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SOME FIXED POINT RESULTS IN MENGER SPACES USING A CONTROL FUNCTION

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Abstract. Here we prove a probabilistic contraction mapping principle in Menger spaces. This is in line with research in fixed point theory using control functions which was initiated by Khan et al. [Bull. Austral. Math. Soc., 30(1984), 1-9] in metric spaces and extended by Choudhury et al. [Acta Mathematica Sinica, 24(8) (2008), 1379-1386] in probabilistic metric spaces. An example has also been constructed.

1 Introduction

In metric fixed point theory, the concept of altering distance function has been used by many authors in a number of works on fixed points. An altering distance function is actually a control function which alters the distance between two points in a metric space. This concept was introduced by Khan et al. in 1984 in their well known paper [12] in which they addressed a new category of metric fixed point problems by use of such functions. Altering distance functions are control functions which alters the metric distance between two points. After that triangular inequality is not directly applicable. This warrants special techniques to be applied in these types of problems. Some of the works in this line of research are noted in [7], [15], [16] and [17]. Altering distance functions have been generalized to functions of two variables [1] and three variables [3] and have been used in fixed point theory. Altering distance functions have also been extended to fixed point problems of multivalued and fuzzy mappings [4]. Recently it has also been extended to probabilistic fixed point theory ([5], [7]). In the present paper we make another use of such concept in proving fixed point results in Menger spaces.

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2 Definitions and Mathematical Preliminaries

Definition 1. ***t-norm*** *[10] [18]*

A t-norm is a function $\Delta : [0,1] \times [0,1] \to [0,1]$ which satisfies the following conditions

(i) $\Delta(1, a) = a$,

(ii) $\Delta(a, b) = \Delta(b, a)$,

(iii) $\Delta(c, d) \geq \Delta(a, b)$ *whenever* $c \geq a$ *and* $d \geq b$,

(iv) $\Delta(\Delta(a, b), c) = \Delta(a, \Delta(b, c))$.

Definition 2. *[10] [18]*

A mapping $F : R \to R^+$ is called a distribution function if it is non-decreasing and left continuous with $\inf_{t \in R} F(t) = 0$ and $\sup_{t \in R} F(t) = 1$, where R is the set of real numbers and R^+ denotes the set of non-negative real numbers.

Definition 3. ***Menger Space*** *[10] [18]*

A Menger space is a triplet (M, F, Δ) where M is a non empty set, F is a function defined on $M \times M$ to the set of distribution functions and Δ is a t-norm, such that the following are satisfied:

(i) $F_{xy}(0) = 0$ *for all* $x, y \in M$,

(ii) $F_{xy}(s) = 1$ *for all* $s > 0$ *and* $x, y \in M$ *if and only if* $x = y$,

(iii) $F_{xy}(s) = F_{yx}(s)$ *for all* $x, y \in M, s > 0$ *and*

(iv) $F_{xy}(u + v) \geq \Delta(F_{xz}(u), F_{zy}(v))$ *for all* $u, v \geq 0$ *and* $x, y, z \in M$.

A sequence $\{x_n\} \subset M$ converges to some point $x \in M$ if for given $\epsilon > 0, \lambda > 0$ we can find a positive integer $N_{\epsilon,\lambda}$ such that for all $n > N_{\epsilon,\lambda}$,

$$F_{x_n x}(\epsilon) > 1 - \lambda.$$

Fixed point theory in Menger spaces is a developed branch of mathematics. Sehgal and Bharucha-Reid first introduced the contraction mapping principle in probabilistic metric spaces [19]. Hadzic and Pap in [10] has given a comprehensive survey of this line of research. Some other recent references in this field of study are noted in [2], [11], [13], [20] and [22].

Definition 4. ***Altering Distance Function*** *[12]*

A function $h : [0, \infty) \to [0, \infty)$ is an altering distance function if

(i) *h is monotone increasing and continuous and*

(ii) *$h(t) = 0$ if and only if $t = 0$.*

Khan et al. proved the following generalization of Banach contraction mapping principle.

