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ON IRREDUCIBLE PROJECTIVE REPRESENTATIONS OF FINITE GROUPS

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Abstract. The paper is a survey type article in which we present some results on irreducible projective representations of finite groups.

Section 2 includes Curtis and Reiner's theorem ([8]) in which is proved that a finite group has at most a finite number of inequivalent irreducible projective representations in an algebraically closed field K. Theorem 15 ([16]) gives an alternative proof of the main theorem of Morris ([15]), where the structure of a generalized Clifford algebra was determined. Similarly, Theorem 16 ([16]) gives the structure theorem for a generalized Clifford algebra which arises in the study of the projective representations of the generalized symmetric group. Section 2 is also dedicated to the study of degrees of irreducible projective representations of a finite group G over an algebraically closed field K. In Theorem 20, H. N. NG proved a generalization of Schur's result and showed that the degree of an irreducible projective representation of a finite group G belonging to $c \in H^2(G; K^*)$, where K is an algebraically closed field such that char$K$ does not divide $|G|$, divides the index of a class of abelian normal subgroups of G, which depends only on the 2-cohomology class c. In Theorem 27, Quinlan proved ([19]) that the representations theory of generic central extensions for a finite group G yields information on the irreducible projective representations of G over various fields.

In Section 3 we give a necessary and sufficient condition for a nilpotent group G to have a class of faithful faithful irreducible projective representation ([18]).

Section 4 includes NG's result in the case of a metacyclic group G with a faithful irreducible projective representation π over an algebraically closed field with arbitrary characteristic, which proved that the degree of π is equal to the index of any cyclic normal subgroup N whose factor group G/N is also cyclic and also a necessary and sufficient conditions for a metacyclic group to have a faithful irreducible projective representation ([18]).

In Section 5 we remind Barannyk's results ([2], [5]) in which he obtained conditions for a finite p-group to have a class of faithful irreducible projective representations.

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Section 6 contains the most important results of the adaptation to projective representations of Clifford's theory of inducing from normal subgroups ([21], [13]).

Full text

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