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SHOCK WAVES IN GAS DYNAMICS

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Abstract. Shock wave theory was studied in literature by many authors. This article presents a survey with references about various topics related to shock waves: Hyperbolic conservation laws, Well-posedness theory, Compactness theory, Shock and reaction-diffusion wave, The CJ and ZND theory, Existence of detonation in Majda's model, Premixed laminar flame, Multidimensional gas flows, Multidimensional Riemann problem.

1 Introduction

Examine a gas initially at rest, with constant density and pressure ρ_0, p_0 bounded on the left by a plane piston, and assume that the gas is compressed at an initial time by the piston moving into the gas with a constant velocity, which is denoted by u. It is known that an attempt to find a continuous solution to this problem leads to a physically meaningless result. Since the problem is self-similar, the only solutions satisfying the gas dynamics equations are the trivial solutions, in which the quantities u, ρ and p are constant, and the centered simple wave solution. Thus, there remains only one possibility for constructing a solutions that would satisfy the boundary conditions of the problem in the undisturbed gas, $u = 0,\ p = p_0$ and $\rho = \rho_0$, while having in the region next to the piston the gas velocity equal to the piston velocity.

Generally speaking, the laws of conservation of mass, momentum, and energy that form the basis for the equations of inviscid flow of a nonconducting gas do not necessarily assume continuity of the flow variables. These laws were originally formulated in the form of differential equations simply because it was assumed at the beginning that the flow is continuous. These laws, however, can also be applied to these flow regions where the variables undergo a discontinuous change. From a mathematical point of view, a discontinuity can be regarded as the limiting case of very

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large but finite gradients in the flow variable across a layer whose thickness tends to zero. Since in the dynamics of an inviscid and nonconducting gas (with molecular structure disregarded) there are no characteristic lengths, the possibility of the existence of arbitrarily thin transition layers is not excluded. In the limit of vanishing thickness these layers reduce to discontinuities. Such discontinuities represent shock waves.

The rest of the paper is organized as follows: Section 1 is devoted to study hyperbolic conservation laws. The well-posedness theory will be discussed in Section 2. Compactness theory is studied in Section 3. Moreover, in Section 4, one kind of nonlinear waves which are called high-speed detonation waves and low-speed deflagration waves are investigated. The CJ and ZND theories and the difference between them are investigated in Section 5. Also, the existence of weak, strong and CJ detonation waves in Majda's model (which is a simplified model for the qualitative study of one dimensional combustion waves, i.e. for the interaction between chemical reactions and compressible fluid dynamics) is shown in section 6. In addition, Section 7 is devoted to the existence of premixed laminar flames. Finally, in Sections 8 and 9 the existence of explicit solution of the compressible Euler equations in more than one space dimensional are considered.

Before ending this section, it is important to mention that, the beauty of the filed presented here lies in the variety of problems to be considered and the broad set of relevant mathematical techniques: functional analysis, dynamical systems, numerical analysis, pseudo-differential operators, differential geometry, etc. Physical intuition is also a great help in this kind of research.

2 Hyperbolic Conservation Laws

Conservation laws are partial differential equations of the abstract form $div_{x,t}F = 0$, where (x, t) are space-time coordinates. Relevant examples of conservation laws include fluiddynamics, electro-magnetism, magneto-hydrodynamics (MHD) with either a classical or a relativistic context, elasticity, thermoelasticity, combustion, electrophoresis and chromatography. They also occur in social science, for example, in the study of car traffic or of crowd flows in large buildings or in a stadium. They even appear in biology, for example, when bacteria or rabbits migrate. Note that hyperbolic conservation laws which are as

$$u_t + f(u)_x = 0, \tag{2.1}$$

are the simplest equations to describe the shock wave. The theory started with the investigation of the Euler equations in the gas dynamics

$$\begin{cases} \partial_t\rho + \partial_x(\rho v) = 0, \\ \partial_t(\rho u) + \partial_x(\rho v^2 + p) = 0, \\ \partial_t(\rho E) + \partial_x(\rho v E + vp) = 0. \end{cases} \tag{2.2}$$

Courant-Friedrichs's book [19] gives the account of the efforts on the equations by many of the leading mathematicians before 1948.

A basic feature of hyperbolic systems of conservation laws is that there are rich phenomena of wave interactions involving shock waves and contact discontinuities. For example, in gas dynamics, pressure waves are shocks, whereas entropy waves and shear waves are contact discontinuities. The classical works of Glimm [41], Glimm and Lax [42] study the nonlinear wave interactions and qualitative behavior of the solutions. There are several subsequent works: Diperma [27], [28], and Liu [70], [71], [73] on regularity and large-time behavior. There is the wave tracing technique (Liu [72]), which yields the more definite form of the principal of nonlinear superposition and is useful for the study of solution behavior. Note that the nonlinearity of these systems generally forbids the existence of classical solutions. In addition, the strong coupling between fields considerably restricts the tools coined for the study of simpler hyperbolic equations. In particular, it does not permit a comparison principal. Amazingly, we do not even know of a functional space in which the Cauchy problem might be well-posed.

Linear theories have been designed for this class, involving appropriate functional spaces. By Duhamel's principal, these theories have been extended to most semi-linear problems, where the principal part is still linear. However, fully nonlinear problems, or even quasi-linear ones, are more difficult to deal with and require other ideas. Solutions of reasonable problems might have poor regularity, in which case they are called weak. Then, an criterion such as, Lax shock inequality [56] (also see Riemann [103] for gas dynamics), the E-criterion of Liu [76], the E-criterion of Chang and Hiso [13] in the case of gas dynamic, the E-criterion is due to Wendroff [21], entropy inequality (which was first considered by Jouguet [48] for gas dynamics, then by Kruzhkov [52] for scalar equation (n=1), and it is due to Lax [57] in general form), viscosity criterion of a admissibility, Olinik condition [95], shock profile and finally entropy condition for shock waves, may be needed to select a unique, relevant solution among the weak one. In fact, a system of hyperbolic conservation laws (2.1) need to be supplemented by the admissibility criterion.

