

MAXIMAL CANONICAL GRAPHS WITH SEVEN NONZERO EIGENVALUES

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Communicated by Slobodan Simić

ABSTRACT. In [3] and [4] A. Torgašev described all finite and infinite connected graphs having 3, 4 or 5 nonzero eigenvalues (not necessarily distinct). In the same papers he has given a general method how to describe all connected graphs with any fixed number of nonzero eigenvalues. In [2] M. Lepović applying his method described all finite connected graphs which have exactly 6 nonzero eigenvalues. We here describe all finite connected graphs with exactly 7 nonzero eigenvalues.

1. Introduction

Throughout the paper, G will denote a connected finite graph, $|G|$ the order of G (i.e., the number of its vertices), $V(G)$ will be the set of vertices of G , and $n(G)$ will be the number of all nonzero eigenvalues of G (including multiplicities). Consider the following equivalence relation on $V(G)$: two vertices $x, y \in V(G)$ are equivalent if they are not adjacent and they have exactly the same neighbors. Let g be the corresponding quotient graph of G . We call g “the canonical graph” of G .

The canonical graph of a connected graph is also connected. A graph is called canonical if no pair of its vertices has the same neighbors. The importance of canonical graphs is given by the following theorem.

THEOREM 1. [3, 4] *For any graph G we have $n(g) = n(G)$.*

Let $n \geq 2$ be a fixed integer. Denote by $T(n)$ the set of all nonisomorphic graphs with exactly n nonzero eigenvalues, and by $T_c(n)$ the set of all canonical graphs belonging to $T(n)$. By Theorem 1 is clear that describing the class $T(n)$ is reduced to describing the class $T_c(n)$. In [3] and [4] it is also proved that the class $T_c(n)$ is finite for any $n \geq 2$, and a general method for finding all graphs from the class $T_c(n)$ is given. In particular, it is proved that any graph $G \in T_c(n)$ has an induced subgraph H with n vertices, belonging to $T_c(n)$. Such a subgraph H can

2010 *Mathematics Subject Classification*: Primary 05C50.

Key words and phrases: Spectra of graphs, Maximal canonical graphs.

not have 0 in this spectrum, and we call it the "co-kernel" of G , or a "basic graph" of G . Clearly, each graph $H \in T(n)$ of order n is canonical, and the set \mathcal{H} of all such nonisomorphic graphs is evidently finite. Of course, a graph can have different co-kernels.

Since every vertex $x \in V(G) \setminus V(H)$ is adjacent to some vertex in H , we have that $|G| \leq 2^n - 1$, and there are no two vertices in $V(G) \setminus V(H)$ which have the same neighbors.

2. The main result

By above properties, a possible method to generate all the graphs from the class $T_c(n)$ is clear. We start from the class \mathcal{H} , and for any fixed graph $H \in \mathcal{H}$, we apply the method of extension, so that each new added vertex is adjacent to some vertices of H and no two vertices have the same neighbors. Then we investigate all possible cases related to the adjacency relation of the new vertex and all added previously, and separate all the graphs of this type which belong to class $T_c(n)$. The previous procedure is obviously finite.

For $n = 7$ we get the following results. Among all 853 connected graphs with 7 vertices, there are exactly 342 graphs without zero in the spectrum. Hence, there are exactly 342 basic graphs in the class $T_c(7)$.

Creating a computer program for mentioned method of extension of these 342 graphs, after a long computer work we got the following result.

THEOREM 2. *There are exactly 23413 nonisomorphic canonical graphs with exactly 7 nonzero eigenvalues. Their orders run over the set $\{7, 8, \dots, 18\}$.*

Since it is almost impossible to expose this whole list, we have some statistics about nonisomorphic canonical graphs with seven nonzero eigenvalues (Table 1). In the table n is the number of vertices of graphs, m is the number of their edges and k is the number of nonisomorphic graphs with exactly seven nonzero eigenvalues which have n vertices and m edges.

From Table 1 we have that $(n, k) \in \{(7, 342), (8, 1384), (9, 3466), (10, 5400), (11, 5656), (12, 4031), (13, 2037), (14, 778), (15, 238), (16, 65), (17, 13), (18, 3)\}$, where k is the number of nonisomorphic canonical graphs with exactly 7 nonzero eigenvalues with n vertices.

We have made a condensation of this result. Namely, the set $T_c(7)$ can be obviously ordered by the relation "to be induced subgraph", and we can separate only the corresponding *maximal* graphs. The set of all maximal graphs from this set is of course finite.

Making a computer program for separating maximal graphs from the class $T_c(7)$, we have got the following result.

