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Fluorescence-enhanced optical tomography in small volume: Telegrapher and Diffusion models

Ranadhyr Roy

Abstract. Small animal fluorescence-enhanced optical tomography has possibility for restructuring drug discovery and preclinical investigation of drug candidates. However, accurate modeling of photon propagation in small animals is critical to quantitatively obtain accurate tomographic images. The diffusion approximation is commonly used for biomedical optical diagnostic techniques in turbid large media where absorption is low compared to scattering system. Unfortunately, this approximation has significant limitations to accurately predict radiative transport in turbid small media and also in a media where absorption is high compared to scattering systems. A radiative transport equation (RTE) is best suited for photon propagation in human tissues. However, such models are quite expensive computationally. To alleviate the problems of the high computational cost of RTE and inadequacies of the diffusion equation in a small volume, we use telegrapher equation (TE) in the frequency domain for fluorescence-enhanced optical tomography problems. The telegrapher equation can accurately and efficiently predict ballistic as well as diffusion-limited transport regimes which could simultaneously exist in small animals. The accuracy of telegrapher-based model is tested by comparing with the diffusion-based model using stimulated data in a small volume. This work demonstrates the use of the telegrapher-based model in small animal optical tomography problems.

Full text

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Ranadhyr Roy Department of Mathematics,
The University of Texas-Pan American,
1201 West University, Edinburg, Tx, 78541, USA.
e-mail: rroy@utpa.edu