Viscous shock profiles are travelling waves $(x,t) \to U((x-st)/\epsilon)$ satisfying $\partial_t u + \partial_x f(u) = \epsilon \partial_x (B(u)\partial_x u)$, with $U(\pm\infty) = u^{\pm}$, where s is the speed of combustion wave [115]. A profile is thus a heteroclinic orbit of the vector field,

$$g(u) := B(u)^{-1}(f(u) - f(u^-) - s(u - u^-)).$$

For shocks of moderates strength, the E-criterion is equivalent to the existence of a shock profile (Majda and Pego [89]). The proof is a nice application of the center manifold theory (see [107]).

More recently, it has been understood that the admissibility of a shock requires not only the existence of a profile, but also its dynamical stability. Weak Lax shock waves have been proved to be stable by Liu *et al.* [77], [43], [118], [51], [34] by means of energy estimates. The stability of large shock profiles has been investigated by

Gardner, Howard and Zumburn [36], [139], [140], using an Evans function technique. All these aspects have a counterpart at the level of numerical approximations [109]. Multidimensional aspects have been considered in [141].
Also in the entropy condition for shock waves, the role of viscosity is important and has been recognized since the time of Stokes. Consider the viscous conservation laws

$$u_t + f(u)_x = (B(u,\epsilon)u_x)_x, \tag{2.3}$$

where $B(u,\epsilon)$ is the viscosity matrix. An important example is the Navier-Stokes equations in gas dynamics

$$\begin{cases} \rho_t + (\rho v)_x = 0, \\ (\rho v)_t + (\rho v^2 + p)_x = (\mu v_x)_x, \\ (\rho E)_t + (\rho v E + vp)_x = (\lambda T_x)_x + (\mu v v_x)_x. \end{cases} \tag{2.4}$$

The dissipation parameters ϵ, here are the viscosity μ, and the heat conductivity λ. When the dissipation parameters ϵ are turned off, the inviscid system becomes the hyperbolic conservation laws

$$B(u,0) = 0.$$

When the hyperbolic conservation law is strictly hyperbolic (for non-strictly hyperbolic conservation laws, the situation is more complicated and interesting, Liu [68]), such as the Euler equations, a shock (u_-, u_+) satisfies the entropy condition if and only if it is viewed as the limit of the travelling wave

$$u^\epsilon(x,t) = \phi(x - st)$$

of the viscous conservation laws

$$-s\phi' + f(\phi)' = (B(\phi,\epsilon)\phi')',$$
$$\phi(\pm\infty) = u_\pm,$$

as the viscosity parameters ϵ, tend to zero. Note that, in most scalar cases, a comparison principle holds, and monotonicity encodes the entropy condition; this happen when we apply the nonlinear semigroup theories. Conley and Smoller systemically studied the connecting orbits for such system of ordinary differential equations in series of papers. Particular structures of physical systems, e.g. Navier-Stokes and magneto-hydrodynamics equations, are used and ideas involving Conley index are employed [115]. These works are generalized to other physical systems such as combustions with refined techniques from dynamical systems with small parameters, see [37].

The stability of viscous waves for systems with artificial viscosity

$$u_t + f(u)_x = \epsilon u_{xx}, \tag{2.5}$$

have been studied in literature by many authors such as Liu [68], [75], Yu [134], Zumburn and Howard [139] and references therein. The pointwise estimate is the main technique, which yields the solution behavior on and off characteristic directions. It originates from the study of the perturbation of a constant state, see Liu [74]. Also Liu [68] claim that, the study of viscous contact continuities remains largely open, through there is the preliminary work of Liu and Xin [78]. In addition, Liu and Yu [84] have succeeded in solving the Riemann problem.

Before ending this section we have to recall, although the Navier-Stokes and Euler equations describe the same medium (say, a gas) more or less accurately, are of distinct mathematical nature. The Euler system is first order in both space and time derivatives. It appears to be hyperbolic under rather nature assumptions. The hyperbolic property means that high-frequency waves propagate at a bounded velocity. For instance, an open domain where the solution is constant will survive for a finite time, although it will be deformed. On the other hand, Navier-Stokes systems display parabolic features, due to second order derivatives. Waves propagate with unbounded speed, as in the heat equation $\partial_t u = \Delta u$. The key reference for Navier-Stokes and Euler systems is two-volume book by Lions [65], [66]. See also Temam [120] in the incompressible case.

3 Well-posedness theory

We recall that a problem is well-posed, if it admits a unique solution which depends continuously on the data, in suitable functional spaces. The mathematical theory of well-posedness for scalar equation, $u \in \mathbb{R}$, even for several space dimension $x \in \mathbb{R}^n$, is now well-understood, cf. Krushkov [22]. In fact, Kruzhkov [52] built a unique semigroup in L^∞, which is both monotone and an L^1-contraction (also see Conway and Smoller [20] and Volpert [124]), for study the existence and uniqueness in the scalar case. Lions *et al.* [67] gave regularity results by means of an averaging lemma, applied to their kinetic formulation. This is the only case where the existence is known in space dimensions greater than one. In one dimension, much of the kruzhkov and lax analysis has been extended to the so-called Temple systems, which include chromatography models, see Leveque and Temple [59], Dafermos and Geng [24], Heibig [45] and Serre [110]. The theory for scalar law has been generalized to Hamilton-Jacobi equation.

The well-posedness theory is based on Glimm's construction of solutions and the study of nonlinear wave interactions (Glimm [41]). Through the wave tracing mechanism (Liu [73]), the construction of the nonlinear functional (Liu and Yang [80]) is reduced to the study of the effects of wave coupling between the solutions on their L^1-distance. Central to this study is the introduction of a generalized entropy functional (Liu and Yang [79]). The generalized entropy functional makes effective usage of the nonlinearity of the flux and is new even for the basic inviscid

Burgers equation. The idea in Liu and Yang [80] complements and is orthogonal to the classical work of Glimm [41] on nonlinear wave interactions in a given solution.