THEOREM 3. *There are exactly 183 maximal graphs with 7 nonzero eigenvalues. Their orders run over the set $\{7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 18\}$. All these graphs are represented in Table 2.*

TABLE 1. Statistic about nonisomorphic canonical graphs with 7 nonzero eigenvalues

| n | m | k | n | m | k | n | m | k | n | m | k | n | m | k | |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|---|
| 7 | 7 | 6 | 10 | 19 | 463 | 12 | 21 | 145 | 13 | 42 | 28 | 15 | 43 | 74 | |
| | 8 | 17 | | 20 | 396 | | 22 | 231 | | 43 | 21 | | 44 | 59 | |
| | 9 | 37 | | 21 | 290 | | 23 | 376 | | 44 | 15 | | 45 | 65 | |
| | 10 | 52 | | 22 | 193 | | 24 | 479 | | 45 | 6 | | 46 | 49 | |
| | 11 | 60 | | 23 | 124 | | 25 | 568 | | 46 | 5 | | 47 | 54 | |
| | 12 | 57 | | 24 | 82 | | 26 | 645 | | 48 | 1 | | 48 | 53 | |
| | 13 | 45 | | 25 | 39 | | 27 | 597 | | | | | 49 | 45 | |
| | 14 | 31 | | 26 | 21 | | 28 | 553 | | | | | 50 | 20 | |
| | 15 | 19 | | 27 | 7 | | 29 | 551 | | | | | 51 | 24 | |
| | 16 | 10 | | 28 | 3 | | 30 | 442 | | | | | 52 | 17 | |
| | 17 | 4 | | 29 | 1 | | 31 | 337 | | | | | 53 | 12 | |
| | 18 | 2 | | 30 | 2 | | 32 | 227 | | | | | 54 | 6 | |
| | 19 | 1 | | | | | 33 | 138 | | | | | 55 | 4 | |
| | 20 | 1 | | | | | 34 | 85 | | | | | 56 | 1 | |
| | 8 | 8 | | 1 | 13 | | 2 | 34 | | 85 | 34 | | 122 | 56 | 1 |
| | | | | | 14 | | 8 | 35 | | 65 | 35 | | 160 | 57 | 2 |
| | | | | | 15 | | 30 | 36 | | 45 | 36 | | 197 | 58 | 2 |
| | | | | | 16 | | 71 | 37 | | 33 | 37 | | 190 | 59 | 2 |
| | | | | | 17 | | 161 | 38 | | 14 | 38 | | 177 | 60 | 1 |
| | | | | | 18 | | 263 | 39 | | 8 | 39 | | 173 | 61 | 1 |
| | | | | | 19 | | 423 | 40 | | 3 | 40 | | 146 | | |
| | | | 20 | 553 | 45 | 1 | 41 | 140 | 43 | 6 | | | | | |
| | | | 21 | 633 | | | 42 | 126 | 44 | 7 | | | | | |
| | | | 22 | 614 | 22 | 4 | 43 | 90 | 45 | 9 | | | | | |
| | | | 23 | 660 | 23 | 12 | 44 | 64 | 46 | 15 | | | | | |
| | | | 24 | 600 | 24 | 34 | 45 | 57 | 47 | 22 | | | | | |
| | | | 25 | 470 | 25 | 75 | 46 | 26 | 48 | 20 | | | | | |
| | | | 26 | 350 | 26 | 123 | 47 | 17 | 49 | 19 | | | | | |
| | | 27 | 212 | 27 | 174 | 48 | 18 | 50 | 16 | | | | | | |
| | | 28 | 132 | 28 | 247 | 49 | 5 | 51 | 15 | | | | | | |
| | | 29 | 106 | 29 | 318 | 50 | 8 | 52 | 14 | | | | | | |
| | | 30 | 52 | 30 | 397 | 51 | 6 | 53 | 15 | | | | | | |
| | | 31 | 35 | 31 | 399 | 52 | 6 | 54 | 17 | | | | | | |
| | | 32 | 13 | 32 | 407 | 53 | 1 | 55 | 19 | | | | | | |
| 9 | 10 | 1 | 33 | 9 | 33 | 367 | | | 56 | 16 | | | | | |
| | 11 | 8 | 34 | 1 | 34 | 330 | 14 | 35 | 7 | 57 | 6 | | | | |
| | 12 | 28 | 36 | 1 | 35 | 311 | | 36 | 11 | 58 | 2 | | | | |

