Surveys in Mathematics and its Applications

ISSN 1842-6298 (electronic), 1843-7265 (print)
Volume 12 (2017), 51 – 63

LOCAL CONVERGENCE OF SOME HIGH ORDER ITERATIVE METHODS BASED ON THE DECOMPOSITION TECHNIQUE USING ONLY THE FIRST DERIVATIVE

Ioannis K. Argyros and Santhosh George

Abstract. We present a local convergence analysis of some high order iterative methods based on the decomposition technique using only the first derivative for solving equations in order to approximate a solution of a nonlinear equation. In earlier studies hypotheses on the higher derivatives are used. Thus by using only first derivative, we extended the applicability of these methods. Moreover the radius of convergence and computable error bounds on the distances involved are also given in this study. Numerical examples are also presented in this study.

Full text

References

[1] S. Amat, M.A. Hernández and N. Romero, *A modified Chebyshev's iterative method with at least sixth order of convergence*, Appl. Math. Comput. **206**(1) (2008), 164-174. MR2474962. Zbl 1157.65369.

[2] S. Amat, S. Busquier and S. Plaza, *Dynamics of the King's and Jarratt iterations*, Aequationes. Math. **69** (2005), 212-213. MR2139283. Zbl 1068.30019.

[3] I.K. Argyros, *Convergence and Application of Newton-type Iterations*, Springer, 2008. MR2428779.

[4] I. K. Argyros and Said Hilout, *Computational methods in nonlinear Analysis*, World Scientific Publ. Co. , New Jersey, USA, 2013. MR3134688. Zbl 1279.65062.

[5] I. K. Argyros D.Chen and Q. Quian, *The Jarratt method in Banach space setting*, J.Comput.Appl.Math. **51** (1994), 103-106. MR1286420. Zbl 0809.65054.

2010 Mathematics Subject Classification: 65D10; 65D99
Keywords: Ninth order method; efficiency index; local convergence; nonlinear equation.

[6] V. Candela and A. Marquina, *Recurrence relations for rational cubic methods I: The Halley method*, Computing **44** (1990), 169-184. MR1053497.

[7] J.Chen, *Some new iterative methods with three-order convergence*, Appl. Math. Comput. *181* (2006), 1519-1522. MR2273048. Zbl 1105.65055.

[8] B. Neta, C.Chun and M. Scott, *Basins of attraction for optimal eighth order methods to find simple roots of nonlinear equations*, Appl. Math. Comput. **227** (2014), 567-592. MR3146342.

[9] A. Cordero and J. Torregrosa, *Variants of Newton's method using fifth order quadrature formulas*, Appl.Math.Comput. **190** (2007), 686-698. MR23387472. Zbl 1122.65350.

[10] A. Corderó, J. Maimo, J. Torregrosa, M.P. Vassileva and P. Vindel, *Chaos in King's iterative family*, Appl. Math. Lett. **26** (2013), 842-848. MR3066701. Zbl 06420913.

[11] A. Cordero, A. Magreñán, C. Quemada and J.R. Torregrosa, *Stability study of eight-order iterative methods for solving nonlinear equations*, J. Comput. Appl. Math., **291** (2016), 348–357. MR3383840.

[12] J. A. Ezquerro and M.A. Hernández, *A uniparametric Halley-type iteration with free second derivative*, Int. J.Pure and Appl. Math. **6**(1) (2003), 99-110. MR1975422. Zbl 1026.47056.

[13] J. A. Ezquerro and M.A. Hernández, *New iterations of R-order four with reduced computational cost*. BIT Numer. Math. **49** (2009), 325- 342. MR2507604 . Zbl 1170.65038.

[14] M. Frontini and E. Sormani, *Some variants of Newton's method with third order convergence*, Appl. Math. Comput. **140** (2003), 419-426. MR1953913.

[15] J.M. Gutiérrez and M.A. Hernández, *Recurrence relations for the super-Halley method*, Computers Math. Applic. **36**(7) (1998), 1-8. MR1647688. Zbl 0933.65063.

[16] F.A. He, *A new iteration method for solving algebraic equations*, Appl. math. Comput. *135* (2003), 81–84. MR1934317. Zbl 1023.65039.