Theorem 5. *[12]*
Let (X, d) be a complete metric space, h be an altering distance function and let $f : X \to X$ be a self mapping which satisfies the following inequality

$$h(d(fx, fy)) \leq c\, h(d(x, y))$$

for all $x, y \in X$ and for some $0 < c < 1$. Then f has a unique fixed point.

In fact Khan et al. proved a more general theorem (Theorem 2 in [12]) of which the above result is a corollary.

Definition 6. ***Φ-function*** *[5]*
A function $\phi : [0, \infty) \to [0, \infty)$ is said to be a Φ-function if it satisfies the following conditions:

(i) $\phi(t) = 0$ if and only if $t = 0$,

(ii) $\phi(t)$ is strictly increasing and $\phi(t) \to \infty$ as $t \to \infty$,

(iii) ϕ is left continuous in $(0, \infty)$ and

(iv) ϕ is continuous at 0.

An altering distance function with the additional property that $h(t) \to \infty$ as $t \to \infty$ generates a Φ-function in the following way.

$$\phi(t) = \begin{cases} \sup\{s : h(s) < t\}, & \text{if} \quad t > 0, \\ 0, & \text{if} \quad t = 0. \end{cases}$$

It can be easily seen that ϕ is a Φ-function.

The following result has been established in [5].

Theorem 7. *[5]*
Let (M, F, Δ) be a complete Menger space with $\Delta(a, b) = min\{a, b\}$ and $f : M \to M$ be a self mapping such that the following inequality is satisfied.

$$F_{fxfy}(\phi(t)) \geq F_{xy}(\phi(t/c))$$

where ϕ is a Φ-function, $0 < c < 1$, $t > 0$ and $x, y \in M$. Then f has a unique fixed point.

It has been established in [5] that the result of Khan et al. noted in Theorem 5 follows from the above theorem. Φ-functions play the role of altering distance functions in probabilistic metric spaces. Further fixed point results by use of Φ-functions have been established in [6].

There are several notions of completeness and convergence in fuzzy metric spaces which are generalisations of Menger spaces. Mihet in [14] has described these concepts in fuzzy metric spaces in a comprehensive manner and has also provided examples to show their differences. Since in this paper we confine our considerations to Menger spaces, we describe them correspondingly in the context of Menger spaces.

Throughout the paper $\mathbb{N}$ denotes set of all natural numbers.

Definition 8. ***Cauchy Sequence***

A sequence $\{x_n\}$ in a Menger space (M, F, Δ) is called a Cauchy sequence if for each $\epsilon \in (0,1)$ and $t > 0$, there exists $n_0 \in \mathbb{N}$ such that $F_{x_n x_m}(t) > 1 - \epsilon$ for all $m, n \geq n_0$.

The Menger space (M, F, Δ) is said to be complete if every Cauchy sequence in M is convergent.

Definition 9. ***G-Cauchy Sequence [8] [9]***

A sequence $\{x_n\}$ in a Menger space (M, F, Δ) is called a G-Cauchy if

$$\lim_{n\to\infty} F_{x_n x_{n+m}}(t) = 1 \text{ for each } m \in \mathbb{N} \text{ and } t > 0\,.$$

We call a Menger space (M, F, Δ) G-complete if every G-Cauchy sequence in M is convergent.

It follows immediately that a Cauchy sequence is a G-Cauchy sequence. The converse is not always true. This has been established by an example in [21].

The following concept of convergence was introduced in fuzzy metric spaces by Mihet [14]. Here we describe it in the context of Menger spaces.

Definition 10. ***Point Convergence or p-convergence [14]***

Let (M, F, Δ) be a Menger space. A sequence $\{x_n\}$ in M is said to be point convergent or p-convergent to $x \in M$ if there exists $t > 0$ such that

$$\lim_{n\to\infty} F_{x_n x}(t) = 1.$$

We write $x_n \to_p x$ and call x as the p-limit of $\{x_n\}$.