Continued on next page

| n | m | k | n | m | k | n | m | k | n | m | k | n | m | k |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| | 13 | 83 | | 37 | 1 | | 36 | 280 | | 37 | 21 | | 59 | 5 |
| | 14 | 166 | | | | | 37 | 174 | | 38 | 30 | | 60 | 5 |
| | 15 | 276 | 11 | 17 | 3 | | 38 | 137 | | 39 | 50 | | 61 | 3 |
| | 16 | 370 | | 18 | 8 | | 39 | 82 | | 40 | 56 | | 62 | 3 |
| | 17 | 445 | | 19 | 34 | | 40 | 52 | | 41 | 58 | | 63 | 2 |
| | 18 | 468 | | 20 | 68 | | 41 | 32 | | 42 | 54 | | 67 | 2 |
| | | | | 57 | 5 | | 63 | 8 | 17 | 62 | 1 | | 72 | 3 |
| 16 | 52 | 3 | | 58 | 3 | | 64 | 9 | | 63 | 1 | | | |
| | 53 | 2 | | 59 | 1 | | 68 | 2 | | 64 | 3 | 18 | 73 | 2 |
| | 54 | 5 | | 60 | 2 | | 69 | 1 | | 65 | 3 | | 81 | 1 |
| | 55 | 6 | | 61 | 3 | | 70 | 1 | | 70 | 1 | | | |
| | 56 | 10 | | 62 | 4 | | | | | 71 | 1 | | | |
| | | | | 57 | 5 | | 63 | 8 | 17 | 62 | 1 | | 72 | 3 |
| 16 | 52 | 3 | | 58 | 3 | | 64 | 9 | | 63 | 1 | | | |
| | 53 | 2 | | 59 | 1 | | 68 | 2 | | 64 | 3 | 18 | 73 | 2 |
| | 54 | 5 | | 60 | 2 | | 69 | 1 | | 65 | 3 | | 81 | 1 |
| | 55 | 6 | | 61 | 3 | | 70 | 1 | | 70 | 1 | | | |
| | 56 | 10 | | 62 | 4 | | | | | 71 | 1 | | | |

TABLE 2. All maximal canonical graphs
with 7 nonzero eigenvalues

| | | | | | | | | | | | | | | |
|-----|----|----|-------|-------|-------|-------|-------|-------|-------|-------|-------|------|--|--|
| 001 | 18 | 73 | 00080 | 00840 | 008C0 | 0351E | 05633 | 0632D | 1803F | 063AD | 0359E | | | |
| | | | 056B3 | 28747 | 180BF | 03D5E | 06B6D | 05E73 | 05EF3 | 06BED | 03DDE | | | |
| 002 | 18 | 73 | 00040 | 08E33 | 12783 | 03A99 | 0D83C | 06C96 | 1718C | 19329 | 1C526 | | | |
| | | | 08E73 | 0D87C | 201BF | 127C3 | 171CC | 06CD6 | 03AD9 | 19369 | 1C566 | | | |
| 003 | 18 | 81 | 080FF | 00F1F | 20F0F | 03373 | 055B5 | 069D9 | 19636 | 1AA5A | 1CC9C | | | |
| | | | 1F0F0 | 23363 | 255A5 | 269C9 | 37F00 | 39626 | 3AA4A | 3CC8C | 3F0E0 | | | |
| 004 | 16 | 52 | 0100 | 2221 | 4448 | 0096 | 0196 | 2321 | 4548 | 8E0F | 187D | 22B7 | | |
| | | | 44DE | 18EB | 23B7 | 19EB | 197D | 45DE | | | | | | |
| 005 | 16 | 56 | 0100 | 0065 | 060A | 070A | 0165 | 32D6 | 34B9 | 981F | 066F | 4CB9 | | |
| | | | 4AD6 | 076F | 33D6 | 4DB9 | 35B9 | 4BD6 | | | | | | |
| 006 | 16 | 64 | 207F | 02BF | 838F | 0DD3 | 15D5 | 1AB9 | 6476 | 3879 | 7C70 | 9B89 | | |
| | | | C786 | DF80 | E546 | EA2A | F22C | FD40 | | | | | | |
| 007 | 16 | 64 | 047F | 149F | 0BA7 | 49AB | 3355 | C372 | 2C6D | 3C8D | 7159 | 8EA6 | | |
| | | | B654 | CCAA | D392 | EB60 | F458 | FB80 | | | | | | |
| 008 | 16 | 68 | 2000 | 0B1F | 80FD | 0D4F | 52B3 | 1397 | 4C6B | 54E3 | 2D4F | 3397 | | |
| | | | 2B1F | 6C6B | 72B3 | 74E3 | 5FFC | 7FFC | | | | | | |
| 009 | 15 | 43 | 0201 | 000C | 0862 | 1190 | 020D | 4532 | 0A63 | 086E | 119C | 1391 | | |
| | | | 0A6F | 24D7 | 24DB | 139D | 453E | | | | | | | |
| 010 | 15 | 44 | 0005 | 0028 | 0B10 | 10C2 | 002D | 10EA | 10C7 | 0B38 | 0B15 | 269E | | |
| | | | 10EF | 26B3 | 4573 | 0B3D | 455E | | | | | | | |
| 011 | 15 | 46 | 0401 | 0208 | 0066 | 0609 | 4898 | 2916 | 026E | 0467 | 11B3 | 11D5 | | |
| | | | 066F | 2D17 | 13BB | 13DD | 48FE | | | | | | | |

