ON DYNAMICS OF QUADRATIC STOCHASTIC OPERATORS: A SURVEY

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Abstract. We discuss the notion of Volterra, ℓ-Volterra and separable quadratic stochastic operators defined on $(m-1)$-dimensional simplex, where $\ell \in \{0, 1, ..., m\}$. The ℓ-Volterra operator is a Volterra operator if and only if $\ell = m$. We study the structure of the set of all Volterra and ℓ-Volterra operators and describe their several fixed and periodic points. For $m = 2$ and $m = 3$ we describe behavior of trajectories of $(m-1)$-Volterra operators. We also mention many remarks with comparisons of ℓ-Volterra operators and Volterra ones. Also we discuss the dynamics of separable quadratic stochastic operators.

Full text

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