The following lemma was proved in [14].

Lemma 11. *[14]*

In a Menger space (M, F, Δ) *with the condition* $F_{xy}(t) \neq 1$ *for all* $t > 0$ *whenever* $x \neq y$, *p-limit of a point convergent sequence is unique.*

It has been established in [14] that there exist sequences which are p-convergent but not convergent .

In [5] B.S. Choudhury and K. Das has proved the result noted in Theorem 7 for the case of minimum t-norm. An open problem that remains to be investigated is whether the result is valid if other t-norms are used and in that case what additional conditions are to be assumed. Here we prove that if the Menger space is a G-complete Menger space and the t-norm is an arbitrary continuous t-norm then a generalization of this result is possible. Further we have established the existence of a fixed point of a self mapping which satisfies a given inequality in a Menger space under the condition of the existence of a specially constructed p-convergent subsequence of a given sequence.

3 Main Results

Theorem 12. *Let* (M, F, Δ) *be a G-complete Menger space and* $f : M \to M$ *be a selfmapping satisfying the following inequality*

$$\frac{1}{F_{fxfy}(\phi(ct))} - 1 \leq \psi(\frac{1}{F_{xy}(\phi(t))} - 1) \tag{3.1}$$

where $x, y \in M$, $0 < c < 1$, ϕ *is a* Φ*-function satisfying Definition 6 and* $\psi : [0, \infty) \to [0, \infty)$ *is such that* ψ *is continuous,* $\psi(0) = 0$ *and* $\psi^n(a_n) \to 0$, *whenever* $a_n \to 0$ *as* $n \to \infty$ *and* $t > 0$ *is such that* $F_{xy}(\phi(t)) > 0$. *Then* f *has a unique fixed point.*

Proof. Let $x_0 \in M$ and the sequence $\{x_n\}$ is constructed by $x_{n+1} = fx_n$ for all $n \in \mathbb{N} \cup \{0\}$.
We assume that $x_{n+1} \neq x_n$ for all $n \in \mathbb{N}$, otherwise f has a fixed point.
By virtue of the properties of ϕ we can find $t > 0$ such that $F_{x_0\,x_1}(\phi(t)) > 0$.
Then by an application of (3.1) we have

$$\frac{1}{F_{x_1\,x_2}(\phi(ct))} - 1 = \frac{1}{F_{fx_0\,fx_1}(\phi(ct))} - 1 \leq \psi\left(\frac{1}{F_{x_0\,x_1}(\phi(t))} - 1\right). \tag{3.2}$$

Again $F_{x_0\,x_1}(\phi(t)) > 0$ implies $F_{x_0\,x_1}(\phi(\frac{t}{c})) > 0$.
Then again by an application of (3.1) we have

$$\frac{1}{F_{x_1\,x_2}(\phi(t))}-1=\frac{1}{F_{fx_0\,fx_1}(\phi(t))}-1\leq\psi\left(\frac{1}{F_{x_0\,x_1}(\phi(\frac{t}{c}))}-1\right). \tag{3.3}$$

Repeating the above procedure successively n times we obtain

$$\frac{1}{F_{x_n\,x_{n+1}}(\phi(t))}-1\leq\psi^n\left(\frac{1}{F_{x_0\,x_1}(\phi(\frac{t}{c^n}))}-1\right). \tag{3.4}$$

Again (3.2) implies that $F_{x_1\,x_2}(\phi(ct))>0$.
Then following the above procedure we have

$$\frac{1}{F_{x_n\,x_{n+1}}(\phi(ct))}-1\leq\psi^{n-1}\left(\frac{1}{F_{x_1\,x_2}(\phi(\frac{ct}{c^{n-1}}))}-1\right). \tag{3.5}$$

Repeating the above step r times, in general we have for $n>r$,

$$\frac{1}{F_{x_n\,x_{n+1}}(\phi(c^rt))}-1\leq\psi^{n-r}\left(\frac{1}{F_{x_r\,x_{r+1}}(\phi(\frac{(c^rt)}{c^{n-r}}))}-1\right). \tag{3.6}$$