012 15 47 0024 0108 00C3 012C 01CB 00E7 2C13 1655 16B2 4A5D
 01EF 2C37 4ABA 175D 17BA
 013 15 49 0801 1081 200E 40F0 033A 0556 066C 280F 0B3B 0D57
 0E6D 15D7 13BB 16ED 60FE
 014 15 49 1002 00E1 411C 0635 0A8D 0C59 10E3 2572 23A6 29CA
 1A8F 1637 1C5B 41FD 2F1E
 015 15 50 0110 0C6A 0E8A 3285 3065 19C9 428F 1B29 2636 24D6
 406F 3395 0D7A 3175 0F9A
 016 15 51 0011 14C6 2B28 1706 28E8 194B 1A9A 2565 26B4 14D7
 416F 1717 28F9 2B39 42BE
 017 15 51 0082 0B25 0D61 321C 1315 2C68 3458 407F 1397 329E
 0DE3 0BA7 2CEA 40FD 34DA
 018 15 52 081C 1541 22A2 4543 29C6 11AD 2E52 1639 2D2A 12D5
 41AF 42D7 463B 1D5D 2ABE
 019 15 53 0480 1090 202F 032B 4353 0D66 0E4D 6057 07AB 1A5D
 24AF 1976 13BB 1EDD 1DF6
 020 15 53 00F0 1503 2A0C 1603 290C 196A 2695 415F 42AF 425F
 41AF 16F3 15F3 2AFC 29FC
 021 15 54 0840 02D4 0D0A 501F 11A7 21A7 1639 2639 601F 079E
 29E7 19E7 2E79 1E79 0FDE
 022 15 54 0460 0450 0B83 12AD 602F 191E 129D 192E 601F 4C9B
 2367 0FE3 0FD3 16FD 1D7E
 023 15 54 1003 0203 421D 056C 09B4 30E2 0CD8 076F 0BB7 0EDB
 156F 19B7 1CDB 62FC 70FC
 024 15 55 018E 01C6 0638 0670 1A9B 1D2D 629B 652D 283F 1AD3
 1D65 57C1 62D3 6565 07FE
 025 15 59 3000 40BD 407D 054F 0A73 058F 0AB3 2357 1CAB 354F
 358F 3A73 3AB3 0FFC 3FFC
 026 15 60 0C4E 13B0 2177 4177 423F 25D3 3A2D 223F 5A2D 3DC1
 3E89 45D3 5DC1 5E89 1FFE
 027 15 63 047F 099F 02EF 22E7 43AB 1C5D 2477 3C55 5B89 5D19
 63A3 7661 7B81 7D11 7FFE
 028 14 41 080C 0414 2050 1023 0183 02E9 036A 098F 0597 182F
 21D3 3073 06FD 077E
 029 14 41 1018 2024 00C3 0303 0555 0695 096A 0AAA 10DB 2327
 20E7 131B 0CFC 0F3C
 030 14 43 1003 200C 04B2 0871 01D8 02E4 0B4D 078E 0D1B 0E27
 12E7 11DB 287D 24BE
 031 14 43 0301 0921 100E 0056 20F0 30A8 0357 069D 130F 06CB
 0977 0CEB 0CBD 30FE
 032 14 43 0601 008E 0194 2170 206A 0CD9 1971 0795 068F 0DC3
 1327 186B 123D 21FE
 033 14 45 0421 091C 109A 2076 01F1 12C3 0B45 0789 260E 1E0F
 1877 14BB 0D3D 23DE
 034 14 45 0070 0848 1207 03A3 0D0D 0696 0535 300F 249E 21AB
 1277 0BEB 0D7D 0EDE
 035 14 45 0250 0454 02AA 110B 28A5 04AE 0B63 309D 0B99 3067
 0D67 135B 0D9D 06FE
 036 14 45 0064 0834 104C 0303 0599 069A 0367 28B3 30CB 134F