As Liu [68] claimed, there are recent important developments in the well-posedness theory. One theory is based on Bressan [9] on the L^1-stability for infinitesimal variation of the initial data, cf. Bressan, Crasts and Picolli [10]. The other is to construct a time-decreasing nonlinear functional which is equivalent to the L^1-distance of two solutions (Liu and Yang [79, 80]). One can formulate as an easy consequence of the well-posedness theory the function class for which the uniqueness theory holds, cf. [11]. Also, Kong and Yang [50] generalized the recent result on L^1 well-posedness theory for strictly hyperbolic conservation laws to the nonstrictly hyperbolic system of conservation laws whose characteristic are with constant multiplicity.

4 Compactness theory

There are two theories for hyperbolic conservation laws. There is the aforementioned qualitative theory originated with Glimm's seminal works and followed up with the recent well-posedness theory. The other theory is the existence theory base on the theory of compensated compactness, starting with the work of Tartar [119]. The programme was achieved for genuinely nonlinear 2×2 systems by Diperna [29], see [30] for the singular context of isotropic gas dynamics. The method is versatile, working also for numerical (see Chen [14]) or relaxation approximation [108] and [123]. The proof uses a maximum principal (K. Chuey *et al.* [16]), and thus provides a solution in L^∞. Uniqueness in this class is uncertain. The more natural context studied by Shearer [111], with only energy estimates, is more difficult to deal with and even less hopeful. Another weakness is that the method cannot be extend beyond the class of rich systems (see, however, the attempt by Benzoni-Gage et al. [4]). It has therefore been more or less abandoned, in spite of its impressive success. Also there have been intensive activities on the existence theory for two conservation laws and some works on large-time behaviors, cf. [15] and references therein. The solutions obtained by this approach of compensated compactness have not been shown to posses the nonlinear wave behavior. As a consequence, the aforementioned well-posedness theory is not applicable to these solutions.
As Serre claimed is his paper [107], one must recognize that the Cauchy problem is still mainly open. The appropriate functional spaces are not yet known. This places the hyperbolic conservation laws in a strange position among the partial differential equations. This also provides extensive opportunities for future researcher.
Also Liu claimed in his paper [68] that, a fundamental open problem is to study the qualitative behavior of a general weak solution, not necessarily the one constructed by a particular method. So far, it is not known, for instance, if a weak solution satisfies such a basic property of hyperbolic equations as the finite speed of propagation. In fact, the verification of this would lead to the understanding of the nonlinear

wave interaction property. Ultimately, one hopes to study the wave behaviors, such as local regularity (cf. [28] and [73]), and large-time behaviors ([70] and [71]) of the solutions. Sufficient regularity of the solutions would allow the application of the aforementioned well-posedness theory.

The compactness theory of Glimm and Lax [42] applies to 2×2 systems. It says that solutions with small sup norm infinite variation norm initial data become of local bound variation for any positive time. This is easily seen for scalar equations; cf. Lax [55]. For 2×2 systems, wave interactions are of third order due to the existence of the coordinates of Riemann invariants. This makes it possible to generalize the scalar compactness result to 2×2 systems. It is not known whether such a strong compactness result holds for more general systems, such as the full 3×3 gas dynamics equations.

5 Shock and reaction-diffusion wave

In this section, we will investigate the compressive and expansive shock waves which are one kind of nonlinear waves and call detonation and deflagration waves. Two other important classes of nonlinear waves are dispersion waves and reaction-diffusion waves. It is, of course, important to relate these distinct classes because a general physical situation often exhibits waves of mixed types. For the study of the limit of the dispersive equations to hyperbolic conservation laws, see Lax, Levermore and Venakids [58]. Liu [68] illustrate with an important physical situation of combustions, which relate the shock waves with reaction-diffusion waves.
accurately, there are two classes of combustion waves in gas: the high-speed detonation waves and low-speed deflagration waves. Detonation waves are physico-chemical propagating structures that are composed of a lead shock wave which initiates chemical reaction in the reactive material. In turn, the release of chemical energy sustains the lead shock wave. In other words, detonation waves are compressive, exothermically reacting shock waves. The steady one-dimensional structure of a detonation wave was first determined by Zeldovich, Von-Neumann and Döring (see [33]) and is known as the ZND structure. The minimum sustainable steady detonation speed is the Chapman-Jouguet (CJ) detonation velocity and is the speed at which the equilibrium or burnt zone flow is sonic relative to the lead shock wave. The presence of the sonic point causes a decoupling of the gas dynamic evolution of the equilibrium flow from the main detonation wave structure. Detonation waves travelling at speeds greater than CJ are called overdriven and have the property that the flow in the burnt region is subsonic relative to the detonation shock. Typical detonation speeds are of the order of 1000-2000 ms^{-1} in gases and 6000-8000 ms^{-1} in condensed solid explosives (see [113]). Also for velocities above CJ, two distinct steady detonation solutions are predicted by a standard, one-dimensional Rankine-Hugoniot analysis. Strong detonations terminate on the subsonic branch of the equilibrium Hugoniot

curve, while weak detonations terminate on the supersonic branch. For a single-step mole-preserving reaction, steady strong detonation waves are possible; however, it is known that no steady travelling weak detonation wave can exist (see [33] and [112]). The strong detonation wave is led by a gas dynamics shocks, which raises the temperature beyond the ignition point, and is then followed by a reaction zone. Thus the strong detonation wave is dominated by the shock wave. The weak detonation wave is supersonic and precedes gas dynamics waves. Thus it is a reaction-diffusion wave and the existence and stability of a weak detonation wave are mainly the result of chemical nonlinearity (this is shown in [83] for a 2×2 model).
Deflagration waves are expansive shock waves. The slow-speed weak deflagration wave usually is given by the reaction-diffusion equations. This is done by assuming that the underlying gas flow is given and not affected by the reaction. Also, the weak deflagration waves are the common occurrence and therefore important in applications. As Liu and Yu [82] have demanded, There has been no stability analysis for deflagration waves.

