

**L^1 -SOLUTIONS OF BOUNDARY VALUE
PROBLEMS FOR IMPLICIT FRACTIONAL ORDER
DIFFERENTIAL EQUATIONS**

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Abstract. The aim of this paper is to present new results on the existence of solutions for a class of boundary value problem for fractional order implicit differential equations involving the Caputo fractional derivative. Our results are based on Schauder's fixed point theorem and the Banach contraction principle fixed point theorem.

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References

- [1] R. P. Agarwal, M. Belmekki and M. Benchohra, A survey on semilinear differential equations and inclusions involving Riemann-Liouville fractional derivative. *Adv Differ. Equat.* **2009**(2009) Article ID 981728, 1-47. [MR2505633](#)(2010f:34113). [Zbl 1182.34103](#).
- [2] R.P Agarwal, M. Benchohra and S. Hamani, A survey on existence result for boundary value problems of nonlinear fractional differential equations and inclusions, *Acta. Appl. Math.* **109** (3) (2010), 973-1033. [MR2596185](#)(2011a:34008).
- [3] S. Abbas, M. Benchohra and G.M. N'Guérékata, *Topics in Fractional Differential Equations*, Springer, New York, 2012. [MR2962045](#).
- [4] S. Abbas, M. Benchohra and G.M. N'Guérékata, *Advanced Fractional Differential and Integral Equations*, Nova Science Publishers, New York, 2015. [MR3309582](#). [Zbl 1314.34002](#).
- [5] D. Baleanu, K. Diethelm, E. Scalas, J.J. Trujillo, *Fractional Calculus Models and Numerical Methods*, World Scientific Publishing, New York, 2012.

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- [6] M. Benchohra, J. Henderson, S.K. Ntouyas and A. Ouahab, Existence results for functional differential equations of fractional order, *J. Math. Anal. Appl.* **338** (2008), 1340-1350. [MR2386501](#).
- [7] M. Benchohra, S. Hamani, and S.K. Ntouyas, Boundary value problems for differential equations with fractional order and nonlocal conditions, *Nonlinear Anal.* **71** (2009), 2391-2396. [MR2532767](#).
- [8] M. Benchohra, S. Hamani and S. K. Ntouyas, Boundary value problems for differential equations with fractional order, *Surveys Math. Appl.* **3** (2008), 1-12. [MR2390179](#).
- [9] L. Byszewski, Theorems about existence and uniqueness of solutions of a semilinear evolution nonlocal Cauchy problem, *J. Math. Anal. Appl.* **162** (1991), 494-505.
- [10] L. Byszewski, Existence and uniqueness of mild and classical solutions of semilinear functional-differential evolution nonlocal Cauchy problem. Selected problems of mathematics, 25-33, **50** th Anniv. Cracow Univ. Technol. Anniv. Issue, 6, Cracow Univ. Technol., Krakw, 1995
- [11] L. Byszewski and V. Lakshmikantham, Theorem about the existence and uniqueness of a solution of a nonlocal abstract Cauchy problem in a Banach space, *Appl. Anal.* **40** (1991), 11-19. [MR1121321](#)(92h:34121). [Zbl 0694.34001](#).
- [12] K. Deimling, *Nonlinear Functional Analysis*, Springer-Verlag, 1985.
- [13] A. M. A. El-Sayed, Sh. A. Abd El-Salam, L^p -solution of weighted Cauchy-type problem of a diffre-integral functional equation, *Intern. J. Nonlinear Sci.* **5** (2008) 281-288. [MR2410798](#)(2009d:34007).
- [14] A.M.M. El-Sayed, H.H.G. Hashem, Integrable and continuous solutions of a nonlinear quadratic integral equation, *Electron. J. Qual. Theory Differ. Equ.* 2008, No. 25, 10 pp. [MR2443206](#)(2009e:45014).
- [15] R. Hilfer, *Applications of Fractional Calculus in Physics*, World Scientific, Singapore, 2000. [MR1890104](#)(2002j:00009).
- [16] A.A. Kilbas, Hari M. Srivastava, and Juan J. Trujillo, *Theory and Applications of Fractional Differential Equations*. North-Holland Mathematics Studies, 204. Elsevier Science B.V., Amsterdam, 2006. [MR2218073](#)(2007a:34002).
- [17] V. Lakshmikantham, S. Leela and J. Vasundhara, *Theory of Fractional Dynamic Systems*, Cambridge Academic Publishers, Cambridge, 2009.
- [18] F. Mainardi, *Fractional Calculus and Waves in Linear Viscoelasticity. An introduction to mathematical models*. Imperial College Press, London, 2010.

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<http://www.utgjiu.ro/math/sma>

- [19] M. D. Ortigueira, *Fractional Calculus for Scientists and Engineers*. Lecture Notes in Electrical Engineering, 84. Springer, Dordrecht, 2011. [MR2768178\(2012b:26003\)](#). [Zbl 1251.26005](#).
- [20] I. Podlubny, *Fractional Differential Equations*, Academic Press, San Diego, 1999.
- [21] V. E. Tarasov, *Fractional Dynamics: Application of Fractional Calculus to Dynamics of Particles, Fields and Media*, Springer, Heidelberg; Higher Education Press, Beijing, 2010.

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