| | | | |
|-----|----|----|--|
| | | | 0B37 389B 05FD 06FE |
| 037 | 14 | 47 | 0248 0270 048D 0996 300F 04B5 0D23 3037 11DE 156B 2A37 0F6B 06FD 0BDE |
| 038 | 14 | 51 | 1240 201F 05A3 083D 20CF 08ED 0B35 2317 0D9E 14EB 1733 17E3 1A7D 1FDE |
| 039 | 13 | 32 | 0201 0424 0842 1068 0099 0116 0625 0A43 018F 0317 08DB 04BD 117E |
| 040 | 13 | 33 | 0009 0444 0882 0116 0231 044D 088B 011F 032E 10F1 0AB3 0675 11EE |
| 041 | 13 | 33 | 0011 0022 000C 002E 001D 0033 003F 0AC7 11C7 0747 0778 0AF8 11F8 |
| 042 | 13 | 35 | 0105 0411 0842 008A 103C 026C 049B 018F 0947 03E3 11B3 067D 187E |
| 043 | 13 | 35 | 0201 008A 0116 1068 0425 0856 028B 0317 04AF 0A57 05B9 0CF9 117E |
| 044 | 13 | 35 | 0048 0421 08C4 011A 0235 0469 1483 0996 032F 1297 053B 027D 09DE |
| 045 | 13 | 36 | 0081 0411 084A 0146 02AC 1133 063C 01C7 08CB 0557 036B 06BD 18BE |
| 046 | 13 | 36 | 0103 020C 00C4 0813 10B1 056A 04AD 01C7 030F 08D7 0C7A 12BD 137A |
| 047 | 13 | 36 | 000A 0441 0886 0138 0265 044B 09B4 0357 026F 1297 0579 14B9 09BE |
| 048 | 13 | 36 | 0821 100E 028A 0416 00F0 0555 01B3 114D 03C9 0C37 0AAB 0E4D 10FE |
| 049 | 13 | 36 | 0210 0023 0154 1087 0469 0233 051E 098E 14CD 0177 0679 0AE9 0B9E |
| 050 | 13 | 36 | 0030 0089 0252 0407 00B9 0907 0437 115B 156C 0937 02DB 06EC 0BEC |
| 051 | 13 | 36 | 0042 0422 080D 0189 0215 0257 10B3 084F 01CB 0637 05AB 18FC 07FC |
| 052 | 13 | 36 | 0081 0416 084C 010E 02AA 1171 0497 018F 08CD 0733 0E71 0B69 10FE |
| 053 | 13 | 36 | 0812 1105 0189 0078 0C06 046C 02B3 02CB 1247 06A7 099B 117D 0C7E |
| 054 | 13 | 36 | 0501 0056 10A8 0096 1068 0623 090D 0597 0A2F 0557 0A79 0AB9 10FE |
| 055 | 13 | 36 | 0489 0485 180A 0078 0074 1806 0333 034B 0347 1627 099B 04FD 187E |
| 056 | 13 | 37 | 008C 020C 0143 0869 04B2 1115 0632 034F 01CF 1A33 18B3 057D 06BE |
| 057 | 13 | 37 | 00D0 0207 018A 0864 0407 1439 1239 02D7 10EE 04D7 0B39 0D39 09EE |
| 058 | 13 | 37 | 020C 0414 0843 1116 02A9 016A 04B1 01CF 0A4F 11B3 1A33 06BD 057E |
| 059 | 13 | 37 | 0828 1050 0207 041A 00E1 0107 092F 1157 0A2F 1257 04FB 06FC 05FC |
| 060 | 13 | 37 | 0041 0086 0129 00C7 04B2 0B1C 122B 055C 01AF 04F3 0AB3 0B5D 165E |