The classical Chapman-Jouguet inviscid theory has been used to study the combustion waves in gas (Courant-Friedrichs [19]). The theory is adequate for the strong detonations. To study the weak detonations and weak deflagrations, the role of diffusion becomes important. In fact, the structure and existence of these waves depends sensitively on the dissipation parameters, a common feature of undercompression waves [69]. This is so because the profile is a saddle-saddle connection. In this case, the wave is stable uniformly with respect to the strength of dissipation parameters, but its admissibility depends sensitively on the relative strength of these parameters. For the analysis of the stability of viscous waves see [69] and references therein. Also Liu and Yu [83] studied the weak detonations for a simple model of Rosales and Majda [106] which we will talk about it more in section 6. As indicated above, the analysis necessarily contains thinking of reaction-diffusion waves. The chemical nonlinearity is more important than the gas dynamics nonlinearity for the stability analysis [68].

The simplest realistic physical model for the gas combustions is the reactive Navier-Stokes equations. It would be interesting to study the stability of weak detonations and weak deflagrations for the reactive Navier-Stocks equations. Besides the reaction-diffusion nature of these waves, one needs also to consider the interactions with the gas dynamics waves. The combustion waves for the reactive Navier-Stocks equations have been constructed in Gardner [35], Wagner [125], Gasser-Szmolyan [37], Hesaaraki and Razani [46] and [47].

6 The CJ and ZND theories

Fickett [32] and Majda [86] proposed independently a model as a simple mathematical analogous for the equations describing one-dimensional compressible flow in a chemically reacting fluid and is as follows:

$$\partial_t(u+v)+\partial_x f(u)=0, \tag{6.1}$$

$$\partial_t v=-\kappa\Phi(u)v, \quad (\kappa>0), \tag{6.2}$$

for $(x,t)\in\mathbb{R}\times(0,\infty)$, with the initial conditions

$$u(x,0)=u_0(x), \ \ v(x,o)=v_0(x) \ \ (x\in\mathbb{R}), \tag{6.3}$$

when the reaction rate κ goes to infinity.

Also the function f will be some given smooth function and

$$\Phi \text{ is smooth and monotone increasing, with } \Phi(u)=0 \text{ for } u\le 0. \tag{6.4}$$

Let us recall that at least two different mathematical theories are used to describe the propagation of combustion waves in reacting flows: the Chapman-Jouguet (CJ) theory and the Zeldovich-Von Neumann-Döring (ZND) theory (see [19], [131] and [87]). Both are formulated starting from the classical Euler equations for gas-dynamics. The CJ theory assumes that the reaction region is *infinitely thin* or, equivalently, that the reaction rate is *infinitely large*; moreover if we neglect all diffusion effects such as viscosity, heat conduction, and diffusion of species, and any external forces such as gravity, then we obtain the following system of differential equations:

$$\begin{cases} \partial_t\rho+\partial_x(\rho u)=0, \\ \partial_t(\rho u)+\partial_x(\rho u^2+p)=0, \\ \partial_t(\rho E)+\partial_x(\rho uE+up)=0, \end{cases} \tag{6.5}$$

Here, ρ is the density, u the fluid velocity, p the pressure and E the total specific energy, namely

$$E=e+qZ+\frac{1}{2}u^2,$$

the quantity e, being the specific internal energy, while q, denotes the amount of heat released by the chemical reaction and Z, is the mass fraction of the reactant. The internal energy e and temperature T, are given through equations of state

$$e=e(\rho,p), \ T=T(\rho,p),$$

which depend on the gas mixture under consideration. To specify the variable Z, the CJ theory introduce a further relation, namely

$$Z(x,t)=\begin{cases} 0, & if \ \sup_{0\le s\le t} T(x,s)>T_i, \\ Z(x,0), & if \ \sup_{0\le s\le t} T(x,s)\le T_i, \end{cases} \tag{6.6}$$

where T_i is a given ignition temperature. If we consider the steady plane waves, then (6.5) becomes:

$$\begin{cases} (a)\ (\rho u)_x = 0, \\ (b)\ (\rho u^2 + p)_x = 0, \\ (c)\ (\rho u E + up)_x = 0, \end{cases} \tag{6.7}$$

As is standard practice, we have represented the extremely complicated chemical reaction by a simplified, one-step chemistry: reactant→ product. From (6.7 a) we see that the mass flux, ρu, has a constant value; we denote this value by m. The fluxes of momentum (6.7 b) and energy (6.7 c) are also constant; from this fact we obtain the *Rankine-Hugoniot conditions* for a shock wave, which in the inviscid theory is represented by a jump discontinuity in the unknowns. The difference between inert gas dynamics, and the exothermic reactive theory discussed here, lies in the fact that Z varies from a positive value on the unburned side of the wave, which we take to lie on the left side, to a zero value on the burned, or right side. Because the internal energy e depends on Z, the change in Z causes the classical Hugoniot curve (the solution locus of (6.7 c)) of gas dynamics to move. As a consequence, we find that, for a given value of m, a given shock state on the left may now be connected by a shock wave to two possible state, the *Chapman-Jouguet point*. In addition, the curve of possible burned states, parameterized by m, has two components. One component, corresponding to compressive waves, is called the detonation branch, and the other component, corresponding to expansive waves, is called the deflagration branch. By way of contrast, in an inert gas, for a given value of m, a state is usually connected to only one state on the right, and the curve of possible terminal shock states is usually connected.
The combustive shock wave of the CJ theory are classified as follows. A wave connecting the unburned state to the closer detonation point is called a *weak* detonation wave, and a connection to the farther detonation point is called a *strong* detonation. A detonation wave terminating at the Chapman-Jouguet point is called a *Chapman-Jouguet* detonation. Deflagration waves are similarly classified. In other words, Strong detonations are supersonic upstream and subsonic downstream. Weak detonations are supersonic upstream and downstream. Weak deflagrations are subsonic upstream and downstream. Strong deflagrations are subsonic upstream and supersonic downstream. For the exothermic, irreversible reactions considered here, strong deflagration violate the second law of thermodynamics and are unphysical, and weak detonations are rare. if we permit an endothermic region then strong deflagration and weak detonation are possible and perhaps even probable [33].

