\int_S\left(\int_S\frac{|Rf(\rho\xi)|^p d\sigma(\xi)}{|1-\langle r\zeta,\rho\xi\rangle|^{n+a+1/2}}\right)d\sigma(\zeta)\right)^{1/p}d\rho\right)^p dr\\ =& C\int_0^1(1-r)^{s+p/2-n-1}\left(\int_0^1\frac{(1-\rho)^a}{(1-r\rho)^{(a+1/2)/p'}}\right.\\ &\left.\cdot\left(\int_S|Rf(\rho\xi)|^p d\sigma(\xi)\int_S\frac{d\sigma(\zeta)}{|1-\langle r\zeta,\rho\xi\rangle|^{n+a+1/2}}\right)^{1/p}d\rho\right)^p dr\\ \le& C\int_0^1(1-r)^{s+p/2-n-1}\left(\int_0^1\frac{(1-\rho)^a}{(1-r\rho)^{a+1/2}}\right.\\ &\left.\cdot\left(\int_S|Rf(\rho\xi)|^p d\sigma(\xi)\right)^{1/p}d\rho\right)^p dr.\end{aligned}\tag{2.3}$$

A simple observation shows that it is possible to select positive parameters a, t_1, t_2, t_3, t_4 such that

(i) $$a = t_1 + t_2 = t_3 + t_4,$$

(ii) $$\frac{1}{p'} < t_3 - t_1 < \frac{s}{p} + \frac{3}{2} - \frac{n+1}{p},$$ (iii) $$t_2 > 1 + \frac{s}{p} - \frac{n+1}{p}.$$

Note that here again we used the assumption that $s > n - p/2$.

Applying Hölder's inequality on (2.3) and Lemma 2.3 we obtain

$$\begin{aligned}
I \le C \int_0^1 (1-r)^{s+p/2-n-1} &\Bigg[\left(\int_0^1 \frac{(1-\rho)^{t_1 p'} d\rho}{(1-r\rho)^{t_3 p'}} \right)^{p/p'} \\
&\cdot \left(\int_0^1 \frac{(1-\rho)^{t_2 p}}{(1-r\rho)^{(t_4+1/2)p}} \left(\int_S |Rf(\rho\xi)|^p d\sigma(\xi) \right) d\rho \right) \Bigg] dr \\
\le C \int_0^1 (1-r)^{s+3p/2-n-2+(t_1-t_3)p} &\left(\int_0^1 \frac{(1-\rho)^{t_2 p}}{(1-r\rho)^{(t_4+1/2)p}} \right. \\
&\left. \cdot \left(\int_S |Rf(\rho\xi)|^p d\sigma(\xi) \right) d\rho \right) dr \\
= C \int_0^1 \Bigg[(1-\rho)^{t_2 p} &\left(\int_S |Rf(\rho\xi)|^p d\sigma(\xi) \right) \left(\int_0^1 \frac{(1-r)^{s+3p/2-n-2+(t_1-t_3)p} dr}{(1-r\rho)^{(t_4+1/2)p}} \right) \Bigg] d\rho \\
\le C \int_B (1-|z|^2)^{s+p} &|Rf(z)|^p d\nu(z) < \infty.
\end{aligned}$$

If $p = 1$, then

$$\begin{aligned}
I \le& C \int_B (1-|z|^2)^{s+1/2} \left(\int_B \frac{|Rf(w)|(1-|w|^2)^a d\nu(w)}{|1-\langle z, w\rangle|^{n+a+1/2}} \right) d\lambda(z) \\
\le& C \int_0^1 (1-r)^{s+1/2-n-1} \left(\int_0^1 (1-\rho)^a \left(\int_S |Rf(\rho\xi)| d\sigma(\xi) \right) \right. \\
&\left. \cdot \left(\int_S \frac{d\sigma(\zeta)}{|1-\langle r\zeta, \rho\xi\rangle|^{n+a+1/2}} \right) d\rho \right) dr \\
\le& C \int_0^1 (1-r)^{s-n-1/2} \left(\int_0^1 \frac{(1-\rho)^a}{(1-r\rho)^{a+1/2}} \left(\int_S |Rf(\rho\xi)| d\sigma(\xi) \right) d\rho \right) dr \\
=& C \int_0^1 (1-\rho)^a \left(\int_S |Rf(\rho\xi)| d\sigma(\xi) \right) \left(\int_0^1 \frac{(1-r)^{s-n-1/2}}{(1-r\rho)^{a+1/2}} dr \right) d\rho \\
\le& C \int_B (1-|w|^2)^{s-n} |Rf(w)| d\nu(w) < \infty.
\end{aligned}$$

(We may assume that $a > \max\{s-n, 0\}$.)