[17] M.A. Hernández and M.A. Salanova, *Sufficient conditions for semilocal convergence of a fourth order multipoint iterative method for solving equations in Banach spaces*. Southwest J. Pure Appl. Math **1** (1999), 29-40. MR1717585. Zbl 0940.65064

[18] R.F. King, *A family of fourth-order methods for nonlinear equations*, SIAM. J. Numer. Anal. **10** (1973), 876-879. MR0343585. Zbl 0266.65040.

[19] A. K. Maheshwari, *A fourth order iterative method for solving nonlinear equations*, Appl. Math. Comput. **211** (2009), 283-391. MR2524167. Zbl 1162.65346.

[20] M.A. Noor, W. A. Khan and K. I. Noor and E. Al-Said, *Higher-order iterative methods free from second derivative for solving nonlinear equations*, Internat. J. Phys. Sciences, **6** (8) (2011), 1887–1893.

[21] M.A. Noor and K. I. Noor, *Three-step iterative methods for nonlinear equations*, Appl. math. Comput. **183**(1) (2006), 322–327. MR2282814. Zbl 1113.65050.

[22] M.A. Noor and K. I. Noor, *Some iterative schemes for nonlinear equations*, Appl. Math. Comput. **183**(2) (2006), 774–779. MR2290831. Zbl 1113.65051.

[23] S. K. Parhi and D.K. Gupta, *Semilocal convergence of a Stirling-like method in Banach spaces*, Int. J. Comput. Methods **7**(02) (2010), 215-228. MR2654116. Zbl 1267.65057.

[24] M.S. Petkovič, B. Neta, L. Petkovič and J. Džunič, *Multipoint methods for solving nonlinear equations*, Elsevier, 2013. MR3293985. Zbl 1286.65060.

[25] F.A. Potra and V. Pták, *Nondiscrete induction and iterative processes*, Research Notes in Mathematics, Vol. **103**, Pitman Publ., Boston, MA, 1984. MR0754338. Zbl 0549.41001.

[26] L. B. Rall, *Computational solution of nonlinear operator equations*, Robert E. Krieger Publishing Company, New York, 1979. MR0240944. Zbl 0476.65033.

[27] H. Ren, Q. Wu and W. Bi, *New variants of Jarratt method with sixth-order convergence*, Numer. Algorithms **52**(4) (2009), 585-603. MR2563716. Zbl 1187.65052

[28] W.C. Rheinboldt, *An adaptive continuation process for solving systems of nonlinear equations*, In: Mathematical models and numerical methods (A.N.Tikhonov et al. eds.), Banach Center Pub. **3** (1978), 129-142. MR0514377. Zbl 0378.65029.

[29] J.R. Sharma, *Improved Chebyshev-Halley-methods with sixth and eighth order of convergence*, Appl. Math. Comput. **256**(1) (2015), 119–124. MR3316053. Zbl 1338.65134

[30] F. A. Shah and M.A. Noor, *Some efficient iterative methods for nonlinear equations based on the decomposition method*, Appl. Math. Comput. **57**(1) (2009), 101–106. MR2484261.

****

[31] F. A. Shap and M.A. Noor, *Some numerical methods for solving nonlinear equations by using decomposition techniques*, Appl. Math. Comput. **251** (2015), 1378–1386. MR3294725. Zbl 1328.65125.

[32] J.F. Traub, *Iterative methods for the solution of equations*, Prentice Hall Englewood Cliffs, New Jersey, USA, 1964. MR0169356. Zbl 0121.11204.

[33] S. Weerakoon and T.G.I. Fernando, *A variant of Newton's method with accelerated third-order convergence*, Appl. Math. Lett. **13** (2000), 87-93. MR1791767. Zbl 0973.65037.

[34] X. Wang and J. Kou, *Convergence for modified Halley-like methods with less computation of inversion*, J. Diff. Eq. and Appl. **19**(9) (2013), 1483-1500. MR3173499. Zbl 1278.65081.

Ioannis K. Argyros
Department of Mathematical Sciences,
Cameron University,
Lawton, OK 73505, USA.
e-mail: iargyros@cameron.edu

Santhosh George
Department of Mathematical and Computational Sciences,
NIT Karnataka,
India-575 025.
e-mail:sgeorge@nitk.ac.in

License