Since $\psi^n(a_n)\to 0$ whenever $a_n\to 0$, we have from (3.6), for all $r>0$

$$F_{x_n\,x_{n+1}}(\phi(c^rt))\to 1 asn\to\infty. \tag{3.7}$$

Let $\epsilon>0$ be given, then by virtue of the properties of ϕ we can find $r>0$ such that $\phi(c^rt)<\epsilon$. It then follows from (3.7) that

$$F_{x_nx_{n+1}}(\epsilon)\to 1 \text{ as } n\to\infty. \tag{3.8}$$

Again

$$F_{x_nx_{n+p}}(\epsilon)\geq\underbrace{\Delta(F_{x_nx_{n+1}}(\frac{\epsilon}{p}),\Delta((F_{x_{n+1}x_{n+2}}(\frac{\epsilon}{p}),\ldots\ldots,(F_{x_{n+p-1}x_{n+p}}(\frac{\epsilon}{p})))\ldots.)}_{p-times}.$$

Making $n\to\infty$ and using (3.8) we have for any integer p,
$F_{x_n,x_{n+p}}(\epsilon)\to 1$ as $n\to\infty$.
Hence $\{x_n\}$ is a G-Cauchy sequence.
As (M,F,Δ) is G-complete, $\{x_n\}$ is convergent and hence $x_n\to z$ as $n\to\infty$ for some $z\in M$.
Again

$$F_{fz,z}(\epsilon)\geq\Delta(F_{fz,x_{n+1}}(\frac{\epsilon}{2}),F_{x_{n+1},z}(\frac{\epsilon}{2})). \tag{3.9}$$

Using the properties of Φ-function, we can find a $t_2>0$, such that $\phi(t_2)<\frac{\epsilon}{2}$.
Again $x_n\to z$ as $n\to\infty$. Hence there exists $N\in\mathbb{N}$ such that for all $n>N$,

$$F_{x_nz}(\phi(t_2))>0.$$

Then we have for $n > N$,

$$\begin{aligned}\frac{1}{F_{fzx_{n+1}}(\frac{\epsilon}{2})} - 1 &\leq \frac{1}{F_{fzfx_n}(\phi(t_2))} - 1 \\ &\leq \psi(\frac{1}{F_{zx_n}(\phi(\frac{t_2}{c}))} - 1).\end{aligned}$$

Making $n \to \infty$, utilizing $\psi(0) = 0$ and continuity of ψ, we obtain

$$F_{fzx_{n+1}}(\frac{\epsilon}{2}) \to 1 \quad \text{as} \quad n \to \infty. \tag{3.10}$$

Making $n \to \infty$ in (3.9), using (3.10), by continuity of Δ and the fact that $x_n \to z$ as $n \to \infty$ we have,

$$F_{fzz}(\epsilon) = 1 \quad \text{for every} \quad \epsilon > 0.$$

Hence $z = fz$.

Next we establish the uniqueness of the fixed point. Let x and y be two fixed points of f.
By the properties of ϕ there exists $s > 0$ such that $F_{xy}(\phi(s)) > 0$.
Then by an application of (3.1) we have

$$\frac{1}{F_{xy}(\phi(cs))} - 1 = \frac{1}{F_{fxfy}(\phi(cs))} - 1 \leq \psi\left(\frac{1}{F_{xy}(\phi(s))} - 1\right). \tag{3.11}$$

Again $F_{xy}(\phi(s)) > 0$ implies $F_{xy}(\phi(\frac{s}{c})) > 0$.