061 13 37 0203 0068 014C 1096 041E 026B 0CB1 08C7 034F 0D95
 0739 13B1 10FE
 062 13 37 00C4 0144 0229 0413 0871 100F 193A 0557 036D 04D7
 18BA 02ED 07BA
 063 13 37 010A 0464 0851 00CD 10B2 030D 0E32 095B 056E 12B5
 0A9B 1175 06AE
 064 13 37 0C05 140A 1830 0159 02A6 0166 0299 058F 064F 0A75
 11BA 09B5 127A
 065 13 37 040C 0430 1803 018D 024E 0272 01B1 08D7 08EB 1317
 132B 05BD 067E
 066 13 38 0110 0432 0858 0185 120E 026B 04A7 08CD 1C0F 05B7
 037B 09DD 02FE
 067 13 38 0070 0434 0898 0143 028D 0507 122E 184B 1C0F 0577
 09DB 02FD 03BE
 068 13 38 0148 022C 00D1 08A3 102D 0616 1417 0B16 04EB 1917
 09EB 02FD 075E
 069 13 38 0109 0206 00A9 08D4 106A 0633 030F 0595 02AF 1556
 0A7B 09DD 14F6
 070 13 38 0070 0492 0911 006C 048E 090D 122B 1247 1C0F 03D7
 03BB 097D 04FE
 071 13 38 088A 1109 0264 042B 0896 1115 0437 0653 01EC 1A17
 05DB 136D 0AEE
 072 13 39 0415 00E1 101A 01A2 029B 0B0E 0837 0A4D 154C 05B7
 10FB 07CD 196E
 073 13 39 00A1 0493 0A1C 054A 0265 1816 031D 1117 0CEA 0657
 05EB 0ABD 196E
 074 13 39 0506 00E8 1211 0456 1095 090F 0A33 085F 076A 08B7
 13A9 12F9 05EE
 075 13 39 0243 00B8 01C2 101C 04AB 0D25 140F 098E 0A0F 0B75
 02FB 1575 11DE
 076 13 39 0470 02A3 110D 0896 040F 0A4A 10B7 185E 126B 05B5
 0769 0BF0 0D5C
 077 13 39 0C02 104C 101C 00B1 00E1 031F 0937 034F 06CB 0CB3
 0CE3 10FD 03FE
 078 13 39 0046 0121 0216 0429 08CB 0167 1193 0A9B 046F 0337
 149B 15FC 0BFC
 079 13 40 0431 020B 1058 0886 00F0 036D 05A7 11CE 150F 0CB7
 02FB 0A7D 18DE
 080 13 40 0065 0618 081B 0986 02AD 0355 10AB 1153 0CAE 0D56
 139B 067D 15E6
 081 13 40 084A 1304 00B1 092A 0A52 044F 11AD 0657 052F 12D5
 08FB 13B5 04FE
 082 13 40 0034 020B 010B 0865 0496 013F 023F 12C7 11C7 0CC7
 13F8 0DF8 0EF8
 083 13 40 010E 0096 0168 00F0 145B 0A5B 07A5 065B 0BA5 15A5
 185B 19A5 01FE
 084 13 40 0402 002D 10D0 0165 021D 042F 061F 0A9B 0567 09E3
 0BD3 10FD 0BFE
 085 13 40 0818 1060 0107 0087 04B5 034B 089F 091F 1167 10E7

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0779 06F9 07FE
086 13 41 0488 0233 100F 0855 00F1 1166 03AB 09CD 0B0F 1476
06BB OCDD OF76
087 13 41 0148 0087 0207 049B 1079 0A65 11B6 01CF 034F 1336
OCF9 OE79 OFB6
088 13 41 0104 004B 0263 041B 10B5 014F 0C9D 051F 0367 0AE5
11FA OEB5 OFFA
089 13 41 004B 01B0 0263 041B 08B5 094E 149D OD1E 0B66 12E5
01FB 16B5 174E
090 13 41 0099 0162 021B 0463 08D5 1156 0BAC 0A57 15AC 1457
01FB OEAD 172E
091 13 42 0113 04A8 080F 0171 120F 086D 02BB 0AD6 126D ODD6
05BB 156D 17D6
092 13 42 0848 1017 0293 0427 00EB 052D 111D 0399 0C6F 11F6
OADB OF1D OFF6
093 13 42 0427 0217 1093 0954 02E9 052E 031E 119A 0E6D 01F7
1AD9 1CE9 1D1E
094 13 42 0810 100F 0267 04AB 00F3 0355 0599 070D 11CE 0A77
OCBB OF1D OFEE
095 13 42 0302 008D 0095 106B 1073 0C6B 038F 0397 052F 0AD3
OC73 13FC OFFC
096 13 44 0870 100F 0393 05A5 068E 0749 10B7 125B 146D 09B7
0B5B OD6D OEFE
097 13 44 008D 0462 081D 01A7 02CB 1313 0937 0A5B 04EF 1537
165B OBFC 17FC
098 13 45 020F 040F 0871 1192 02DB 0337 0537 04DB 07EC 1937
18DB 1BEC 1DEC
099 13 45 0309 0432 08C7 10C7 106F 066B 0793 0997 086F 1197
073B OFFC 17FC
100 13 48 082D 10D2 0363 0517 068B 0977 0C9F 149F 0AEB 1177
12EB OFFC 17FC
101 13 52 005F 003F 02AF 0537 02CF 0557 11BB 0E5D 1AEE 1D76
1FB9 1FD9 1FE6
102 12 26 001 306 498 468 262 194 195 263 307 469 499 83E
103 12 29 038 034 14A 289 146 285 C0B C07 B23 4D3 2BD 17E
104 12 30 004 20B 413 0C3 123 127 0C7 20F 417 879 7F8 7FC
105 12 31 018 092 14C 229 465 2A3 1C6 COF B17 2BB 47D 1DE
106 12 32 202 210 C0C 0A7 14B 0B5 159 46F 2B7 35B 99D 1FE
107 12 33 101 23A 456 80F 073 2A9 4C5 68C 557 33B 78D 9EE
108 12 36 21F 0AF 837 07B 43D 13E 7C8 BC2 DE0 EC1 F50 F84
109 12 36 606 A28 C50 09B 165 2AF 4D7 32F 557 8F9 979 1FE
110 12 36 03F 0CF 197 26B 639 535 9C6 ACA D94 E68 F30 FC0
111 12 36 60F A17 C27 0F9 17A 1BC 3D8 5E8 9F0 E43 E85 F06
112 12 39 01F 16D OE7 4B5 34B 693 719 96E CB6 F1A ADD FE2
113 12 46 400 81F 1F7 2FB 37D 3BE 3CF 7CF 5F7 6FB 77D 7BE
114 12 48 1DF OFF 1BF A6F E73 DB5 7DA EEC F33 F53 FAC FCC
115 11 22 041 00A 092 10C 434 04B 14D 2B1 OD3 325 43E
116 11 22 021 041 098 106 234 44A 147 127 OD9 0B9 61E
117 11 23 012 0C1 061 226 30C 198 42D 073 OD3 48D 31E