The CJ theory for detonation wave is useful for deriving the Rankine-Hugoniot conditions, and for classifying the types of wave. However, this theory is physically flawed, because in reality the reaction zone is much thicker than the shock layer. This is due to the fact that the chemical reaction depends on molecular collisions and requires a distance much longer than the mean free path to achieve signification completion. The shock layer, however, has been experimentally observed mean free path thick. Consequently the appropriate inviscid model in the one space developed independently by Zeldovich, Von Neumann, and Doring [135], [136], [92], [93] and [31] and which is known as the ZND model.

In order to describe more precisely the internal structure of combustion waves, the ZND theory assumes that the reaction rate is *finite*, although the effect of viscosity and heat conduction are disregarded. This corresponds to replace the equation (6.6) by the following supplementary equation for the mass fraction of unburnt gas:

$$\partial_t(\rho Z) + \partial_x(\rho u Z) = -\kappa\rho\Phi(T)Z. \tag{6.8}$$

The rate function $\Phi(T)$ has typically the following form:

$$\Phi(T) = \begin{cases} 0, & \text{if } T < T_i, \\ T^\alpha \exp\{-\frac{A}{T-T_i}\}, & \text{if } T \geq T_i, \end{cases}$$

where T_i is the ignition temperature, A is the activation energy and k the reaction rate $(T_i, A, k > 0)$; as for α, this is a dimensionless parameter in the range $(-1, 2]$.

Systems (6.5) and (6.8) are the hyperbolic systems of balance laws, whose local existence of solutions is established by the results in [25], at least for initial data of small bounded variation (see also [133]). Global existence of solutions (even for the Riemann problem) seems to be still an open problem.

Wagner [125] proved that (6.5) and (6.8) can be solved explicitly, the only detonation wave solutions are strong or CJ detonations. These waves, which are known as ZND waves, begin with a jump discontinuity which is an inert shock wave. This shock wave heats the gas above the ignition temperature; the reaction proceeds, with the velocity and temperature following a curve of equilibrium states for (6.7 b,c), parameterized by Z. One of the interesting feature of these waves is the peak in the pressure and density which is known as the *von Neumann spike*. By way of contrast, in inert shock waves the pressure and density are usually monotone [39].

The equations of inert, inviscid, non-heat-conducting gas dynamics are an example of a nonlinear hyperbolic system of conservation laws. In the theory for such systems it is standard practice to set admissibility criteria to distinguish physical from unphysical shock waves. One of the criteria in which much faith is put is to accept a shock wave as physical if it is *structurally stable*. A shock wave is structurally stable if it is the limit of solutions to models which include more physical effects, such as viscosity and heat conduction, as these models tend to the original inviscid model in which these effects are neglected. For steady plane detonation waves the effects of viscosity, heat conduction, and species diffusion may be considered to obtain the (steady) *reacting compressible Navier-Stokes equations:*

$$\begin{cases} (\rho u)_x = 0, \\ (\rho u^2 + p)_x = (\mu u_x)_x, \\ (\rho u E + up)_x = (\lambda T_x)_x + (\mu u u_x)_x + (q\rho D Z_x)_x, \\ (\rho u Z)_x = -\kappa\rho\Phi(T)Z + (\rho D Z_x)_x. \end{cases} \tag{6.9}$$

Here μ is the coefficient of viscosity, λ is the heat conductivity, D is the diffusion rate for the reactant, and q is the difference in the heats of formation of the reactant and the product [131].
Gardner [35] proved the existence of travelling plane detonation wave solutions to the Lagrangian reacting compressible Navier-Stokes equations. He made no assumption on the Lewis ($L = \frac{\lambda}{\rho D c_p}$) and Prandtl (when Prandtl number is $\frac{3}{4}$, then $\mu = \frac{\lambda}{c_p}$) numbers, where c_p is the specific heat at constant pressure. However, he assumed that the gas is ideal and he omitted the species diffusion term from the energy balance equation, and this term is not usually neglected. It may be that the effect of this term on the solution is very small. Moreover, he assumed that $\Phi(T)$ is nondecreasing smooth function. Wagner [125] and [128] proved a necessary condition and a sufficient condition for the existence of steady plane wave solutions to the Navier-Stockes equations for a reacting gas. These solutions represent plane detonation waves, and converge to **ZND** detonation waves as the viscosity, heat conductivity, and species diffusion rates tend to zero. He assumed that the Prandtl number is $\frac{3}{4}$, but arbitrary Lewis numbers are permitted. he hasn't made any assumptions concerning the activation energy. he showed that the stagnation enthalpy and the entropy flux are always monotone for such solutions, and that the mass density and pressure are nearly always not monotone, as predicted by the **ZND** theory. Gasser and Szmolyan [37] analyzed the existence of steady plane wave solution of the Navier-Stokes equations for a reacting gas. Under the assumption of an ignition temperature the existence of detonation and deflagration waves close to the corresponding waves of the ZND model has proved in the limit of small viscosity, heat conductivity, and diffusion. Their method was constructive, since the classical solutions of the ZND model serve as singular solutions in the context of geometric singular perturbation theory. The singular solutions consist of orbits on which the dynamics are slow-driven by chemical reaction and of orbits on which the dynamics are fast-driven by gasdynamic shocks. The approach was geometric and leads to a clear, complete picture of the existence, structure, and asymptotic behavior of detonation and deflagration waves. They considered a three-component gas with chain branching mechanism in [38], as a specific example, and proved the existence of travelling wave solutions for the resultant system. They discussed the qualitative difference between the already known case and a simple one-step reaction. Finally, a general method to prove the existence of combustion waves for a multi-component gas was presented in [38]. Hesaaraki and Razani [46] and [47] proved that for the general discontinues reaction rate function $\Phi(T)$ the travelling wave solutions for weak, Strong and Chapman-Jouguet detonation waves exist.