Then replacing s by $\frac{s}{c}$ in (3.11) we obtain

$$\frac{1}{F_{xy}(\phi(s))} - 1 \leq \psi\left(\frac{1}{F_{xy}(\phi(\frac{s}{c}))} - 1\right).$$

Repeating the above procedure n times we have

$$\frac{1}{F_{xy}(\phi(s))} - 1 \leq \psi^n\left(\frac{1}{F_{xy}(\phi(\frac{s}{c^n}))} - 1\right) \to 0 \text{ as } n \to \infty \text{ (by the properties of } \psi).$$

This shows that $F_{xy}(\phi(s)) = 1$ for all $s > 0$.
Again from (3.11) it follows that $F_{xy}(\phi(cs)) > 0$.
Repeating the same argument with s replaced by cs we have $F_{xy}(\phi(cs)) = 1$ and in general we have,

$F_{xy}(\phi(c^n s)) = 1$ for all $n \in \mathbb{N} \cup \{0\}$.

By the properties of ϕ for any given $\epsilon > 0$ there exists $r \in \mathbb{N} \cup \{0\}$ such that $\phi(c^r s) < \epsilon$, so that from the above we have

$F_{xy}(\epsilon) = 1$ for all $\epsilon > 0$, that is $x = y$.

This establishes the uniqueness of the fixed point.

□

Theorem 13. *Let (M, F, Δ) be a Menger space with the condition $F_{xy}(t) \neq 1$ for all $t > 0$ whenever $x \neq y$, and $f : M \to M$ be a self mapping which satisfies the inequality (3.1) in the statement of Theorem 12. If for some $x_0 \in M$, the sequence $\{x_n\}$ given by $x_{n+1} = fx_n$, $n \in \mathbb{N} \cup \{0\}$ has a p-convergent subsequence then f has a unique fixed point.*

Proof. Let $\{x_{n_k}\}$ be a subsequence of $\{x_n\}$ which is p-convergent to $x \in X$. Consequently there exists $s > 0$ such that

$$\lim_{k\to\infty} F_{x_{n_k} x}(s) = 1. \tag{3.12}$$

Further, following (3.8) we have
$\lim_{i\to\infty} F_{x_{n_i} x_{n_i+1}}(s) = 1.$
Therefore given $\delta > 0$ there exist $k_1, k_2 \in \mathbb{N} \cup \{0\}$ such that for all $k' > k_1$ and $k'' > k_2$ we have,

$$\begin{aligned} F_{x_{n_{k'}} x}(s) &> 1 - \delta \\ \text{and } F_{x_{n_{k''}} x_{n_{k''}+1}}(s) &> 1 - \delta. \end{aligned}$$

Taking $k_0 = \max\{k', k''\}$, we obtain that for all $j > k_0$,

$$F_{x_{n_j} x}(s) > 1 - \delta \tag{3.13}$$

and

$$F_{x_{n_j} x_{n_j+1}}(s) > 1 - \delta. \tag{3.14}$$

So we obtain

$$\begin{aligned} F_{x_{n_j+1} x}(2s) &\geq \Delta(F_{x_{n_j+1} x_{n_j}}(s), F_{x_{n_j} x}(s)) \\ &\geq \Delta(1 - \delta, 1 - \delta) \qquad [\text{ by (3.13) and (3.14)}]. \end{aligned}$$

Let $\epsilon > 0$ be arbitrary. As $\Delta(1, 1) = 1$ and Δ is a continuous t-norm, we can find $\delta > 0$ such that

$$\Delta(1 - \delta, 1 - \delta) > 1 - \epsilon.$$

It follows from (3.13) and (3.14) that for given $\epsilon > 0$ it is possible to find a positive integer k_0 such that for all $j > k_0$,
$$F_{x_{n_j+1} x}(2s) > 1 - \epsilon.$$
Hence