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118 11 23 022 030 0C1 11C 10E 60D 28B 0F1 0E3 455 13E
 119 11 23 042 021 085 109 41C 213 063 0C7 14B 43D 3BE
 120 11 23 021 041 098 106 215 40B 147 127 0D9 0B9 67E
 121 11 24 005 021 0C2 138 11C 293 0E3 0C7 44B 13D 63E
 122 11 24 10E 093 425 219 035 151 2E0 483 541 609 3EE
 123 11 25 106 083 418 205 031 049 14F 137 49B 61D 2FE
 124 11 25 08B 0A5 0B4 740 09A 30B 147 465 31A 474 638
 125 11 25 018 012 0C6 123 129 0CC 60F 475 13B 395 0DE
 126 11 26 090 146 229 40F 21D 11E 463 1E2 2E1 2B9 1D6
 127 11 26 044 083 023 20F 439 153 067 0C7 499 3B8 3FC
 128 11 26 10C 031 423 059 095 313 2E2 487 44B 13D 3EE
 129 11 26 034 109 203 055 0A5 45A 4AA 237 4CB 13D 3CE
 130 11 26 1C2 125 618 425 40F 28E 10F 171 2F0 471 2DA
 131 11 27 003 164 298 136 239 2C9 1C6 167 29B 49D 46E
 132 11 28 022 115 219 053 0B1 44F 3CC 137 23B 4AD 3EE
 133 11 29 086 051 0E5 529 338 1C3 24F 0D7 51B 43D 3BE
 134 11 31 201 402 03F 05F 0AF 157 1A7 1C7 1F8 3F9 5FA
 135 11 31 017 00F 0AB 14D 0B3 155 467 387 6BA 75C 7F8
 136 11 31 00F 017 0B3 153 0AB 14B 38E 475 61B 7F4 7EC
 137 11 32 186 078 497 50F 23B 25D 369 2F1 5A3 5C5 1FE
 138 11 32 02E 1D0 237 26B 2AD 5C9 30F 4F1 553 595 1FE
 139 11 33 08F 117 227 447 0BB 15D 26D 473 7B2 7CC 7F8
 140 11 33 06F 0AF 0D7 31B 535 656 1E9 696 768 7A8 7D0
 141 11 34 0E0 11E 21D 46B 4B3 4C7 34F 397 33B 2FD 1FE
 142 11 35 140 0B7 44F 23D 50F 29E 2AB 1F7 3EB 37D 3DE
 143 11 35 14F 0B7 45B 23D 52B 2D5 3A5 5C3 6B1 749 7FE
 144 11 36 300 50F 617 07B 0BD 0DE 1EF 2F7 37B 3BD 3DE
 145 11 37 0E0 11B 217 46F 4BD 4DE 36F 2F7 1FB 3BD 3DE
 146 11 40 107 0F0 47F 27F 3BB 3DD 3EE 1F7 5BB 5DD 5EE
 147 10 27 038 0C5 146 183 21F 22F 237 1BB 0FD 17E
 148 10 33 09F 06F 237 13B 1CB 2C7 363 393 3FD 3FE
 149 09 14 018 006 00C 012 125 143 0D1 0A9 01E
 150 09 16 038 052 094 111 1E0 107 04B 08D 02E
 151 09 16 023 00B 015 105 049 091 061 181 1FE
 152 09 17 011 00A 023 043 107 087 01B 0FC 17C
 153 09 17 018 00F 033 055 161 186 0AA 0CC 0F0
 154 09 18 081 102 017 00F 047 027 078 0F9 17A
 155 09 18 081 101 017 00F 04B 035 069 071 1FE
 156 09 18 02B 00F 01B 115 066 1C4 0B8 1D0 1E0
 157 09 19 049 00E 130 055 063 187 0AB 09D 13E
 158 09 21 0C0 10B 107 01D 02E 0B7 07B 0DD 0EE
 159 09 22 088 10B 035 056 06F 197 07B 0BD 0DE
 160 09 22 08B 10B 035 056 06C 197 07B 1AD 1CE
 161 09 22 00D 070 08B 097 0AE 157 13B 07D 16E
 162 09 22 081 104 03A 05F 06F 077 0BB 07D 13E
 163 09 24 0F0 11B 127 14D 18E 0B7 07B 0DD 0EE
 164 09 25 0A3 11C 03F 14F 0CF 0F5 0FA 175 17A
 165 09 26 04B 035 11F 09F 0ED 173 0F3 16D 1FE
 166 09 30 05F 03F 12F 0D7 1A7 1C7 1FB 1FD 1FE