7 Detonation in Majda's model

The study of the existence of detonation waves is one of the fundamental problems in combustion. Majda [86] proposed a simplified model for the qualitative study of one dimensional combustion waves, i.e., for the interaction between chemical reactions and compressible fluid dynamics. The model was derived from the one-dimensional combustion equations written in Lagrangian coordinates for a simple Reactant→Product mechanism (see [86]). Majda neglected the diffusion coefficient in his model. Larrouturou [53] extended Majda's model by adding a positive diffusion coefficient. The model is as follows:

$$\begin{cases} (U + q_0 Z)_t + (f(U))_x = \beta U_{xx}, \\ Z_t = -\kappa\Phi(U)Z + DZ_{xx}, \end{cases} \tag{7.1}$$

where U is a lumped variable having some features of density, temperature and velocity, Z mass fraction of unburned gas (note that the completely unburned gas corresponds to $Z = 1$ and a totally burned gas corresponds to $Z = 0$), $q_0 > 0$ is the effective heat release from the chemical reaction, $\beta > 0$ a lumped viscosity-thermal-conductivity coefficient and $D > 0$ is diffusion coefficient. The independent variables t and x are the time and space variables, respectively.
From now on, we assume $U = T$ (the temperature). Also we encounter the well known cold boundary difficulty, that is, the unburned state is not a stationary point of (7.1) since the "reaction rate function" $\Phi(T) \neq 0$, for $T > 0$. One resolution of the cold boundary difficulty can be based on activation energy asymptotic (see [131]). However, in our analysis we use the common mathematical idealization of an ignition temperature, Φ is modified such that (see [98])

$$\Phi(T) = \begin{cases} 0 & \text{for } T < T_i, \\ \Phi_1(T) & \text{for } T \geq T_i, \end{cases} \tag{7.2}$$

where $\Phi_1(T)$ is a smooth positive function and T_i is the "ignition temperature" of the reaction. A typical example for $\Phi_1(T)$ is the Arrhenius law, *i.e.* $\Phi_1(T) = T^\gamma e^{-\frac{A}{T}}$ for some positive constants γ and A. Note that $\Phi(T)$ is discontinues at the point T_i. A careful discussion of this assumption and its consequences for detonation and deflagration wave (with one-step chemistry) can be found in [49] and [98]. Finally, $f(T)$ is a convex strongly nonlinear function satisfying (see [86])

$$\begin{array}{l} \frac{\partial f}{\partial T} = a(T) > 0, \ \frac{\partial^2 f}{\partial T^2} > \delta > 0, \\ \lim\limits_{T\to+\infty} f(T) = +\infty, \end{array} \tag{7.3}$$

and for example you can choose $f(T) = \frac{1}{2}aT^2$ $(a > 0)$ (see [106] page 1100).
System (7.1) was proposed by Majda [86] as a model for dynamic combustion, i.e. for the interaction between chemical reactions and compressible fluid dynamics.

He proved the existence of weak and strong detonations, for q_0 independent of T, together with a very simple form of $\Phi(T)$. His results are described in terms of liberated energy, q_0, that is, for fixed $\kappa > 0$, he proved that there is a critical liberated energy, q_0^{CR}, such that when $q_0 > q_0^{CR}$, (7.1) admits a strong detonation combustion profile and when $q_0 = q_0^{CR}$, (7.1) admits a weak detonation combustion profile. Also Rosales and Majda [106] investigated the qualitative model (7.1) in a physical context. Colella *et al* [18] have used fractional step methods based on the use of a second order Godunov method. They demonstrated that (7.1) has dynamically stable weak detonations which occur in bifurcating wave patterns from strong detonation initial data. They used (7.1) to emphasize both to predict and analyze the theoretical and numerical phenomena. Also Li [60] studied (7.1) when $\beta = 0$. He established global existence of the solution to the problem and studied the asymptotic behavior of the solution. He also proved that the solution converges to a self-sustaining detonation wave and if the data are small, the solution decays to zero like an N-wave. Liu and Ying [81] studied strong detonation waves for (7.1) and proved these waves are nonlinearly stable by using energy method for the fluid variable and a pointwise estimate for the reactant. Ying *et al* [132] continued the nonlinear stability of strong detonation waves for (7.1). In fact they showed that if q is sufficiently small, then a perturbation of a travelling strong detonation wave leads to a solution which tends to a shifted travelling strong detonation wave as $t \to +\infty$, and the rate is determined by the rate of initial perturbation as $|x| \to \infty$. System (7.1) was also investigated by Hanouzet *et al* [44] when $\beta = 0$. They studied the limiting behavior of solutions of (7.1) when the reaction rate tends to infinity. Roquejoffre and Vila [105] studied the stability of detonation waves of (7.1) when $\kappa = 1$. Also Liu and Yu [83] considered (7.1) when $\beta = 0$. They proved that the weak detonation waves of the model are nonlinear stable. Szepessy [117] studied the nonlinear stability of travelling weak detonation waves of (7.1) when $f(T) = \frac{1}{2}T^2$. Finally, Razani [98], [99], [100] and [101] proved the existence of weak, strong and CJ detonation waves for model (7.1). Recently, using Evans function technique, Lyng and Zumbrun [85] developed a stability index for weak and strong detonation waves (for (7.1) when $D = 0$) yielding useful necessary conditions for stability. In fact, they showed, in the simple context of the Majda model, that the methods introduced in [5] and [36] for the study of stability of viscous shock wave, may, with slight modifications, be applied also in the study of stability of detonations to :(i) construct an analytic Evans function on the set $Re\, \lambda \geq 0$, (where λ is an eigenvalue) for the linearized operator about the detonation wave, and (ii) in both the strong and weak detonation cases, compute and expression for $\Gamma(:= sgnD'(0)D(\infty))$, where D is the Evans function) in terms of quantities associated with the travelling wave ordinary differential equations. In this simple setting, the connection problem is planar, and they can in fact do more, obtaining a complete evaluation of Γ; the result, for both weak and strong detonations, is $\Gamma > 0$, consistent with stability.