For the case $0 < p < 1$ the following lemma will be needed.

LEMMA 2.4. *Let $0 < r < 1$ and $0 < p < \infty$. There is a constant C such that if $f \in \mathcal{M}$ then*

(i) $$\frac{|Rf(w)|^p}{|1-\langle z, w\rangle|^p} \le C \int_{E_r(w)} \frac{|Rf(\xi)|^p}{|1-\langle z, \xi\rangle|^p} d\lambda(\xi), \; z, w \in B$$

(ii) $$\left(\frac{|\overline{R}f(w)|}{|1-\langle z,w\rangle\,|}\right)^p \le C\int_{E_r(w)}\left(\frac{|\overline{R}f(\xi)|}{|1-\langle z,\xi\rangle\,|}\right)^p d\lambda(\xi),\ z,w\in B$$

Note that the constant C is independent of z and w.

We will prove (i). The proof of (ii) is similar. By the formula (1.3) in [**1**]

$$Rf(w)=\int_S\frac{Rf(\varphi_w(\rho\xi))d\sigma(\xi)}{1-\langle\rho\xi,w\rangle},\ w\in B,\ 0<\rho<1.$$

Multiplying this equality by $2n\rho^{2n-1}(1-\rho^2)^{-n-1}h(\rho)d\rho$, where h is a radial function which belongs to $C^\infty(B)$ with compact support in B such that $\int_B h(z)d\lambda(z)=1$, then integrating from 0 to 1 and using the invariance of the measure λ, we get

$$Rf(w)=\int_B h(\varphi_w(z))\frac{Rf(z)}{1-\langle\varphi_w(z),w\rangle}d\lambda(z)=\int_B h(\varphi_w(z))\frac{1-\langle z,w\rangle}{1-|w|^2}Rf(z)d\lambda(z)$$

by Theorem 2.2.5 [**9**, p. 28]. By a suitable choice of a function h we obtain

$$|Rf(w)|\le C\int_{E_r(w)}|Rf(\xi)|d\lambda(\xi),\ w\in B,\ \text{for some } 0<r<1.$$

Since $|1-\langle z,w\rangle\,|\simeq|1-\langle z,\xi\rangle\,|$, if $\xi\in E_r(w)$, we have

$$\frac{|Rf(w)|}{|1-\langle z,w\rangle\,|}\le C\int_{E_r(w)}\frac{|Rf(\xi)|}{|1-\langle z,\xi\rangle\,|}d\lambda(\xi),$$

and consequently,

$$\left(\frac{|Rf(w)|}{|1-\langle z,w\rangle\,|}\right)^p\le C\int_{E_r(w)}\left(\frac{|Rf(\xi)|}{|1-\langle z,\xi\rangle\,|}\right)^p d\lambda(\xi),\ z,w\in B$$

(see [**8**]).

To finish the proof of Theorem 1.4 assume that $0<p<1$. Applying Theorem 3.2 (iii) [**3**] to the function

$$F(w)=\left(\frac{|Rf(w)|}{|1-\langle z,w\rangle\,|^{a+n+1/2}}\right)^{p/2},\quad w\in B\qquad (z\in B\text{ - fixed})$$

and replacing p, r, k, q by 2, $2/p$, $2/p$, $p(a+n+1)-n$ respectively and using Lemma 2.4 we find that

$$\left(\int_B\frac{|Rf(w)|(1-|w|^2)^a d\nu(w)}{|1-\langle z,w\rangle\,|^{a+n+1/2}}\right)^p\le C\int_B\frac{|Rf(w)|^p(1-|w|^2)^{p(a+n+1)-n-1}d\nu(w)}{|1-\langle z,w\rangle\,|^{p(a+n+1/2)}}.$$

Thus, assuming that $a>s/p-n$,

$$\begin{aligned}I&\le C\int_B(1-|z|^2)^{s+p/2}\left(\int_B\frac{|Rf(w)|^p(1-|w|^2)^{p(a+n-1)-n-1}d\nu(w)}{|1-\langle z,w\rangle\,|^{p(a+n+1/2)}}\right)d\lambda(z)\\&=C\int_B|Rf(w)|^p(1-|w|^2)^{p(a+n+1)-n-1}\left(\int_B\frac{(1-|z|^2)^{s+p/2-n-1}d\nu(z)}{|1-\langle z,w\rangle\,|^{p(a+n+1/2)}}\right)d\nu(w)\\&\le C\int_B(1-|w|^2)^{p+s}|Rf(w)|^p d\lambda(w)<\infty.\end{aligned}$$

This finishes the proof of Theorem 1.4.

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(Received 01 11 1995)