$\lim_{j\to\infty} F_{x_{n_j+1}x}(2s) = 1$,
that is

$$x_{n_j+1} \longrightarrow_p x. \tag{3.15}$$

Again, following the properties of ϕ-function we can find $t > 0$ such that

$$\phi(t) \leq 2s < \phi(\frac{t}{c}).$$

Also from (3.15) it is possible to find a positive integer N_1 such that for all $i > N_1$

$$F_{x_{n_i+1}x}(2s) > 0.$$

Consequently for all $i > N_1$,

$$\begin{aligned}\frac{1}{F_{x_{n_i+1}fx}(2s)} - 1 &\leq \frac{1}{F_{fxfx_{n_i}}(\phi(t))} - 1 \\ &\leq \psi\left(\frac{1}{F_{xx_{n_i}}(\phi(\frac{t}{c}))} - 1\right) \leq \psi\left(\frac{1}{F_{xx_{n_i}}(2s)} - 1\right).\end{aligned}$$

Making $i \to \infty$ in the above inequality, and using (3.12) and the continuity of ψ we obtain $F_{x_{n_i+1}fx}(2s) \to 1$ as $i \to \infty$,
that is,

$$x_{n_i+1} \to_p fxasi \to \infty. \tag{3.16}$$

Using (3.15), (3.16) and Lemma 11 we have

$$fx = x$$

which proves the existence of the fixed point.
The uniqueness of the fixed point follows as in the proof of Theorem 12.

□

Remark 14. *Theorem 13 has close resemblance with a result of Mihet (Theorem 2.3 of [14]).*

Corollary 15. *Let (M, F, Δ) be a G-complete Menger space and let $f : M \to M$ satisfy*

$$F_{fxfy}(\phi(ct)) \geq F_{xy}(\phi(t)) \tag{3.17}$$

where $t > 0$, $0 < c < 1$, $x, y \in M$, Δ is any continuous t-norm, and ϕ is a Φ-function. Then f has a unique fixed point.

If we take $\psi(t) = t$ for all $t > 0$ then (3.1) implies (3.17). Then by an application of the Theorem 12 the corollary follows.

Example 16. *Let* (M, F, Δ) *be a complete Menger space where* $M = \{x_1, x_2, x_3, x_4\}$ *,* $\Delta(a, b) = min\{a, b\}$ *and* $F_{xy}(t)$ *be defined as*

$$F_{x_1x_2}(t) = F_{x_2x_1}(t) = \begin{cases} 0, & if \quad t \leq 0, \\ 0.9, & if \quad 0 < t \leq 3, \\ 1, & if \quad t > 3, \end{cases}$$

$$F_{x_1x_3}(t) = F_{x_3x_1}(t) = F_{x_1x_4}(t) = F_{x_4x_1}(t) = F_{x_2x_3}(t)$$

$$= F_{x_3x_2}(t) = sF_{x_2x_4}(t) = F_{x_4x_2}(t) = F_{x_3x_4}(t) = F_{x_4x_3}(t) = \begin{cases} 0, & if \quad t \leq 0, \\ 0.7, & if \quad 0 < t < 6, \\ 1, & if \quad t \geq 6. \end{cases}$$

$f : M \to M$ *is given by* $fx_1 = fx_2 = x_2$ *and* $fx_3 = fx_4 = x_1$ *. If we take* $\phi(t) = t^2$*,* $\psi(t) = 2t^3$ *and* $c = 0.8$*, then it may be seen that* f *satisfies the inequality* (3.1) *and* x_2 *is the unique fixed point of* f*.*

References

[1] B.S. Choudhury and P.N. Dutta, *A unified fixed point result in metric spaces involving a two variable function*, FILOMAT, **14**(2000), 43-48. MR1953993. Zbl 1012.54047.

[2] B.S. Choudhury, *A unique common fixed point theorem for a sequence of self-mappings in Menger spaces*, Bull. Korean Math. Soc., **37**(2000), No.3, 569-573. MR1779246 Zbl 0959.54026.

[3] B.S. Choudhury, *A common unique fixed point result in metric spaces involving generalised altering distances*, Math. Communications, **10**(2005), 105-110. MR2199098. Zbl 1089.54514.

[4] B.S. Choudhury and P.N. Dutta, *Common fixed points for fuzzy mappings using generalised altering distances*, Soochow J. Math., **31**(2005), 71-81. MR2130497. Zbl 1068.54043.