8 Premixed Laminar Flame

One of the most important problems of combustion is the planar premixed flames, that is, the one-dimensional deflagration wave. In a case of a single-step reaction involving one reactant, it reduces to a system of two reaction diffusion equations [53] and [90]. The existence of travelling wave solutions for this system has been established for both positive and zero ignition temperature ([6], [7], [17], [90], [104], [114], [122], [126] and [127]). In [102] we discussed the existence of travelling wave solutions in a model for slow, "constant density" combustion. Alternatively, the travelling wave solutions for this model may be derived from a more complicated system, assuming only a slow speed of propagation, and weak temperature and pressure variation ([6], [7] and [127]). The model is a simple model of an exothermic chemical reaction in a gas and is as follows (for a background on the physical motivation and derivation of the following model, see Buckmaster and Ludford [12], Larrouturou [53], Wagner [126], [127] and Williams [131]):

$$\left\{ \begin{array}{l} Y_t = (\nu(Y,T)Y_x)_x - DY\Phi(T), \\ T_t = (\lambda(Y,T)T_x)_x + qDY\Phi(T), \end{array} \right. \tag{8.1}$$

where T is the temperature and Y the mass fraction of the unburned gas. Note that the completely unburned state corresponds to $Y = Y_f$ and a totally burned state corresponds to $Y = 0$. Also the physically desirable values of the unknowns Y and T are non-negative. In fact, we consider $0 \le Y \le Y_f$ and $0 \le T \le T_b$ (see [127]). The parameters ν, λ, D and q are positive. The independent variables t and x are the time and space variables, respectively. Finally, the "reaction rate function" $\Phi(T)$ is as (7.2).

System (8.1), with $\Phi(T)$ in a very simple form, has received extensive mathematical treatment in recent years. Berestycki *et al* [6] proved the existence of a solution of (8.1). Also in [7] they considered the deflagration wave problem for a compressible reacting gas, with one reactant involved in a single-step chemical reaction. They showed how the one-dimensional travelling wave problem reduces to a system of two reaction-diffusion equations. Wagner [127] obtained a sufficient condition for the existence of travelling waves representing premixed laminar flames. In order to do this, he used a topological method in his article. The necessary condition has been given by Marion [90]. The existence of travelling wave solutions of (8.1) was established by Terman [122] in the case $T_i = 0$. Also stability and instability results for the travelling waves, where

$$\Phi(T) = \left\{ \begin{array}{ll} 0 & \text{for } T < 0, \\ Be^{-\frac{E}{T}} & \text{for } T \ge 0, \end{array} \right.$$

have been obtained by Clavin [17], Sivashinsky [114], and Roquejoffre and Terman [104]. Avrin [1] studied the equations with initial data that are bounded, uniformly continuous, and nonnegative but otherwise arbitrary. He established the existence of unique global strong solutions satisfying appropriate a priori estimates. With a positivity condition imposed on the initial data for the temperature, he showed that the concentration decays exponentially. Also in [2] he studied the qualitative behavior of solutions to the initial-boundary value problem for the reaction-diffusion equations (8.1). In both cases where T and Y satisfy zero Neumann boundary conditions or fixed Dirichlet boundary conditions, extensive qualitative results have been given concerning complete asymptotic burning and eventual quenching. Weber *et al* [130] considered (8.1) and assumed that the chemical reaction can be represented by the Arrhenius rate law. They applied a different non-dimensionalization, and used the ratio of the activation energy to the heat of the reaction as a large parameter around which an asymptotic analysis was based. Mercer *et al* [91] then used this non-dimensionalization to study the effects of heat loss on the routes to extinction of the combustion wave, given that the activation energy is large or that the heat of the reaction is small. In addition, Avrin [3] considered models of laminar flames with Arrhenius kinetics in long thin tubes, and under certain conditions imposed on the initial temperature, a rough sense of flame propagation resulting in complete asymptotic burning of the fuel for one step reaction ($A \to B$) is guaranteed. He studied a model of a two-step reaction ($A \to B \to C$) and some models of one-step reactions with multiple species, and identified sufficient conditions on the initial temperature that guarantee a rough sense of flame propagation and complete asymptotic burning. Moreover, Billingham and Mercer [8] investigated (8.1). They used the method of matched asymptotic expansions to obtain asymptotic approximations for the permanent form travelling wave solutions and their results were confirmed numerically. Finally, Razani [102] studied the existence of premixed laminar flames for (8.1).

9 Multidimensional Gas Flows

Systems of hyperbolic conservation laws for one space dimension have been investigated by many authors. The compressible Euler equations in more than one space dimensions are as follows (see Liu [68] and Courant-Friedrichs [19]):

$$\begin{aligned} &\rho_t + \textstyle\sum_{j=1}^{m}(\rho v_j)_{x_j}(\rho v_i) + \sum_{j=1}^{m}(\rho v_i v_j)_{x_j} + p_{x_j} = 0,\ i = 1, \cdots, m, \\ &(\rho E)_t + \textstyle\sum_{j=1}^{m}(\rho E v_j + p v_j)_{x_j} = 0. \end{aligned} \tag{9.1}$$

Particular solutions such as self-similar solution have been constructed, see Zhang-Zheng [138]. This kind of solution explain that the gas flows around a solid boundary. As Liu [68] claimed, one interesting problem is studying the nonlinear stability of these self-similar flows.

(i) In the case of one dimensional supersonic stationary flows, one may study the stability by many methods such as the random choice method (see Liu [68] and Courant-Friedrichs [19] for more details).

(ii) The stability of two-dimensional supersonic stationary flows will be studied as a one space dimension (because it is possible to view one space dimension as time).
(iii) Courant-Friedrichs [19] prove that For straight cone, the flow is self-similar on three dimensional. In this case, Chen [14] obtained the linearized approach for the study of the flow when the tip of the cone is perturbed. On the other hand, Lien and Liu [64] studied the global stability when the cone, away from the tip, is perturbed by using the locally self-similar flows as building blocks in the approximation.

