

COMPUTING OPTIMAL CONTROL WITH A QUASILINEAR PARABOLIC PARTIAL DIFFERENTIAL EQUATION

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Abstract. This paper presents the numerical solution of a constrained optimal control problem (COCP) for quasilinear parabolic equations. The COCP is converted to unconstrained optimization problem (UOCP) by applying the exterior penalty function method. Necessary optimality conditions for the considered problem are established. The computing optimal controls are helped to identify the unknown coefficients of the quasilinear parabolic equation. Numerical results are reported.

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