| | | | | | | | | | | |
|-----|----|----|----|----|----|----|----|----|----|----|
| 167 | 08 | 13 | 25 | 26 | C2 | 0B | 13 | C1 | 78 | 9C |
| 168 | 08 | 15 | 0A | 29 | 54 | 27 | C3 | 33 | 9D | 5E |
| 169 | 08 | 17 | 1B | 27 | 4D | 8B | B4 | 6A | D5 | F2 |
| 170 | 08 | 17 | 49 | 93 | 0F | 47 | A5 | 39 | 71 | FE |
| 171 | 07 | 09 | 06 | 03 | 0A | 15 | 49 | 70 | 2C | |
| 172 | 07 | 09 | 09 | 05 | 03 | 41 | 21 | 11 | 7E | |
| 173 | 07 | 11 | 34 | 4A | 45 | 23 | 51 | 29 | 1E | |
| 174 | 07 | 11 | 03 | 06 | 0D | 15 | 3A | 65 | 5A | |
| 175 | 07 | 12 | 15 | 0B | 45 | 23 | 51 | 29 | 7E | |
| 176 | 07 | 12 | 31 | 49 | 45 | 23 | 13 | 0D | 7E | |
| 177 | 07 | 12 | 30 | 49 | 46 | 27 | 1B | 1D | 2E | |
| 178 | 07 | 12 | 23 | 58 | 25 | 26 | 1B | 4D | 56 | |
| 179 | 07 | 13 | 21 | 41 | 0F | 17 | 1B | 1D | 7E | |
| 180 | 07 | 14 | 25 | 51 | 2B | 17 | 4B | 1D | 7E | |
| 181 | 07 | 15 | 23 | 43 | 0F | 17 | 1B | 7D | 7E | |
| 182 | 07 | 17 | 17 | 0F | 47 | 27 | 7B | 7D | 7E | |
| 183 | 07 | 21 | 3F | 5F | 6F | 77 | 7B | 7D | 7E | |

We notice that all the graphs in the table are represented in the form:

$$n_1 \quad n_2 \quad n_3 \quad a_1 \quad a_2 \quad a_3 \quad \dots \quad a_{n_2},$$

where n_1 is the ordering number of maximal graph, n_2 the number of its vertices, n_3 the number of its edges and $a_1, a_2, a_3, \dots, a_{n_2}$ denote the rows of adjacency matrix. The coding is made by converting the row of adjacency matrix from binary cod into hexadecimal.

We notice that for each graph from the above table the corresponding basic graph is always induced by its first 7 vertices. So it can be easily found.

From Table 2 we have that there exist exactly 13 maximal graphs of order 7, 4 graphs of order 8, 18 graphs of order 9, 2 graphs of order 10, 32 graphs of order 11, 13 graphs of order 12, 63 graphs of order 13, 11 graphs of order 14, 19 graphs of order 15, 5 graphs of order 16 and 3 graphs of order 18.

Any other canonical graph with 7 nonzero eigenvalues is then an induced subgraph of the 183 maximal graphs and an overgraph of the corresponding basic graph of this maximal graph. Hence, the complete list of all canonical graphs with 7 nonzero eigenvalues can be easily generated by Table 1.

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(Received 05 02 2008)