[5] B.S. Choudhury and K. Das, *A new contraction principle in Menger spaces*, Acta Mathematica Sinica, English Series, **24**(8) (2008), 1379-1386. MR2438308. Zbl pre05497467.

[6] B.S. Choudhury, P.N. Dutta and K. Das , *A fixed points result in Menger space using a real function*, Acta. Math. Hungar., 122 (2009), 203-216.

[7] P.N. Dutta and B.S. Choudhury *A generalization of contraction mapping principle*, Fixed Point Theory and Applications, Volume **2008**, Article ID 406368, doi: 10.1155/2008/406368.

[8] A. George and P. Veeramani, *On some results in fuzzy metric spaces*, Fuzzy Sets and Systems, 64(1994), 395-399. MR1289545. Zbl 0843.54014.

[9] A. George and P. Veeramani, *On some results of analysis for fuzzy metric spaces*, Fuzzy Sets and Systems, **90**(1997), 365-368. MR1477836. Zbl 0917.54010.

[10] O. Hadzic and E. Pap, *Fixed Point Theory in Probabilistic Metric Spaces*, Kluwer Academic Publishers, 2001. MR1896451. Zbl 0994.47077.

[11] O. Hadzic and E. Pap, *A fixed point theorem for multivalued mappings in probabilistic metric spaces and an application in fuzzy metric spaces*, Fuzzy Sets and Systems, **127**(2002), 333-344. MR1899066. Zbl 1002.54025.

[12] M.S. Khan, M. Swaleh and S. Sessa , *Fixed point theorems by altering distances between the points*, Bull. Austral. Math. Soc., **30**(1984), 1-9. MR0753555. Zbl 0553.54023.

[13] D. Mihet, *Multivalued generalisations of probabilistic contractions*, J. Math. Anal. Appl., **304**(2005), 464-472. MR2126543 Zbl 1072.47066 .

[14] D. Mihet, *On fuzzy contractive mappings in fuzzy metric spaces*, Fuzzy Sets and Systems, **158**(2007), 915-921. MR2302646. Zbl 1117.54008.

[15] S.V.R. Naidu, *Some fixed point theorems in metric spaces by altering distances*, Czechoslovak Mathematical Journal, **53**(2003), 205-212. MR1962009. Zbl 1013.54011.

[16] K.P.R. Sastry and G.V.R. Babu, *Some fixed point theorems by altering distances between the points*, Ind. J. Pure. Appl. Math., **30**(6),(1999), 641-647. MR1701042. Zbl 0938.47044.

[17] K.P.R. Sastry, S.V.R. Naidu, G.V.R. Babu and G.A. Naidu, *Generalisation of common fixed point theorems for weakly commuting maps by altering distances*, Tamkang Journal of Mathematics, **31**(3),(2000), 243-250. MR1778222. Zbl 0995.47035 .

[18] B. Schweizer and A. Sklar, *Probabilistic Metric Spaces*, Elsevier, North-Holland, 1983. MR0790314. Zbl 0546.60010.

[19] V.M. Sehgal and A.T. Bharucha-Reid, *Fixed point of contraction mappings on PM space*, Math. Sys. Theory, **6**(2)(1972), 97-100. MR0310858. Zbl 0244.60004.

[20] B. Singh and S. Jain, *A fixed point theorem in Menger space through weak compatibility*, J. Math. Anal. Appl., **301**(2005), 439-448. MR2105684. Zbl 1068.54044.

** ******************************

[21] R. Vasuki and P. Veeramani, *Fixed point theorems and Cauchy sequences in fuzzy metric spaces*, Fuzzy Sets and Systems, **135**(2003), 415-417. MR1979610 Zbl 1029.54012.

[22] Tatjana Zikic-Dosenovic, *A multivalued generalization of Hick's C-contraction*, Fuzzy Sets and Systems, **151**(2005), 549-562. MR2126173 Zbl 1069.54025.

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