10 Multidimensional Riemann problem

The Riemann problem is a fundamental problem for quasilinear hyperbolic systems of conservation laws, which is a Cauchy problem with piecewise constant initial data with a jump at the origin in the one-dimensional case. Now, consider a $m \times m$ system of linear partial differential equations

$$\frac{\partial u}{\partial t} + \sum_{i=1}^{n} A_i \frac{\partial u}{\partial x_i} = 0, \tag{10.1}$$

where $(t, x) = (t, x_1, x_2, \cdots, x_n) \in \mathbb{R}_+ \times \mathbb{R}^n$, $u = (u_1, \cdots, u_m) \in \mathbb{R}^m$, A_i $(1 \leq i \leq n)$ are real constant $m \times m$ matrices and m, n are positive integers.

Definition 1. *[61] System (10.1) is called to be hyperbolic, if for all*

$$\alpha = (\alpha_1, \cdots, \alpha_n) \in \S^{n-1}$$

(the unit sphere in $\mathbb{R}^n$), the matrix $\sum_{i=1}^{n} \alpha_i A_i$ is diagonalizable with real eigenvalues. Moreover, (10.1) is strictly hyperbolic, if for all $\alpha = (\alpha_1, \cdots, \alpha_n) \in \S^{n-1}$, the matrix $\sum_{i=1}^{n} \alpha_i A_i$ has n distinct real eigenvalues.

In general, classifying $m \times m$ hyperbolic systems with real constant coefficients and giving all possible canonical forms of $m \times m$ hyperbolic systems are very difficult. In 1967, Strong [116] proved that for every 2×2 hyperbolic system with real constant coefficients, all coefficient matrices can be simultaneously symmetrizable and the system can be reduced to a canonical form ($n \geq 2$). In [54], Lax shows that (10.1) for $m = 2(\text{mod } 4)$ may not be a strictly hyperbolic system ($n \geq 3$). In 1991, Oshime ([96] and [97]) classified 3×3 hyperbolic systems with real constant coefficients and listed all canonical forms. In [40], the authors gave the explicit solution to the Riemann problem in the case m=2, in which Riemann data are given by a piecewise smooth vector function. Li and Sheng [61] were interested in solving the general Riemann problem for (10.1) in the case $m = 2$, in which Riemann data are given by a piecewise smooth vector function. They first gave the canonical forms of 2×2 hyperbolic systems and then the explicit solution to the corresponding general Riemann problem.

Note that the Riemann problem is a problem for quasilinear systems of conservation laws. In the two dimensional case, the corresponding Riemann problem may be considered as the Cauchy problem with constant initial data in each quadrant. However, Li and Sheng [62] further considered the Riemann problem in a more general way in the two-dimensional case: the initial data are piecewise smooth functions of the polar angle θ. This kind of Riemann problem is then called the general Riemann problem. Li and Sheng [62] solved the general Riemann problem for the linearized system of two-dimensional isentropic flow in gas dynamics

$$\begin{cases} \frac{\partial \rho}{\partial t} + \rho_0(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}) = 0, \\ \frac{\partial u}{\partial t} + \frac{\partial P'(\rho_0)}{\partial \rho_0}\frac{\partial \rho}{\partial x} = 0, \\ \frac{\partial v}{\partial t} + \frac{\partial P'(\rho_0)}{\partial \rho_0}\frac{\partial \rho}{\partial y} = 0, \end{cases}$$

$$t = 0: \ (\rho, u, v) = (\rho_0(\theta), u_0(\theta), v_0(\theta)),$$

where ρ is the density, (u, v) is the velocity, ρ_0 is a positive constant, $p = p(\rho)$ is the equation of state satisfying $P'(\rho_0) > 0, \theta$ is the polar angle such that

$$\begin{cases} x = r\cos\theta, \\ y = r\sin\theta, \end{cases} \quad 0 \le r < +\infty, 0 \le \theta \le 2\pi,$$

and $\rho_0(\theta), u_0(\theta), v_0(\theta)$ are bounded piecewise smooth functions. They gave the explicit solution to this general Riemann problem.

Appendix A.

The flow equations which consider the gas dynamic variables as functions of the space coordinates and time are called the Euler equations, or the flow equations in Eulerian coordinates.

Lagrangian coordinates are frequently used to describe one-dimensional flow, that is, plan and cylindrically and spherically symmetric flow. In contrast to Eulerian coordinates, Lagrangian coordinates do not determine a given point is space, but a given fluid particle. Gasdynamic flow variables expressed in terms of Lagrangian coordinates express the changes in density, pressure, and velocity of each fluid particle with time. Lagrangian coordinates are particularly convenient when considering internal processes involving individual fluid particle, such as a chemical reaction whose progress with time depends on the changes of both the temperature and the density of each particle. The use of Lagrangian coordinates also occasionally yields a shorter and easier way of obtaining exact solutions to the gas dynamic equations, or provides a more convenient numerical integration of them. The derivative with respect to time in Lagrangian coordinates is simply equivalent to the total derivative D/Dt. The particle can be described either in terms of the mass of fluid separating it from a given reference particle (in one dimension), or in terms of its position at the initial instant of time. On addition, the use of Lagrangian coordinates is especially simple in the case of plane motion, when the flow is a function of only one cartesian coordinate x.

A tedious calculation using the chain rule and product rule shows that the Euler and Lagrangian equations of gas dynamics on one space dimension are equivalent for classical solutions [19]. In addition, Wagner [129] demonstrated the equivalence of the Euler and Lagrangian equations of gas dynamics on one space dimension for weak solutions which are bounded and measurable in Eulerian coordinates. He assumed all known global solutions on $\mathbb{R} \times \mathbb{R}^+$. In particular, solutions containing vacuum states (zero mass density) were included. Furthermore, he proved that there is a one-to-one correspondence between the convex extensions of the two systems, and the corresponding admissibility criteria are equivalent. In the presences of a vacuum, the definition of weak solution for the Lagrangian equations must be strengthened to admit test functions which are discontinuous at the vacuum. As an application, he translated a large-data existence result of Diperna for the Euler equations for isentropic gas dynamics into a similar theorem for the Lagrangian equation.

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